HW set 2

Problem 1

Consider a nucleus of charge Ze. A single electron in the ground state has wave function $\varphi(r) = Ce^{-Zr/a_0}$ with a_0 the Bohr radius. Assume that for two electrons the wavefunction is: $\psi(r_1, r_2) = \bar{\varphi}(r_1)\bar{\varphi}(r_2)$ with $\bar{\varphi}(r) = De^{-\bar{Z}r/a_0}$

(a) Find the value of $\bar{Z}$ that minimizes the total energy and find the resulting energy.

(b) Find the most probable radius for an electron when the atom has 1 and 2 electrons.

(c) Find the overlap matrix element $<\varphi(r)|\bar{\varphi}(r)> = \int d^3r\ \varphi(r)\bar{\varphi}(r)$

and make a plot it versus Z.

Problem 2

(a) Derive the conductivity sum rule

$$\int_0^\infty d\omega\sigma_1(\omega) = C\frac{ne^2}{m} \qquad (1)$$

using the Drude form for $\sigma_1(\omega)$. Find the value of C, which is independent of τ.

(b) To derive Eq. (1) classically in general (without using the Drude form for $\sigma_1(\omega)$):

(i) Assume an impulsive electric field $E(t) = \delta(t)$ is applied to a system of particles of mass m, charge e and number density n. Fourier-analyze the current density and electric field as

$$J(t) = \int_{-\infty}^{\infty} d\omega J(\omega)e^{-i\omega t} \qquad E(t) = \delta(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} d\omega e^{-i\omega t}$$

and using $J(\omega) = \sigma(\omega)E(\omega)$ show that

$$J(t>0) = \frac{2}{\pi}\int_0^\infty d\omega\sigma_1(\omega)\cos(\omega t) \qquad (2)$$

(ii) Find $J(t=0^+)$ by direct calculation of the change in momentum of the particles under the electric field $E(t) = \delta(t)$.

(iii) Using the result of (ii) and Eq. (2), derive Eq. (1).

Problem 3

AM, 2.1

Problem 4

AM, 2.3