On the ‘Quasilinear Theory’ of the Vlasov Plasma:

Status of Dynamical Friction and Subcritical Growth Theory

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Outlook

- QLT is <u>the</u> classic problem of nonlinear plasma theory, ~ 55 yrs old
- 'QLT' is a catch-all for many, often loosely related, ideas
- Quasilinear approaches constitute <u>the</u> working tool for calculating mean field evolution in turbulence
- As yet, several aspect of QLT remain unresolved.

Outlook, cont'd

- Here:
  - Perspective is that of an applied physicist
  - Approach is historical, though:
    - Emphasizing recent developments
    - Necessarily broad brush
  - Seek identify current issues where/how interdisciplinary approaches contribute? How might this subject be revitalized?

Outline

I) Some (scientific) history

a) – basic ideas, content (Sagdeev et al '60's)

b) – challenges I

- Granulations, dynamical friction (Dupree et al, 70's)
- Mode coupling and/in growth (Laval et al, 80's)

c) – Rejoinders

- Traveling wave tube experiment (Tsunoda et al, 90's)

Outline, cont'd

- Rejoinders, cont'd
  - Momentum constraints on the B-O-T (Liang, P.D. 90's)

II) Recent Times (Enter high resolution simulations)

- Subcritical growth in the Berk-Breizman model (Lesur, P.D. 2013)
- Nonlinear CDIA growth (Lesur, P.D. et al, 2014)

* – Beyond 1D: Darmet model of drift wave turbulence (Kosuga, P.D., 2011-)

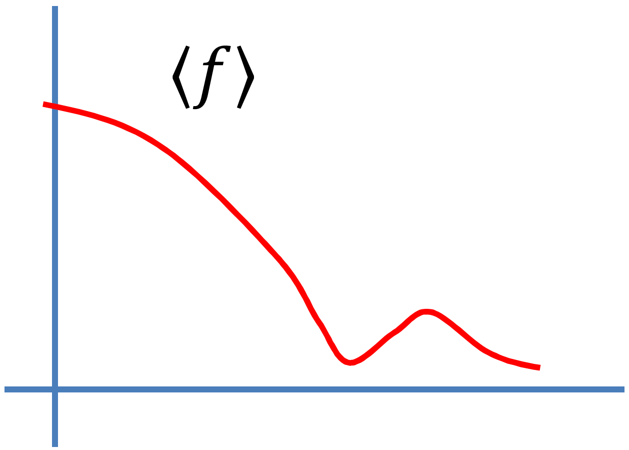
Outline, cont'd

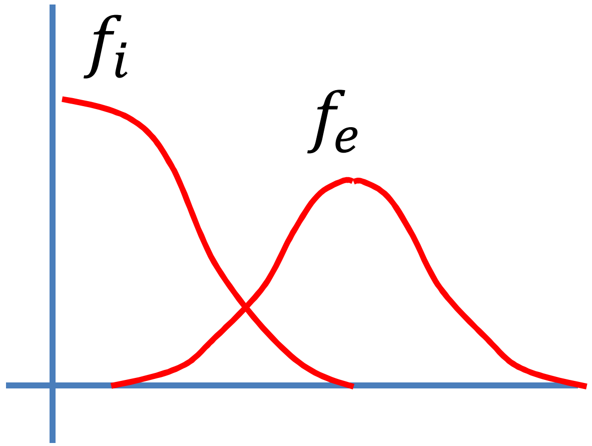
III) Thoughts for Discussion

- Where does all this stand?
- Where to next?

I) Some Scientific History

- Good beginnings: Vedenov, Velikov, Sagdeev; Drummond, Pines
  - 1D Vlasov evolution / relaxation of B-O-T, CDIA





Inputs

- Landau theory
- Stochasticity
- Radiative transfer theory

  - QL system, from mean field approach with linear response

$$\epsilon(k,\omega) = 0, \quad \partial_t \langle f \rangle = \frac{\partial}{\partial v} D \frac{\partial \langle f \rangle}{\partial v} \qquad \partial_t |E_k|^2 = 2\gamma_k |E_k|^2 \qquad D = D(|E|^2)$$

- Key:
  - $D = \frac{q^2}{m^2} \sum_k |E_k|^2 \frac{|\gamma_k|}{(\omega - kv)^2 + |\gamma_k|^2}$
  - Resonant → $\pi\delta(\omega - kv)$ → irreversible
  - Non-resonant → $|\gamma_k| / \omega_k^2$ → reversible / 'fake'
  - Non-resonant diffusion for stationary turbulence is problematic. Energetics?
  - Coarse graining implicit in $\langle \, \rangle$
  - First derivation via RPA, ultimately particle stochasticity is fundamental

- Central elements/orderings:
  - resonant diffusion, irreversibility:
    - "chaos" ←→ coarse graining
    - Island overlap at resonances: $\frac{\omega}{k_{i+i}} - \frac{\omega}{k_i} \leq \sqrt{q\phi/m}$
  - linear response?:
    - $\tau_{ac} < \tau_{tr}$, τ_{decorr} , γ_k
    - $\tau_{ac}^{-1} = \left|\frac{d\omega}{dk} - \frac{\omega}{k}\right| \, |\Delta k|$ → correlation time of wave-particle resonance
    - $\tau_{tr}^{-1} = k\sqrt{q\phi/m}$ → particle bounce time in pattern
    - $\tau_{decorr}^{-1} = (k^2 D)^{1/3}$ → particle decorrelation rate (cf. Dupree '66)

- QLT is Kubo # < 1 theory

i.e. $\frac{q}{m}\tilde{E}\tau_{ac}/\Delta v_T = \Delta v_T\, k\, \tau_{ac} < 1$

but often pushed to Ku ~ 1

- QLT assumes:
  - all fluctuations are eigenmodes (i.e. neglect mode coupling)?
  - all $\delta f \sim \tilde{E}\, \partial\langle f\rangle/\partial v$?

    (resemble $\delta B \sim \tilde{v}\langle B\rangle$ in MF dynamo theory)

- Energetics → 2 component description

  – Resonant Particles vs Waves

$$\partial_t(RPKED) + \partial_t(W\ ED) = 0$$

or

  – Particles vs Fields

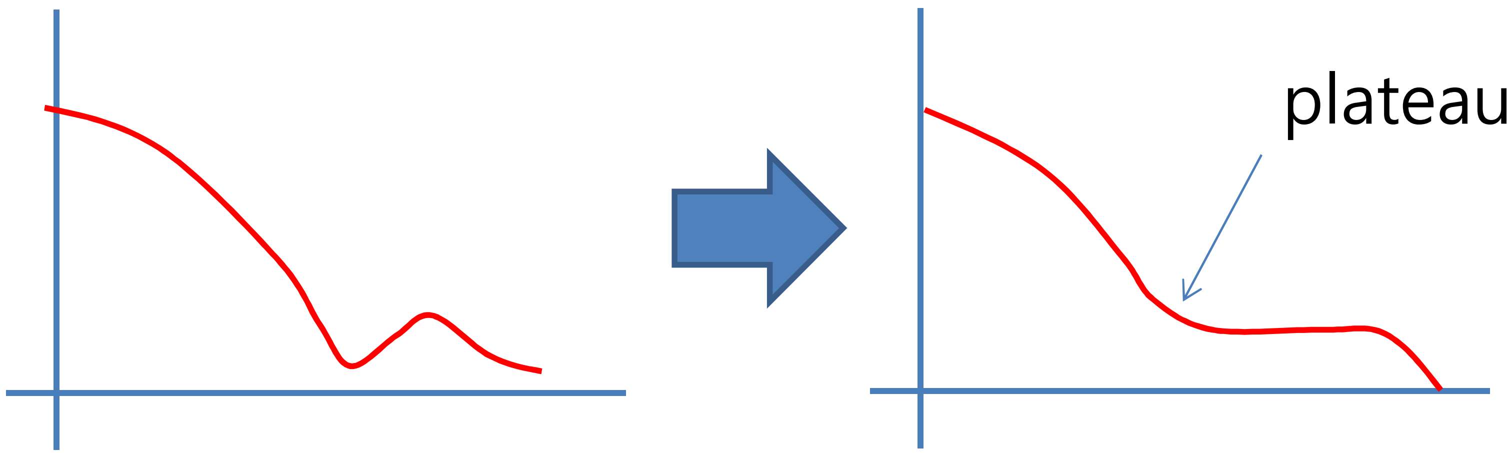
$$\partial_t(PKED) + \partial_t(FED) = 0$$

  – Species coupled via waves, only (CDIA)

  – Issues: how describe stationary state with RP drive?

i.e. $D_R\left(\frac{\partial\langle f\rangle}{\partial v}\right)^2 = d_{col}\ \left\langle\left(\frac{\partial \delta f}{\partial v}\right)^2\right\rangle$, ala' Zeldovich

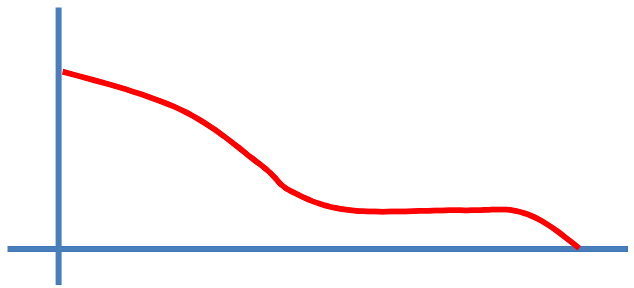
- Outcome:
  - B-O-T: Plateau formation



  - prediction for $|\tilde{E}_{sat}|^2 / 4\pi nT$ when plateau formed
  - CDIA:
    - wave driven momentum transfer e->i
    - anomalous resistivity model (quasi-marginality)

- Why Plateau?
  - In collisionless, un-driven system, need at stationarity: $\int dv\; D_R(\partial\langle f\rangle/\partial v)^2 = 0$
  - So either: (collisions: RHS → $d_{col}\ \langle \left(\frac{\partial \delta f}{\partial v}\right)^2 \rangle$)

    i) $\partial\langle f\rangle/\partial v = 0$, where $D(v) \neq 0$ on interval → plateau with finite amplitude waves



    ii) Or $D_R = 0$ → fluctuation decay everywhere, $\gamma_k < 0$

- If ii), can show from QL system:

- $\langle f(v,t)\rangle = \langle f(v,0)\rangle + \frac{\partial}{\partial v}\left(\frac{D_R(v,t)-D_R(v,0)}{\pi\omega_{pe}^2 v^2}\right)$

- If $D_R \rightarrow 0$ as t increases $\langle f(v,t)\rangle \approx \langle f(v,0)\rangle$

  ($D_R(0)$ feeble)

- But $D_R \rightarrow 0$ requires $\frac{\partial\langle f\rangle}{\partial v} < 0$, while $\frac{\partial\langle f(v,0)\rangle}{\partial v} > 0$ → contradiction!

So

- i) applies → plateau formes

- Experiment: Roberson-Gentle '71
  - Beam → magnetized plasma → B-O-T (1D) "Gentle" B-O-T
  - Punchline: QLT successful where it is predicted to apply
  - N.B.: No studies of mode-coupling, fluctuation spectra
- Major question:
  - why ~ linear growth, $\delta f \approx \delta f^{c}$ relevant in turbulent state?

II) Challenges 70's

- Mode coupling
- Resonance broadening

↓

- Phase space eddies
- Dynamical friction

→ Stochastic view

→ Dupree, Kadomtsev…

- BGK modes
- Phase space holes, water bag models

↓

- Phase space vorticities
- Drag, wake

→ Coherent view

→ Lynden-Bell, Berk, Roberts, Feix, Schamel

<u>Phase space granulations</u>

Fluctuation constituent in addition to waves → major impact on dynamics

- <u>Granulations</u>
  - Mode coupling mediated by resonant particles
  - Distorts distribution, so: (akin eddy, vortex)
  - $\delta f = f^c + \tilde{f}$ granulation
  - Calculate $\langle \tilde{f} \rangle^2$ via $\langle \delta f^2 \rangle$+extraction
  - Poisson equation → $\tilde{f}$ induces dynamical friction (i.e. drag), as for discreteness

Granulations alter relaxation

$$\partial_t \langle \delta f^2 \rangle + T_{1,2} \langle \delta f^2 \rangle = D \left(\frac{\partial \langle f \rangle}{\partial v} \right)^2 - F \left(\frac{\partial \langle f \rangle}{\partial v} \right)$$

Relative scattering, streaming

$$\frac{\partial \langle f \rangle}{\partial t} = \frac{\partial}{\partial v} \left[D \frac{\partial \langle f \rangle}{\partial v} - F \right]$$

- Implications → mode coupling enter growth dynamics
  - Dynamical friction enters relaxation, and mean ←→ fluctuation coupling
  - Interspecies drag solves stationarity problem

  And:

  - Introduces new routes to relaxation, subcritical growth via collisionless momentum transfer by structures
  - Prediction of subcritical CDIA instability (Dupree '82) → mostly vindicated
  - Supported by Berman simulations ('83)

- A (seemingly) concrete prediction:
  - Enhanced B-O-T growth (Laval, Pesme, …) '80's
  - $\gamma \rightarrow (\#)\gamma_L$; wave only

    $\# > 1$
  - Curiously, $F = 0$ in theory → retained mode coupling in $T_{1,2}$ but not in drive
  - Physics: enhanced phase correlations in Cerenkov emission of plasma waves
  - Attracted wide attention

C) Rejoinders

- TWT experiment (Tsunoda et al late 80's – 90's)
- 'Simulate' B-O-T via
  - Beam → resonant
  - Slow wave helix → non-resonant
- Can program variety of spectral perturbations, and control phase initialization
- Can measure:
  - net growth of perturbations
  - distribution function

- TWT Apparatus

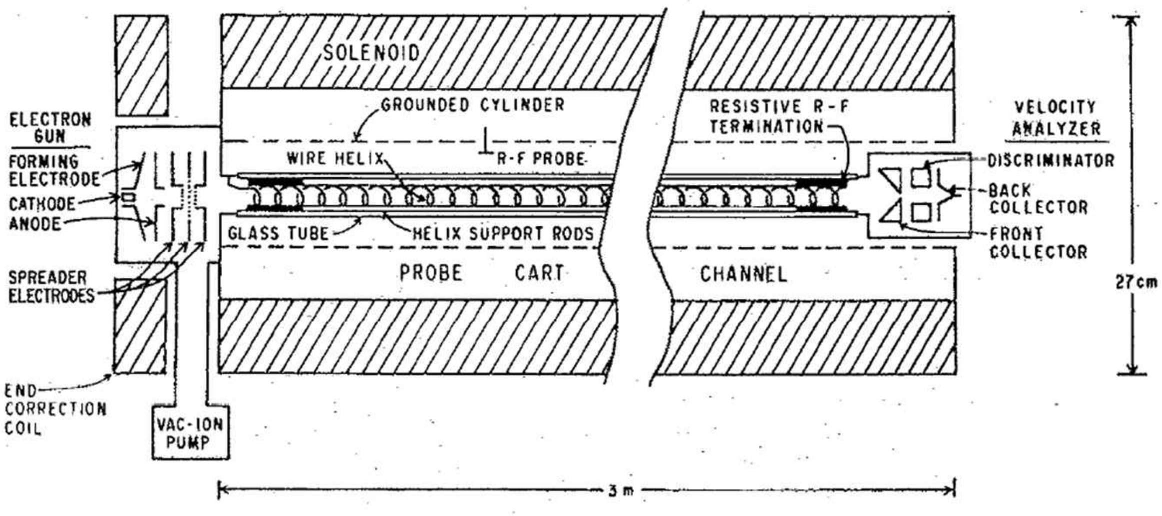
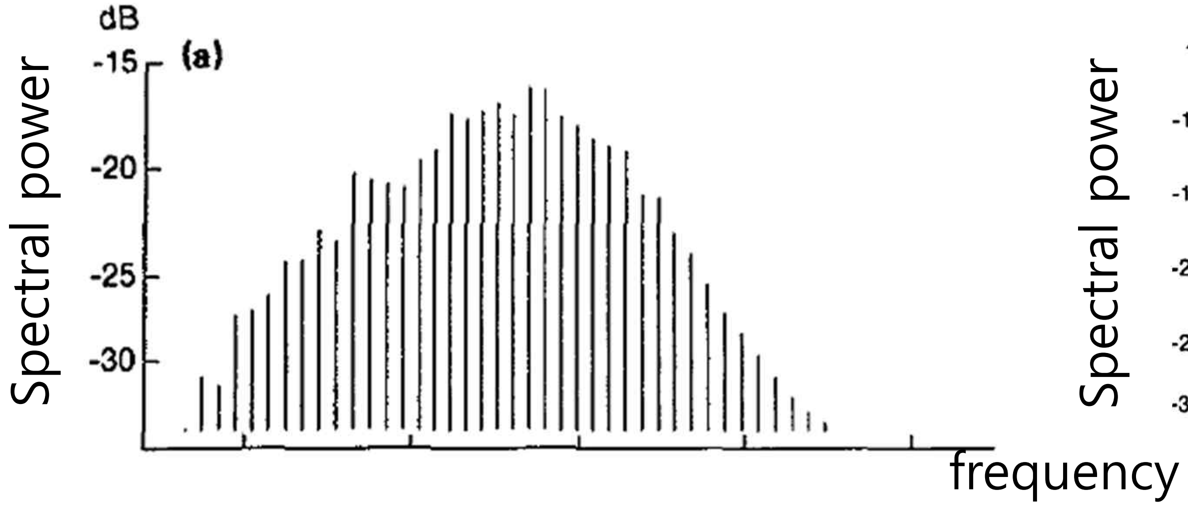
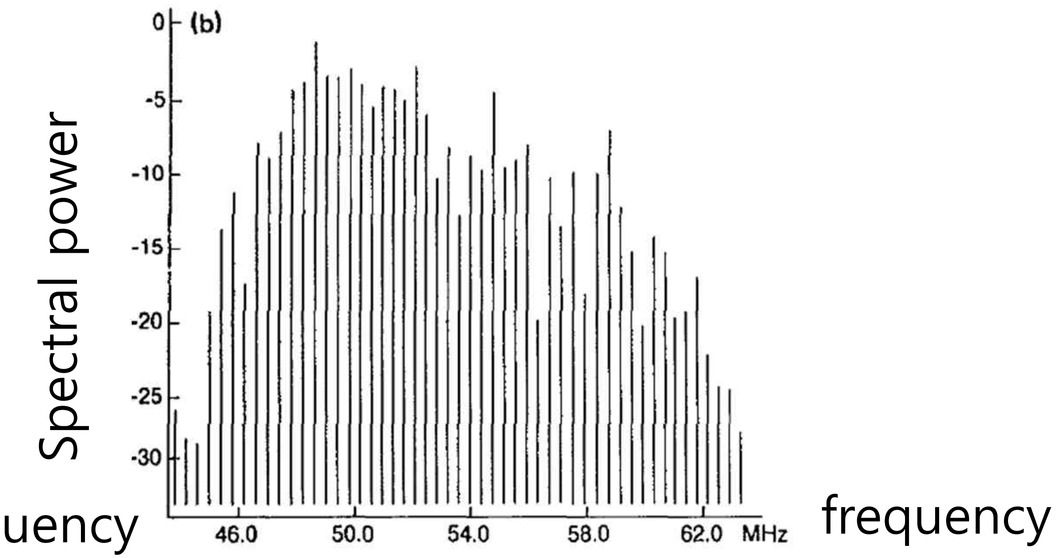


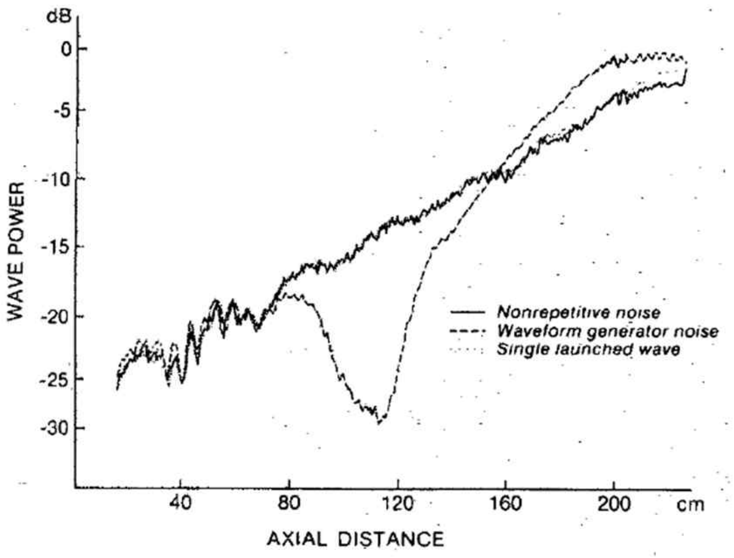
FIG. 1. Schematic diagram of the apparatus (not drawn to scale).

- Spectral evolution → evidence for mode coupling mediated by resonant particles





- The reckoning:



Dashed → one mode in smooth spectrum

Dotted → linear (single, weak mode)

Solid → non-rep noise

- "no deviation of frequency, ensemble averaged growth from Landau, to 10%"

- Message: mode coupling via resonant particles occurs, yet growth tracks linear Landau

- Comments
  - TWT results effectively vindicated QLT ala’ 60’s and demolished ALP
  - Much more might have been extracted by TWT
    - Studies of nonlinear transfer
    - Effect of adjustable dissipation in slow wave structure (see below)
    - Coordinated numerical simulation effort → ideal venue for validation of Vlasov codes
  - Time to re-visit TWT or variant?

- Comments, cont'd
  - Some thoughts on the outcome (Liang, P.D. '93)
  - Gist: momentum conservation

    Well known: Balescu-Lenard evolution of 1D stable plasma leaves $\partial_t \langle f \rangle = 0$

    i.e. Like particle, momentum and energy conserving collision leaves final state = initial state

    ∴ Granulations not effective in enhancing relaxation
  - Complication: here system not stationary → growing waves

- Analysis: key points

$$\left(\partial_t + T_{1,2}\right)\langle \delta f\,(1)\delta f\,(2)\rangle = S(v)$$

$$S(v) = -2\frac{q}{m}\,\langle \tilde{E}\delta f\,\rangle\,\partial\langle f\,\rangle/\partial v$$

- For $S(v)$:

$$\frac{q}{m}\langle \delta E(1)\delta f(1)\rangle = \sum_k{}'\left(-k^2\frac{q^2}{m^2}\langle \phi_k\phi_{-k}\rangle \pi\delta(\omega_k - kv)\frac{\partial f_0}{\partial v} - ik\frac{q}{m}\langle \phi_k\tilde{f}_{-k}\rangle\right)e^{2\gamma_k t}$$
$$= \sum_k{}'\left[-k^2\frac{q^2}{m^2}\pi\delta(\omega_k - kv)\frac{\partial f_0}{\partial v}\left(\frac{4\pi n_0 q}{k^2}\right)^2\int \frac{dv_1\,dv_2}{|\epsilon(k,\omega_k + i\gamma_k)|^2}\langle \tilde{f}_k(v_1)\tilde{f}_{-k}(v_2)\rangle\right.$$
$$\left. -k\frac{q}{m}\left(\frac{4\pi n_0 q}{k^2}\right)\frac{\operatorname{Im}\epsilon(k,\omega_k + i\gamma_k)}{|\epsilon(k,\omega_k + i\gamma_k)|^2}\int dv'\langle \tilde{f}_k(v')\tilde{f}_{-k}(v)\rangle\right]e^{2\gamma_k t}.$$

- Further:

$$\frac{q}{m}\langle \delta E(1)\delta f(1)\rangle = -\sum_k{}'k\frac{q}{m}\frac{\gamma_k\partial\epsilon'(k,\omega_k)/\partial\omega}{|\epsilon(k,kv + i\gamma_k)|^2}$$
$$\times\langle \tilde{\phi}_k\tilde{f}_{-k}(v)\rangle e^{2\gamma_k t}.$$

- N.B.: $S(v) \sim \gamma_k$ as electrons exchange momentum with waves, only here

- Results:
- For $S(v)$:

$$S(v)=2k^2\frac{q}{m}\frac{\gamma_k/\omega_k}{\epsilon''(k,\omega_k)+\gamma_k\partial\epsilon'(k,\omega_k)/\partial\omega}\frac{\partial f_0}{\partial v}$$
$$\times\langle\tilde{\phi}_k\tilde{f}_{-k}(v)\rangle e^{2\gamma_k t}$$
$$=\frac{2}{\pi}\frac{q}{m}\frac{k^4}{\omega_k\omega_p^2}\frac{\gamma_k^L\gamma_k}{\gamma_k-\gamma_k^L}\langle\tilde{\phi}_k\tilde{f}_{-k}(v)\rangle e^{2\gamma_k t},$$

- For γ_k:

$\sim \tau_{ac} < \tau_c < \gamma_k^{-1}$:

$$\gamma_k \approx \gamma_k^L\left(1-\frac{2A(k)}{\pi}\frac{\gamma_k^L}{\omega_k}\right)^{-1} \approx \gamma^L\left(1+O\left(\frac{\gamma^L}{\omega_k}\right)\right)$$

$\sim \tau_{ac} < \gamma_k^{-1} < \tau_c$:

$$\gamma_k \equiv \gamma^L\left(1+\frac{2A(k)}{\pi\beta}\frac{1}{\omega_k\tau_c}\right) \approx \gamma^L\left[1+O\left(\frac{1}{\tau_c\omega_k}\right)\right]$$

- Small <u>additive</u> correction to linear growth rate!

- Comments
  - Compare:
    - ALP: $\gamma \approx \# \, \gamma^{L}$
    - LD: $\gamma \approx \gamma^{L} \, (1 + \epsilon)$

    APL inconsistent with TWT results

    LD within error bars
  - QLT ‘61 (seemingly) vindicated for Gentle B-O-T, single species
  - LD explains how reconcile observation of mode coupling with QL growth

  But
  - Is the B-O-T representative? CDIA?

II) Recent Times (Lesur, Kosuga, P.D.)

- Subcritical growth in the B-B model (Lesur, P.D. 2013; P.D., Lesur, Kosuga Aix Fest 2009)
  - What is B-B (Berk-Breizman) model?
  - B-B ('99) based on reduced model of energetic particles (i.e. alphas) resonant with Alfven wave (TAE). Point is that resonant particle distribution evolves like 1D plasma, near resonance
  - Reduction is somewhat controversial, still
  - Analogy: beam, helix ←→ TWT

    EP's, bulk motion in AW ←→ tokamak

  Both are beam-driven instabilities

- For EP distribution

$$\frac{\partial f}{\partial t} + v\frac{\partial f}{\partial x} + \frac{qE}{m}\frac{\partial f}{\partial v} = -\gamma_a \delta f + \frac{\gamma_f^2}{k}\frac{\partial \delta f}{\partial v} + \frac{\gamma_d^3}{k^2}\frac{\partial^2 \delta f}{\partial v^2}$$

$$E = re\,(Z), \qquad f = f_0 + \delta f$$

$$\frac{dZ}{dt} = -\frac{m\omega_p^2}{4\pi nq}\int f e^{-i\,\varepsilon}\, dv - \gamma_d Z$$ ← key difference

- Note: collisions and 'extrinsic' γ_d

  * γ_d resembles dissipative helix response in TWT

  → momentum, energy exchange channel ?!

- Linearly $\gamma = \gamma_{kin} - \gamma_d$

- Useful to exploit analogy with QG fluid

  - So 'phasetrophy' $\psi_s = \int_{-\infty}^{\infty} dv \langle \delta f_s^2 \rangle$

  - Wave energy $W = nq^2 \langle E^2 \rangle / m\omega_p^2$

- So, for <u>single structure</u> (with single wave)

  – For ψ: $$\frac{d\Psi_s}{dt} = -2\frac{q_s}{m_s}\int_{-\infty}^{\infty}\frac{df_{0,s}}{dv}\langle E\,\delta f_s\rangle\,dv - \gamma_\Psi^{\mathrm{col}}\Psi_s$$

  – For W: $$\frac{dW}{dt} + 2\gamma_d W = -2\sum_s u_s q_s \int \langle E\,\delta f_s\rangle\,dv$$ $u_s = \omega_p/2k$

  – Akin to Charney-Drazin theorem: $$\frac{dW}{dt} + 2\gamma_d W = \sum_s \frac{m_s u_s}{d_v f_{0,s}}\left(\gamma_\Psi^{\mathrm{col}} + \frac{d}{dt}\right)\Psi_s$$

- Approximate solution (granulations + single wave):

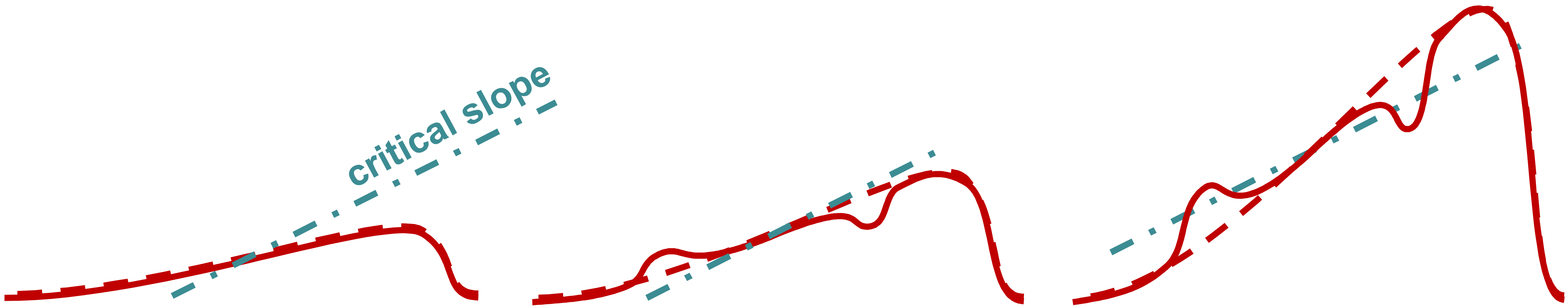
$$\boxed{\gamma_\psi \approx \frac{16}{3\sqrt{\pi}} \frac{\Delta v}{v_R} \frac{\gamma_{L,0}}{\omega_p} \gamma_d}$$

- Nonlinear, $\Delta v \sim (q\phi\,/m)^{1/2}$
- Exploits γ_d (dissipation)

  i.e. can have $\gamma_{L,0} - \gamma_d < 0$ but $\gamma_\psi > 0$
- $\gamma_{L,0} > 0$ ←→ free energy
- Previous study similar (P.D. et al; 2009 Festival de Theorie proceedings), but limited to near marginal

Subcritical instability

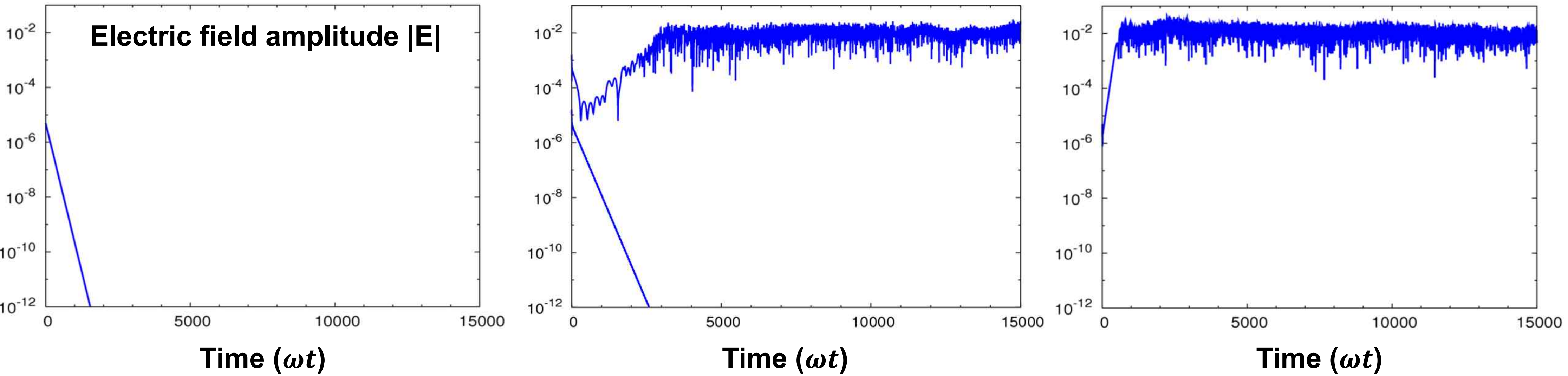
Linear growth rate $\gamma \approx \gamma_L - \gamma_d$ $\Rightarrow$ Critical slope $\gamma_L = \gamma_d$



Stable, $\gamma < 0$

Nonlinearly unstable, $\gamma < 0$ (Subcritical instability)

Unstable, $\gamma > 0$

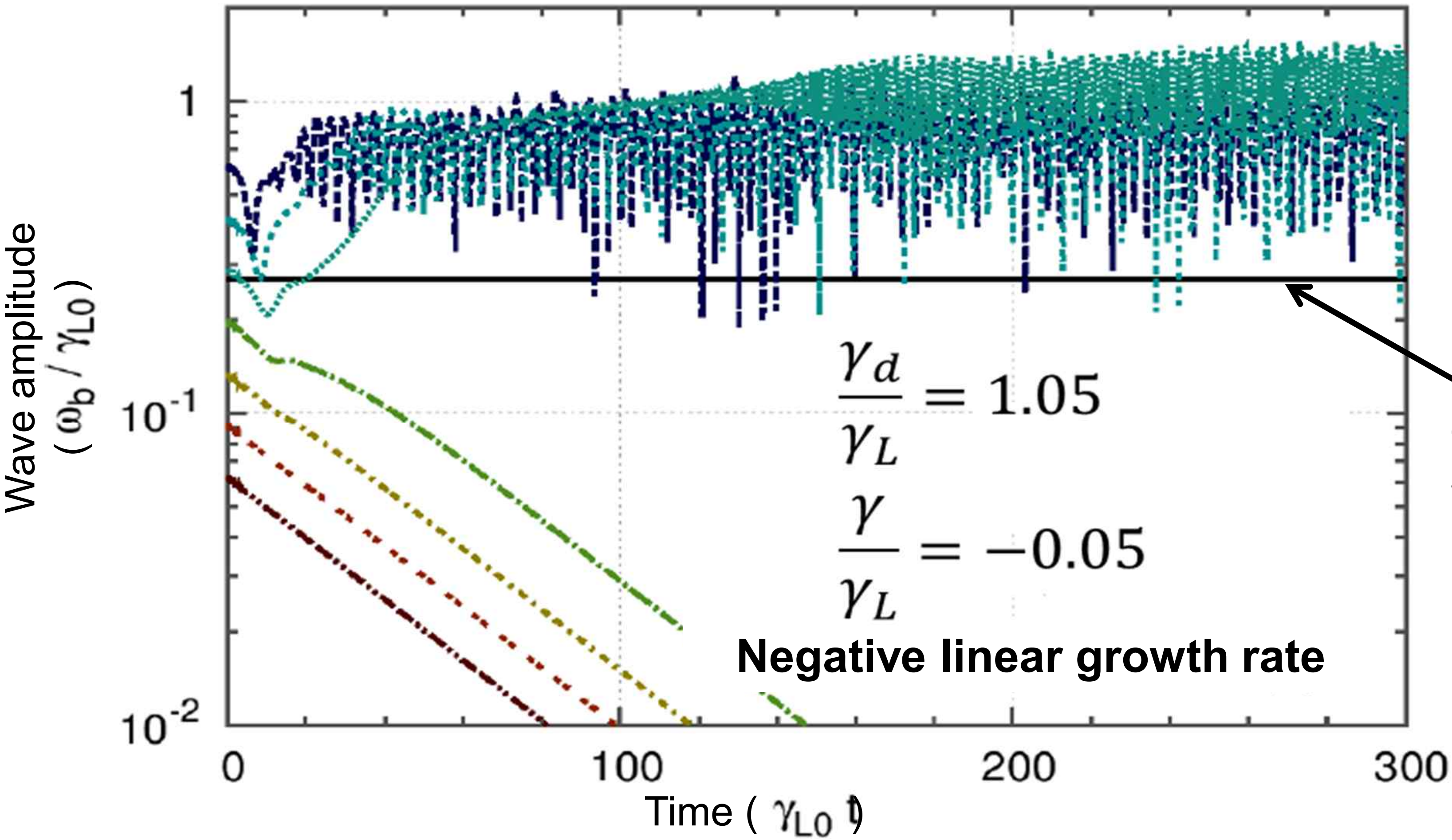


Nonlinear growth rate

$$\gamma_\Psi \approx \frac{16}{3\sqrt{\pi}} \frac{\Delta v}{v_R} \frac{\gamma_L}{\omega_p} \gamma_d$$

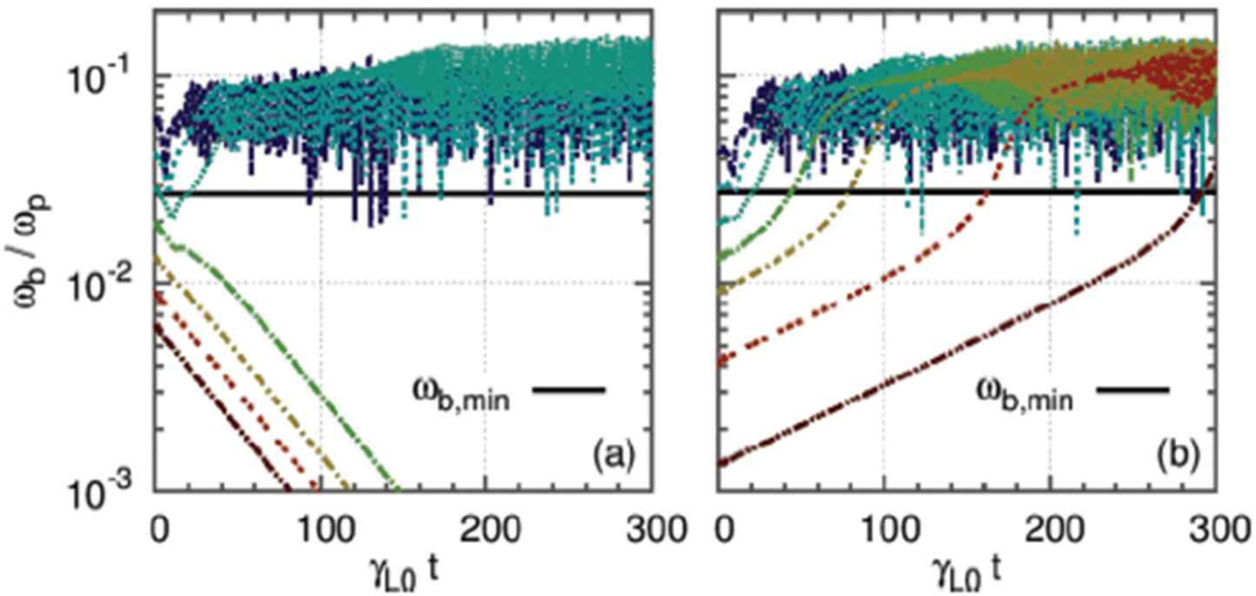
Lesur, Diamond, PRE 2013

$\Rightarrow$ **Nonlinear growth does not require that $\gamma_{L,c} > \gamma_d$**



Predicted by balancing the above growth rate with collisions.

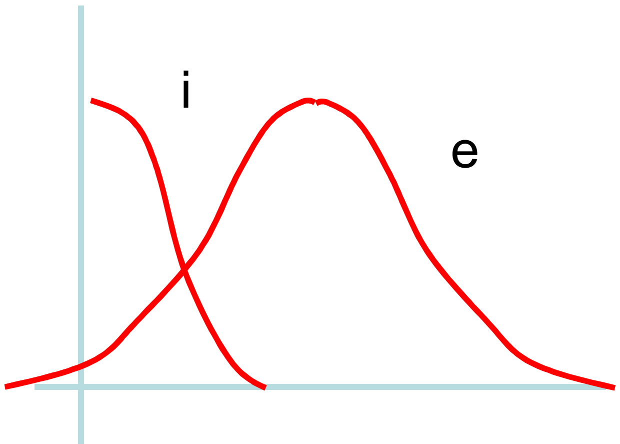
- Perhaps more convincing:



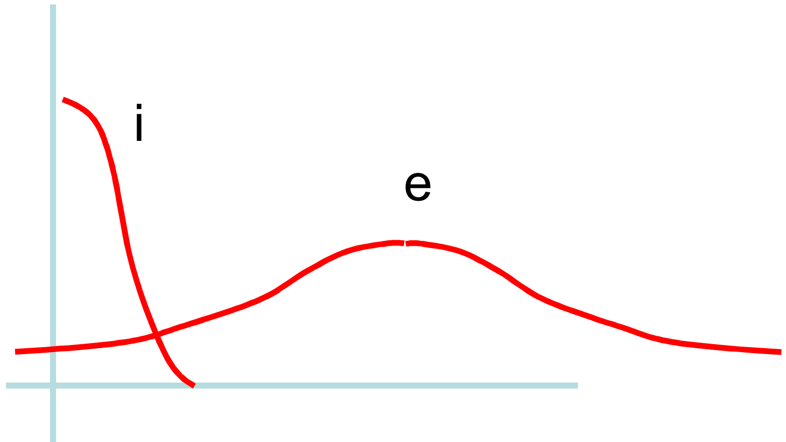
- Point is that even weak linear instability can be swamped by nonlinear growth → note for weak linear instability, saturation levels match those for nonlinear instability

- Establishes existence of robust exception to QLT61 ! Clearly related to γ_d dissipation channel. Limited to single structure.

- CDIA, revisited (Lesur, P.D., et al 2014)
  - Analysis and simulation of B-B model suggest re-visitation of CDIA studies (Dupree, Berman 1982, 1983)
  - Recovers subcritical/nonlinear growth for direct electron interaction?



instead



Substantial overlap,
Ion Landau damping
Kills linear growth

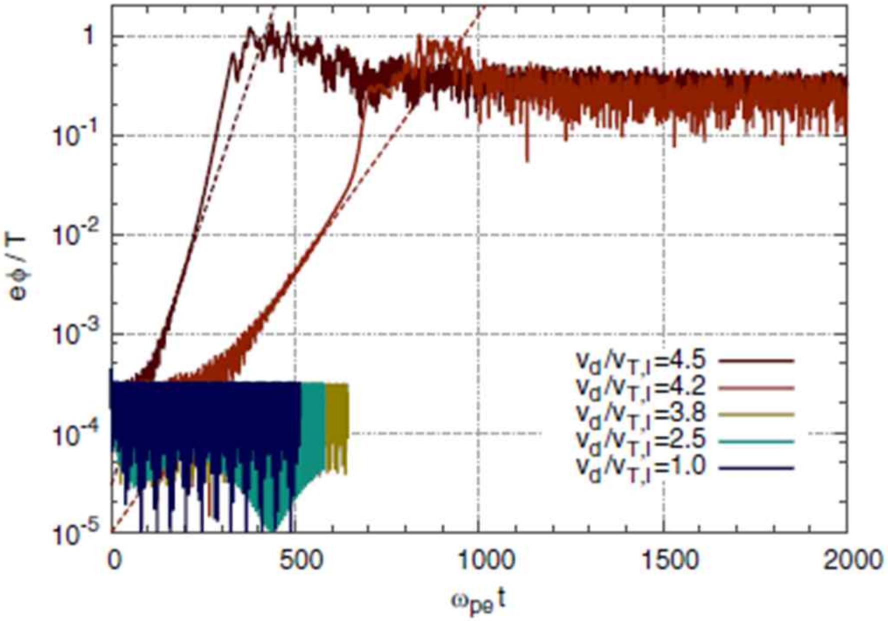
Textbook case,
Linear CDIA

➔ collisionless dynamical friction → NL growth ?

- Relevant to anomalous resistivity and reconnection problems
- Seek compare:
  - CDIA wave regime (weak turbulence)
  - Phase space structure turbulence growth

  i.e. to what extent is statistical theory relevant
- Primarily computational study

- Some key results



$$m_e/m_i = 1/4$$

- For ensemble of waves, no subcritical instability

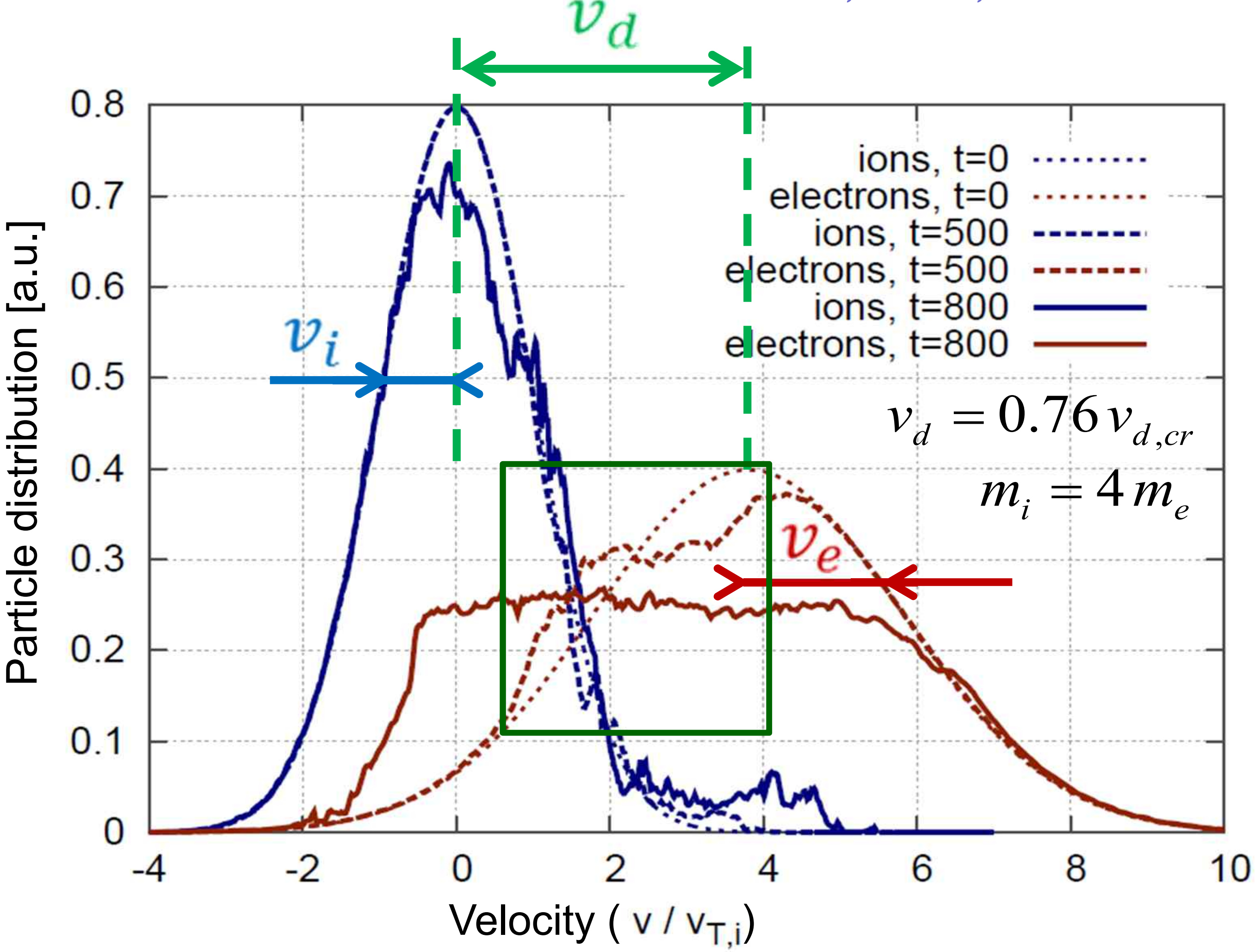
→

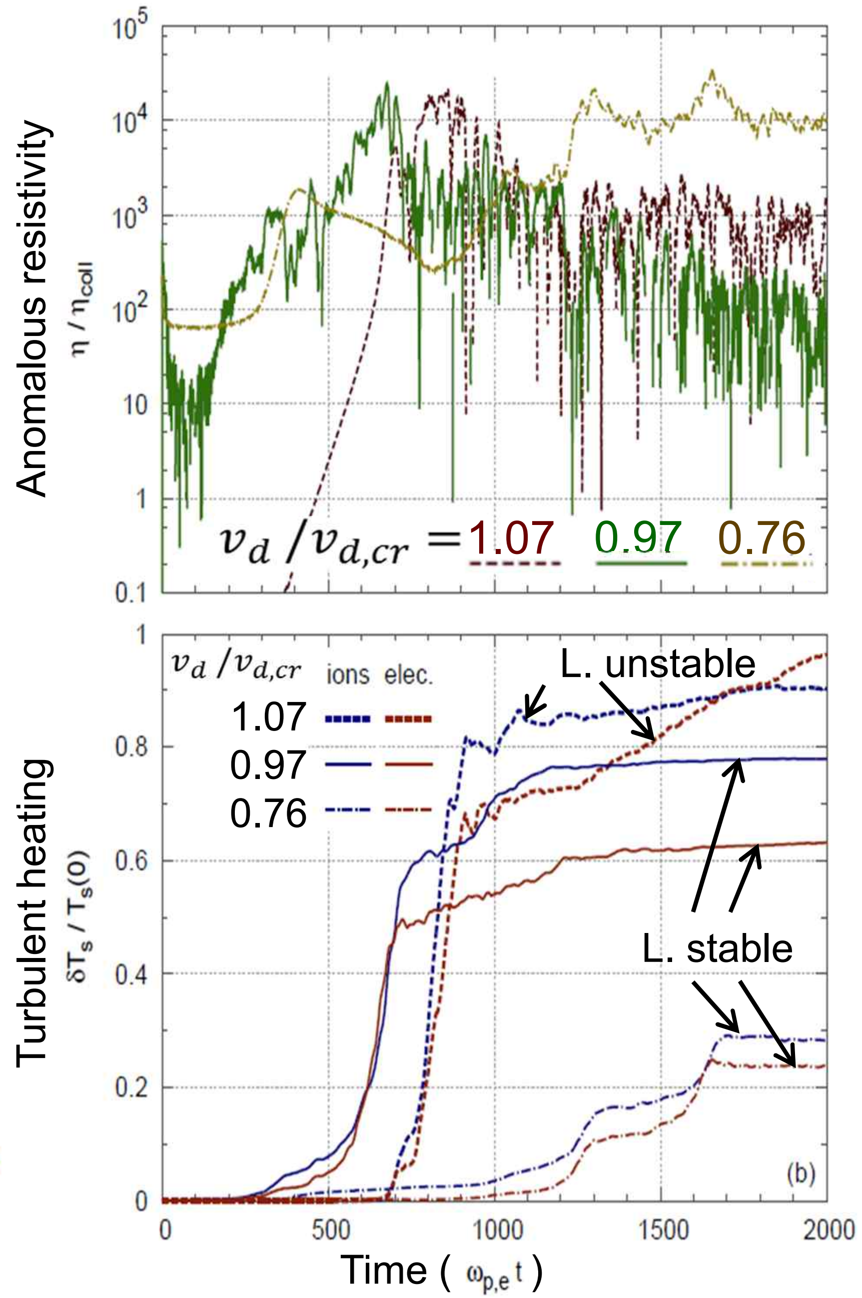
- Quasilinear theory prevails in realm where it is expected to!
- These computations performed with Vlasov code
- Berman et al performed with PIC. Repetition with PIC reveals numerical noise responsible for instability

Current-driven ion-acoustic

Phase-space structures can drive CDIA nonlinearly, eve n far from marginal stability ($v_d = 0.5 v_{d,cr}$) and for smal l initial amplitude ($e\varphi/T \sim 10^{-3}$).

Lesur, et al., PPCF'14





⇒ Linear stability is irrelevant in the steady-state

- Comments
  - Clear departure from QLT61 observed in B-B model
  - Nonlinear growth ~ $\gamma_d \gamma_{L,0} \Delta v$
  - Subcritical growth of phase space <u>structures</u> observed in CDIA studies at large mass ratio
  - Structures can be quite modest in amplitude. Structure → self bound $\Delta v \sim \tilde{f}\ \epsilon$

→ Appears to support Berman '83 simulations

- But:
  - For ensemble of waves, no subcritical growth ?!
  - Earlier cases of subcritical growth for waves linked to PIC noise.

- Where do we stand?

Old Haitian proverb:

"If you are not confused, you don't know what is going on."

- What is difference between "small structure" and a "nonlinear wave"?

  Rigorously, what is a "structure"?

- Is 'ensemble of waves' concept physically meaningful at finite amplitude?

  Should we care?

- Is QLT61 formally correct but limited to a regime of no practical relevance?

* - Is QLT61 incomplete in relevant regimes?! Momentum exchange channels?!

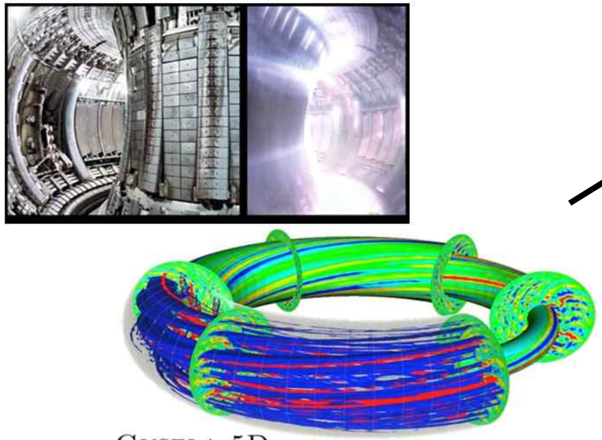
III) Beyond 1D: the 'Darmet Model' (after Pellat, Tagger)

- A reduced model (2D+energy) of kinetic drift wave turbulence driven by resonant particles
- Suggests $Ku \geq 1$ → phase space structures, vortices form
- Readily amenable to simulation

Cf. Y. Kosuga, P.D. 2011 → see also P.D. et al, '82

Impact on transport modeling

- Conventional transport modeling by quasilinear theory (QLT)



GYSELA 5D

Total Heat Flux ← Transport models

e.g. Quasilinear theory (QLT)

Recent comparison studies:

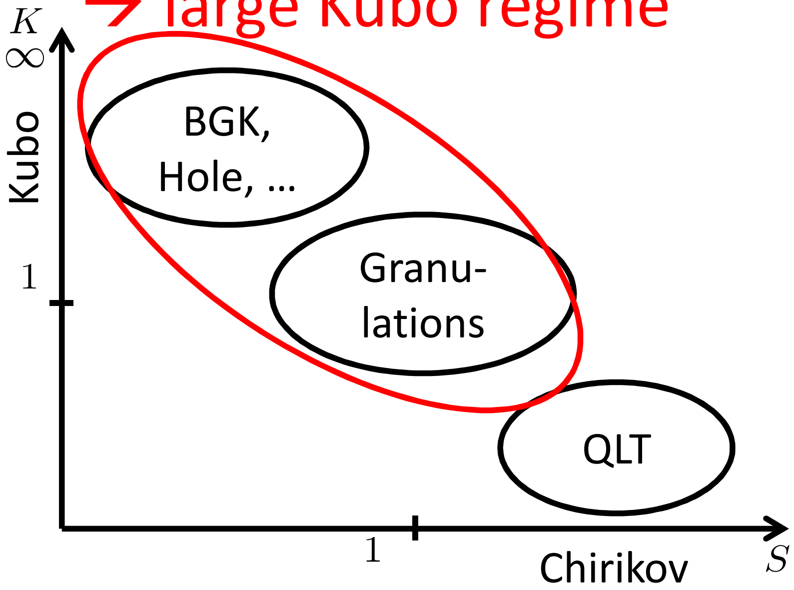
GYRO + TGLF '08

GENE + $\chi \sim \gamma/k_\perp^2$13

→ claim *'QLT transport models agrees with 1st principle simulations'* (?)

- However, applicability of QLT *dubious* for strongly resonant turb.

→ large Kubo regime



CTIM, CTEM, EPM, ...

→ 1D precession resonance, long τ_{ac}

→ e.g. for CTIM $K \equiv \tau_{ac}/\tau_{circ} \sim 10$

CTEM $K \sim 7$ Y. Xiao, '09

Transport by *strongly resonant turbulence*?

Model (formulation as flux driven)

Hydro analogue -
Thermal Rossby wave

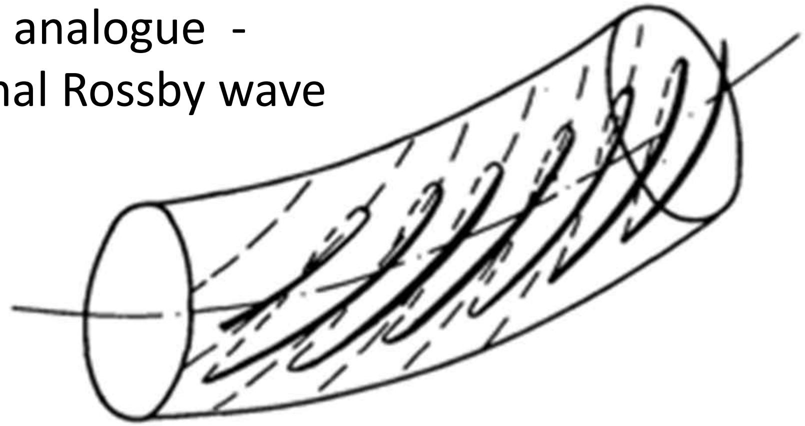


FIG. 3. Banana drift in axisymmetric system.

Trapped Ion ITG turbulence

Ions: $$\partial_t f_i + v_{Di}\bar{E}\partial_y f_i + \{\phi, f_i\} = 0$$

Electrons: $$n_e = (1 - i\delta)\phi$$

G.K. Poisson: $$n_e = \sqrt{2\epsilon_0}\frac{2}{\sqrt{\pi}}\int \sqrt{E}dEf_i + \rho^2\nabla_\perp^2\phi$$

Polarization

Triggered by $\omega_k = \omega_{Di}$
precession resonance

→ reduced models for tokamak turbulence

Vlasov $\xrightarrow[\omega_c > \omega]{}$ Gyrokinetic eq. $\xrightarrow[\omega_b > \omega]{}$ Bounce kinetic eq.

→ arguably the simplest model that captures N.L. ExB mixing + resonance via 1D precession

→ dissipative and hydro instability

→ zonal flow enters

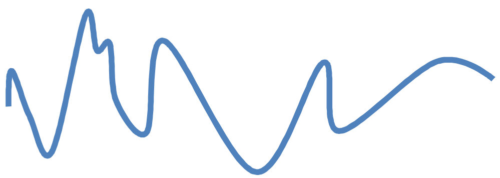
TIM can have high Kubo number

$$K \equiv \tau_{ac}/\tau_{circ}$$

(Due weak dispersion)

Packet dispersal rate

$$\tau_{ac}^{-1} \sim |\frac{d\omega}{dk_\theta} - \frac{\omega}{k_\theta}|\Delta k_\theta$$



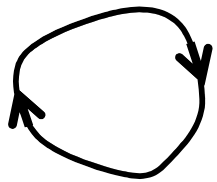
$$\tau_{ac}^{-1} \sim \frac{k_\theta^2 \rho^2 \sqrt{2\epsilon_0}\omega_*}{(1+k_\perp^2\rho^2)^2}\frac{\Delta k_\theta}{k_\theta}$$ for TIM

circulation (eddy turn-over) rate

$$\tau_{circ}^{-1} \sim k_0 v_{Di} \Delta E$$

R.B.T. → $\Delta E \sim 1/(k_0 v_{Di} \tau_c)$

$$\tau_{circ}^{-1} \sim \tau_{E\times B}^{-1}$$

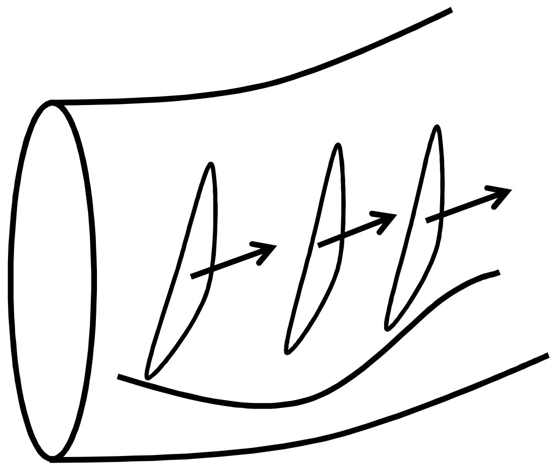


$K \sim 10$ for $\omega_k \tau_{circ} \sim \Delta k_\theta / k_\theta \sim O(1)$

$k\rho_i \sim 0.1$

Field pattern rather coherent and resonant particles produce ExB eddys

∴ trapped ion granulations can form and impact turbulence dynamics, transport!



- - Drift resonance relatively coherent $\rightarrow$ $K > 1$ easily satisfied (P.D. et. al. '82)

  $\leftrightarrow$ 1D structure

$$K = \frac{\tilde{v}}{|d\omega/dk_\theta - \omega/k_\theta||\Delta k_\theta|\Delta_r}$$

$\longrightarrow$ strongly resonant structure formation likely

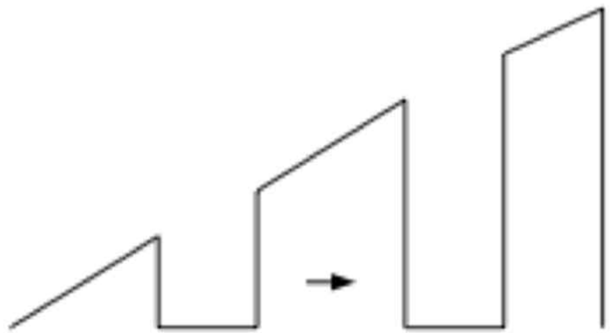
$\longrightarrow$ Dynamics

$$\frac{df}{dt} = \frac{d}{dt}(f_0 + \delta f) = 0$$

$$\partial_t \int dE\sqrt{E}\langle\delta f^2\rangle = -2\frac{d}{dt}\int dE\sqrt{E} f_0 \delta f \quad ; \quad f_0 = f_0(x_0) + (x - x_0)\left.\frac{\partial f_0}{\partial x}\right|_{x_0} + \ldots$$

$$\partial_t \int dE\sqrt{E}\langle\delta f_i^2\rangle = -2\langle\tilde{v}_r \delta n_i\rangle \left.\frac{\partial f_0}{\partial x}\right|_{x_0}$$

and QN $\longrightarrow$ $-\rho^2\nabla^2\tilde{\phi} = \delta n_i - \delta n_e$

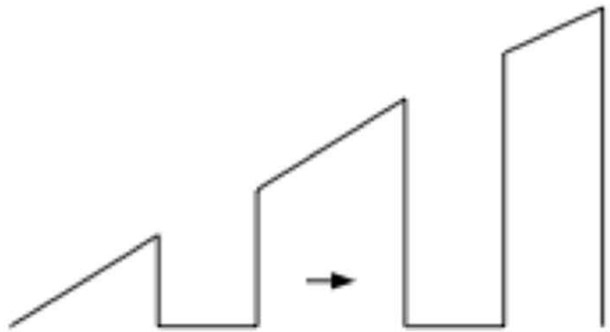


$\delta f_i\left(\frac{x - x_0}{\Delta x}, \frac{E - E_0}{\Delta E}\right)$

- → Physics: Ambipolarity / PV conservation

  - total dipole moment conserved, including polarization charge

$$\int dx \sum_{\alpha} q_\alpha n_\alpha(x) x = const$$



$\delta f_i(\frac{x-x_0}{\Delta x}, \frac{E-E_0}{\Delta E})$

  - Polarization Flux → Reynolds Force

$$\partial_t \left\{ \int dE\sqrt{E} \frac{\delta f_i^2}{2\langle f\rangle'|_{x_0}} + \langle V_\theta \rangle \right\} = -\nu \langle V_\theta \rangle - \langle \tilde{v}_r \delta n_e \rangle$$

- Non-acceleration Thm
- Electron flux critical

Observe:

- even localized phase space structure dynamics → ZF coupling appears
  No need for modulational instability, 4 wave interaction, ...

- For TIM regime, non-adiabatic electrons dissipative (i.e. collisional response)

$$\partial_t \left\{ \int dE\sqrt{E} \frac{\delta f_i^2}{2\langle f\rangle'|_{x_0}} + \langle V_\theta \rangle \right\} = -\nu \langle V_\theta \rangle + D_{DT} \frac{\partial \langle n \rangle}{\partial x}$$

- Observe:

  - structure + Z.F. evolution

$$\partial_t \left\{ \int dE\sqrt{E}\frac{\delta f_i^2}{2\langle f\rangle'|_{x_0}} + \langle V_\theta\rangle \right\} = -\nu\langle V_\theta\rangle + D_{DT}\frac{\partial\langle n\rangle}{\partial x}$$

  - Charney - Drazin Non-Acceleration Theorem for H-W model

$$\frac{\partial}{\partial t}\{\mathrm{WAD} + \langle V_\theta\rangle\} = -\nu\langle V_\theta\rangle - \langle \tilde{V}_r\tilde{n}\rangle - \frac{1}{\langle q\rangle'}\left\{\mu\langle(\nabla\tilde{q})^2\rangle + \partial_r\langle\tilde{V}_r\tilde{q}^2\rangle\right\}$$

$$\mathrm{WAD} = \frac{\langle\delta q^2\rangle}{\langle q\rangle'}$$

$$q = n - \nabla^2\phi$$

$\rightarrow$ $\sim$ Exact correspondence!

- $\int dE\sqrt{E}\delta f_i^2/2\langle f\rangle|_{x_0}$ $\rightarrow$ equivalent to zonal pseudomomentum
- momentum conservation of structure + ZF is fundamental $\longleftrightarrow$ follows from flux of polarization charge
- electron flux drives NET system momentum
- subcritical growth possible

Basic Structure of Theory

Dynamics: Evolution of two point phase space density correlation

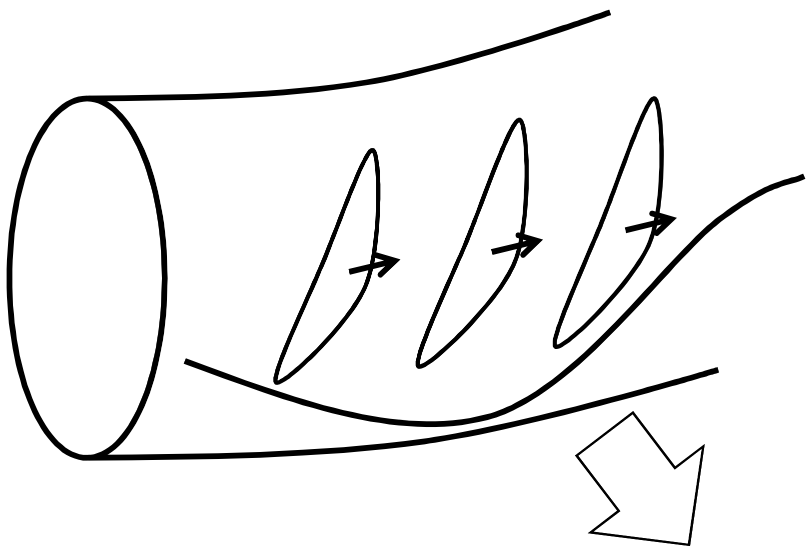
$$\partial_t \langle \delta f(1) \delta f(2) \rangle + T(1,2) = P(1,2)$$

production $\propto -\langle \tilde{v}_x \delta f \rangle \langle f \rangle'$

- related to free energy
- acts as source for turbulence

Triplet Term, life time of correlation via t
urbulent mixing

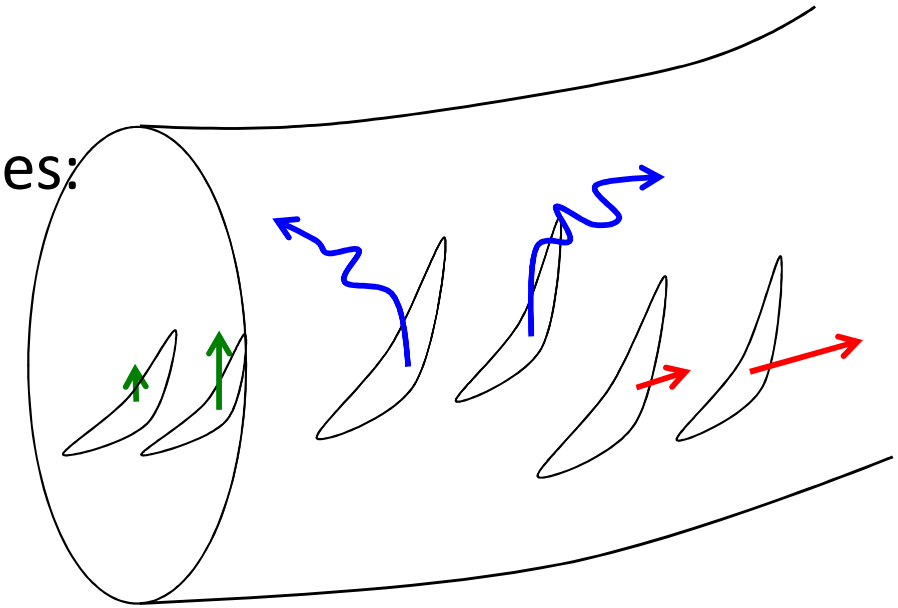
$$T(1,2) = v_{Di} \bar{E}_1 \frac{\partial}{\partial y_1} \langle \delta f(1) \delta f(2) \rangle + v_y' x_1 \frac{\partial}{\partial y_1} \langle \delta f(1) \delta f(2) \rangle$$
$$+ \nabla_1 \cdot \langle \mathbf{v}_{E\times B}(1) \delta f(1) \delta f(2) \rangle + (1 \leftrightarrow 2)$$



Will treat via closure theory !?

Analysis of Mixing

Triplet term after closure, relative coordinates:



$$T(1,2) = v_{Di}\bar{E}_- \frac{\partial}{\partial y_-}\langle \delta f(1)\delta f(2)\rangle$$
$$+ v_y' x_- \frac{\partial}{\partial y_-}\langle \delta f(1)\delta f(2)\rangle$$
$$- \nabla_- \cdot (\mathbf{D}_- \cdot \nabla_- \langle \delta f(1)\delta f(2)\rangle)$$

Moment evolution solved time asymptotically strong shear

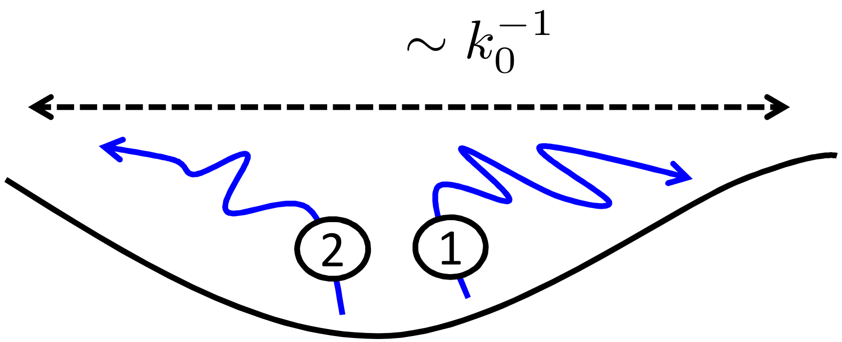
$$\langle\langle y_-^2 \rangle\rangle \cong \frac{e^{\sigma t}}{3}\left(y_-^2 + \frac{\sigma}{\Delta\omega_c}x_-^2 + \sqrt{\frac{2\sigma}{\Delta\omega_c}}x_- y_- + \frac{2v_{Di}^2}{\sigma^2}\bar{E}_-^2 + 4\frac{v_{Di}}{\sigma}\sqrt{\frac{\sigma}{2\Delta\omega_c}}\bar{E}_- x_- + \frac{2v_{Di}}{\sigma}\bar{E}_- y_- \right)$$

$\sigma = (2\Delta\omega_c {v_y'}^2)^{1/3}$: Geometric mean of ExB decorrelation rate and shear

Mixing time (deccorelation rate)

$$\langle\langle y_-^2 \rangle\rangle (t = \tau_{cl}) \cong k_0^{-2}$$

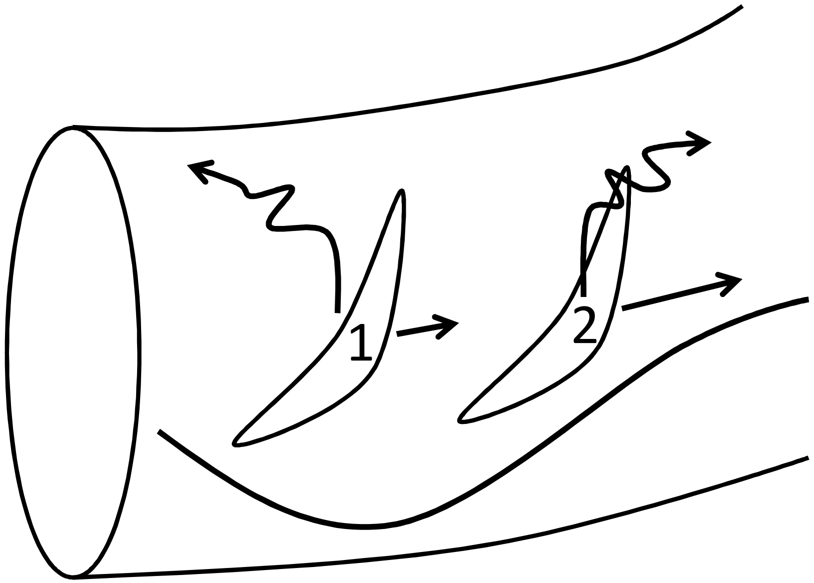
$$\boxed{\sigma\tau_{cl} = \ln\left(\frac{k_0^2 y_-^2}{3} + \frac{k_0^2\sigma}{3\Delta\omega_c}x_-^2 + \frac{k_0^2}{3}\sqrt{\frac{2\sigma}{\Delta\omega_c}}x_- y_- + \frac{2v_{Di}^2 k_0^2}{3\sigma^2}\bar{E}_-^2 + \frac{4v_{Di}k_0^2}{3\sigma}\sqrt{\frac{\sigma}{2\Delta\omega_c}}\bar{E}_- x_- + \frac{2v_{Di}k_0^2}{3\sigma}\bar{E}_- y_- \right)^{-1}}$$



Lifetime of granulations

Relative separation in turbulent field:

$$\langle\langle x_-^2\rangle\rangle + \langle\langle y_-^2\rangle\rangle \cong e^{t/\tau_c}\left(x_-^2 + y_-^2 + \frac{8\tau_c v_{Di}\bar{E}_- y_-}{3} + \frac{8\tau_c^2 v_{Di}^2 \bar{E}_-^2}{3}\right)$$



Life time of clumps:

$$\langle\langle x_-^2\rangle\rangle + \langle\langle y_-^2\rangle\rangle \sim k_0^{-2}$$

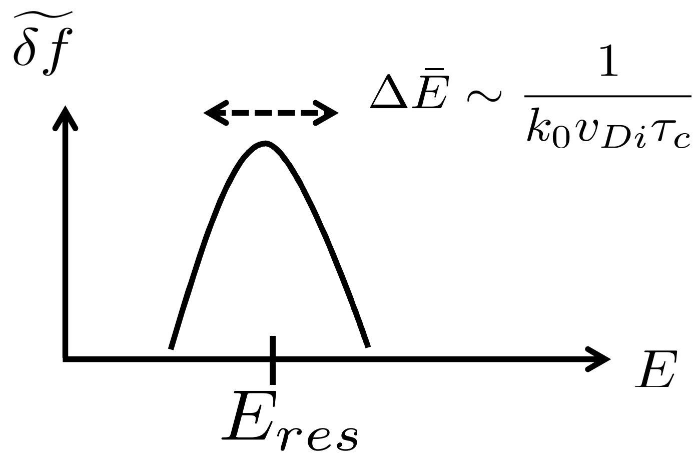
$$\tau_{cl} = \tau_c \ln\left[k_0^2 x_-^2 + k_0^2 y_-^2 + \frac{8k_0^2\tau_c v_{Di}\bar{E}_- y_-}{3} + \frac{8\tau_c^2 k_0^2 v_{Di}^2 \bar{E}_-^2}{3}\right]^{-1}$$

Typical scales:

physical space → $\lesssim k_0^{-1} \sim \Delta_c$

energy space → $\Delta E \sim (k_0 v_{Di}\tau_c)^{-1}$

resonance broadening via ExB scattering



Sharp correlation at small scales

- Steady state correlation:

$$\langle \delta f_i(1)\delta f_i(2)\rangle \cong \tau_{cl}(x_-, y_-, E_-)P(E_+)$$

$$\tau_{cl} = \tau_c \ln F(-)$$

$F \to \infty$ as 1→2 (absent collisional cut-off)

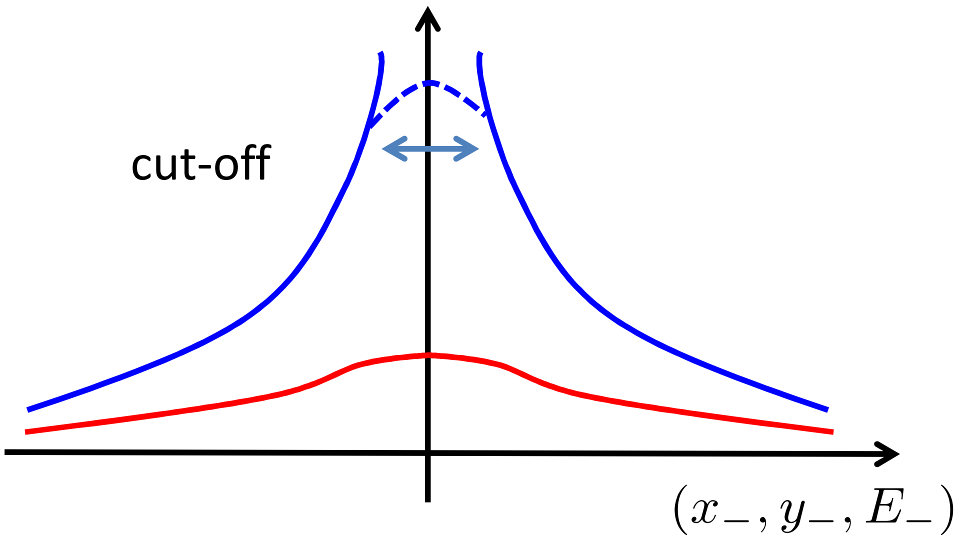
$$\delta f = \delta f^c + \widetilde{\delta f}$$ granulations

in contrast to QLT prediction

$$\delta f = \delta f^c = R\tilde{\phi}$$

$$\langle \delta f^c \delta f^c \rangle = 2\tau_c D_\perp \langle f\rangle'^2$$:finite

Schematically:



→ observed numerically in 70-80's

(Hui '75, Dupree '75, Berman '83)

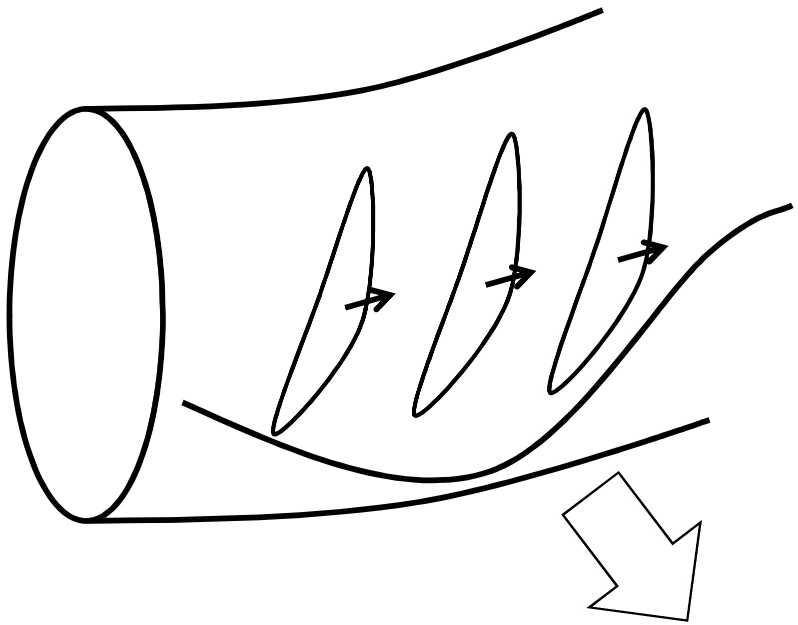
→ Drift wave turbulence, via modern computing scheme and power ???

Access to free energy (Ignore ZF, for now)

- Formation of the clumps of resonant particles (bunch of bananas)

- Scatter off electrons and release free energy

- Interplay, competition of diffusion, DF



- Net production due to electron dynamical friction:

assuming

$$\tilde{\phi} \sim \int d^3 v \widetilde{\delta f}$$

Ion free energy

electron dissipation

$$P(\bar{E}_+) = \sum_{\mathbf{k}} \frac{2\mathrm{Im}\chi_i \mathrm{Im}\chi_e}{|\partial\chi/\partial\omega_{\mathbf{k}}||\mathrm{Im}\chi|} \frac{\delta(\bar{E}_+ - \bar{E}^{res}_{\mathbf{k}})}{8\pi v^3_{thi}\sqrt{\epsilon_0}\sqrt{\bar{E}^{res}_{\mathbf{k}}}} \left\langle \frac{\widetilde{\delta n_i}}{n_0} \widetilde{\delta f}_i(\bar{E}^{res}_{\mathbf{k}}) \right\rangle_{\mathbf{k}}$$

Transport Flux

- Structure of theory (N.F. ’13)

	Quasi-linear theory	Dupree-Lenard-Balescu theory
Kubo number	$K \ll 1$	$K \gtrsim 1$
fluctuation	eigenmodes (waves)	structures (granulations)
Mean Evolution	Quasilinear diffusion $-D\langle f\rangle'$	Lenard-Balescu $-D\langle f\rangle' + F\langle f\rangle$

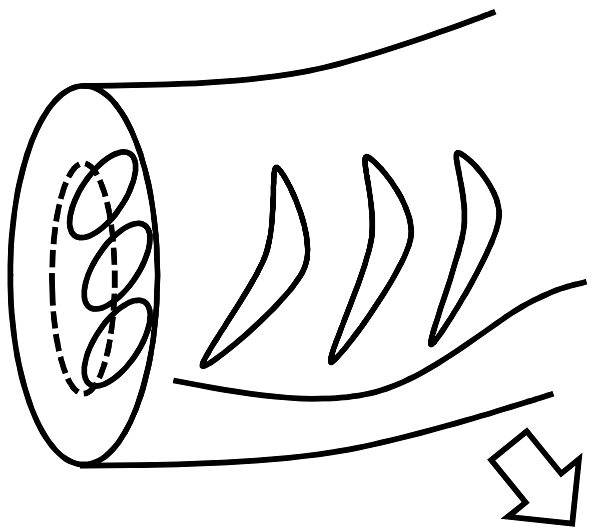
Dynamical Friction

- Transport Flux $\quad \langle \tilde{v}_r \delta f\rangle = J_{i,i} + J_{i,e} + J_{i,pol}$

$J_{i,i}$ QL flux $\sim D_\perp \langle f\rangle'$

$J_{i,e}$ D.F. from electrons $\propto \mathrm{Im}\chi_e$

$J_{i,pol}$ D.F. from zonal flow $\propto \partial_r \langle \tilde{v}_r \tilde{v}_\theta\rangle$



- Flux by D.F. can be comparable to flux by QLT:

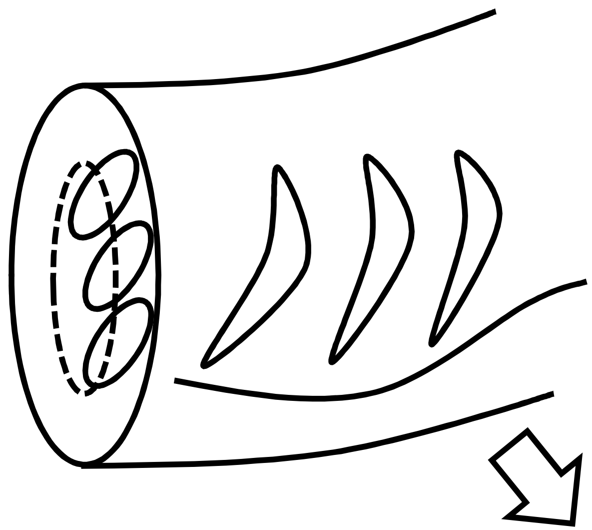
$$\frac{J_{DF}}{J_{QLT}} \sim \frac{\overline{\mathrm{Im}\chi_e k_\theta \rho_s}}{\overline{k_\theta^2 \rho_s^2}(\omega_{\mathbf{k}}/\omega_{Di})(c_s/\omega_{Di})\langle f_i\rangle' \sqrt{\epsilon_0} v_{thi}^3} \sim \frac{2+3\eta_e}{\eta_i} \frac{\omega}{\nu_e/\epsilon_0} \left(\frac{\omega_{di}}{\omega}\right)^2$$

Transport Flux - Detail

$$J_{i,i} = -\,\mathrm{Re}\sum_{\mathbf{k}\omega} k_\theta^2 \rho_s^2 c_s^2 R_{\mathbf{k}\omega} \left\langle \left(\frac{e\tilde{\phi}}{T_e} \right)^2 \right\rangle_{\mathbf{k}\omega} \langle f_i \rangle' + \sum_{\mathbf{k}\omega} k_\theta \rho_s c_s \frac{\mathrm{Im}\chi_i}{|\chi(\mathbf{k},\omega)|^2} \left\langle \frac{\widetilde{\delta n_i}}{n_0} \widetilde{\delta f}_i(2) \right\rangle_{\mathbf{k}\omega}$$

$$J_{i,e} = -\sum_{\mathbf{k}\omega} k_\theta \rho_s c_s \frac{\mathrm{Im}\chi_e}{|\chi(\mathbf{k},\omega)|^2} \left\langle \frac{\widetilde{\delta n_i}}{n_0} \widetilde{\delta f}_i(2) \right\rangle_{\mathbf{k}\omega}$$

$$J_{i,pol} = -\sum_{\mathbf{k}\omega} k_\theta \rho_s c_s \frac{\mathrm{Im}\chi_{pol}}{|\chi(\mathbf{k},\omega)|^2} \left\langle \frac{\widetilde{\delta n_i}}{n_0} \widetilde{\delta f}_i(2) \right\rangle_{\mathbf{k}\omega}$$



Transport by D.F. on electrons

Ion heat flux due to D.F. on electrons

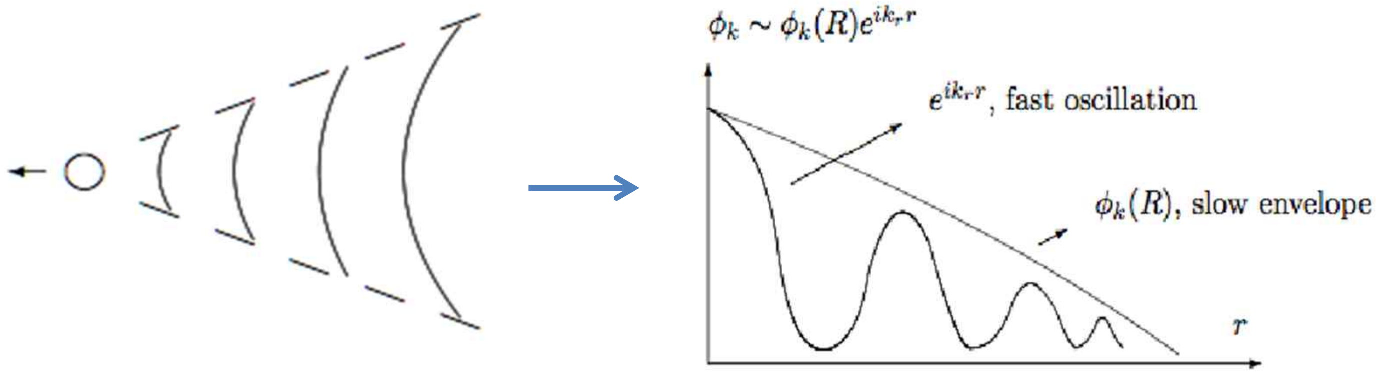
Ion free energy

$$-\frac{d}{dr}Q_i \simeq \sum_k \frac{2\mathrm{Im}\epsilon_e \mathrm{Im}\epsilon_i}{|\mathrm{Im}\epsilon||\partial\epsilon/\partial\omega_k|} T_i \bar{E}_k^{res} \left\langle \frac{\widetilde{\delta n_i}}{n_0} \frac{\widetilde{\delta f}_i}{f_0} (\bar{E}_k^{res}) \right\rangle_k$$

electron dissipation triggers release of ion free energy

Granulations – zonal flow coupling

- Granulations → Pol. charge scatt ering → ZF coupling



→ sets necessary phase fo r flow coupling

$$\langle \tilde{v}_r \nabla_\perp^2 \tilde{\phi} \rangle \sim k_\theta k_r \partial_R |\tilde{\phi}_{\mathbf{k}}|^2$$

- Coupled dynamics:

$$\frac{\partial}{\partial t}\left(\frac{v_*^i \langle v_\theta \rangle}{v_{thi}^2} + \int d^3v \frac{\langle \delta h^2 \rangle}{2\langle f \rangle}\right)$$
$$= \int d^3v \frac{P_{i,i} + P_{i,e}}{2\langle f \rangle} - \int d^3v \frac{\tau_{cl}^{-1}\langle \delta h^2 \rangle}{2\langle f \rangle} - \nu \frac{v_*^i \langle v_\theta \rangle}{v_{thi}^2}$$

akin to Charney-Drazin momentum the orems for PV fluids

- Quantitatively:

$$\frac{(\partial_t \langle \delta f^2 \rangle)_{Z.F.}}{(\partial_t \langle \delta f^2 \rangle)_{D.F.}} \sim \frac{\overline{k_r k_\theta}}{\overline{k_\theta^2}} \frac{\eta_e}{1 + 3\eta_e/2} \frac{\nu_e/\epsilon_0}{\sqrt{\epsilon_0}\omega_{c,i}} \frac{L_{T_e}}{L_{env}}$$

effective for steep intensity gradient region

In other words:

- Hamiltonian advection: $\partial_t f + \{f, H\} = 0$
- Mean + fluctuation conserved

$$f = \langle f \rangle + \delta f \longleftrightarrow q = \beta y + \omega$$

gradients!

- GK Poisson equation

$$\int d^3 v f + \rho_s^2 \nabla^2 \phi = g(\phi, n_e, \ldots)$$

multi-species

(Solvability? : "PV invertibility", MEM)

→ Evolution of δf MUST drive zonal flow

Heuristics of Zonal Flows

Ambipolarity breaking → polarization charge → Reynolds stress : The critical connection

- Schematically:
  - Polarization charge ➡ $\boxed{\rho^2}\nabla^2\phi = \boxed{n_{i,GC}(\phi) - n_e(\phi)}$

    polarization length scale ↲ ↳ *ion, electron guiding center density*

    so $\Gamma_{i,GC} \neq \Gamma_e$ ➡ → $\rho^2\left\langle \tilde{v}_{rE}\nabla_\perp^2\tilde{\phi}\right\rangle \neq 0$ ⬌ 'PV mixing'

    ↳ *polarization flux* → What sets cross-phase?
  - If 1 direction of symmetry (or near symmetry):

    $$\left\langle \tilde{v}_{rE}\nabla_\perp^2\tilde{\phi}\right\rangle = -\partial_r\left\langle \tilde{v}_{rE}\tilde{v}_{\perp E}\right\rangle \quad \text{(Taylor, 1915)}$$

  - Flow drive: $-\rho^2\partial_r\left\langle \tilde{v}_{rE}\tilde{v}_{\perp E}\right\rangle$ ➡ Reynolds force ➡ Flow drive

Summary

Physical quantity	Predictions	Relevant Feature
Basic Scales	$\Delta x \sim \Delta y \lesssim k_0^{-1} \sim \Delta_c$ $\Delta E \lesssim T_i/(\omega_{di}\tau_c)$	need resolve turb. scales and res. broadening
Correlation in phase space	$\langle \widetilde{\delta f_i}(1)\widetilde{\delta f_i}(2)\rangle$ $\cong (\tau_{cl}(x_-, y_-, E_-) - \tau_c)P$	log. div. at small scales $\lim_{1\to 2}\langle \widetilde{\delta f_i}\widetilde{\delta f_i}\rangle \gg \tau_c D_\perp \langle f\rangle'^2$
Frequency Spectrum	$\Delta\omega \sim \dfrac{F\lvert \mathrm{Im}\chi_i\rvert\lvert \mathrm{Im}\chi_e\rvert}{k_0\lvert v_{di}\rvert\lvert \partial\chi/\partial\omega_k\rvert^2}$	Depends both on i-free energy *and* e-diss.
Transport flux	$-D\langle f\rangle' + F\langle f\rangle$	Dynamical Friction appears as flux *not* proportional to gradient

Comments

- Model is simple, clear
- Phase space structures likely
- Numerous predictions to shoot at
- Directions:
  - Subcritical growth via electron scattering

* – Granulations and (flux driven) avalanching

→ does Cerenkov emission enhance avalanching? Hints of yes (Xiao '09)

* – Granulation – ZF interaction

III) Thoughts for Discussion

- Where does this story stand?
  - QLT '61 vindicated for relaxation of single species B-O-T, its paradigmatic example
  - 1D conservation constraints allow reconciliation of mode coupling with observed Landau growth. This interpretation raises (implicitly) the question of how representative the classic B-O-T is.

But

- Significant departures from QLT61 appear in (even 1D) systems with multiple energy-momentum exchange channels, usually associated with multi-species
    - B-B via γ_d
    - CDIA, though structure required.

  Signature of nonlinear growth

- Role of strong wave-particle resonance and phase space structure in even simple drift-zonal systems is not understood and merits further study
    - Subcritical growth?
    - Role of granulations in avalanching? (nucleate?)
    - Granulation interaction with zonal flows?

- What to Do?
  - Revitalize TWT, in coordination with modern simulation program
    - Allow variable slow wave structure dissipation → γ_d as in B&B → test Lesur, P.D. model?
    - Mode coupling, beat resonance (NLLD) phenomena
  - Is a (philosophically) similar CDIA experiment possible? Many testable predictions on the record. Consider multi-ion species to deal with m/M issue. Negative ion plasma to deal with mass ratio?!
  - While corresponding basic experiment dubious, Darmet model simulation program appears doable and interesting. Coordination with GYSELA studies might identify prediction testable in confinement studies.

- A bit philosophical, but:
  - What is the difference between a ‘finite amplitude wave’ and ‘structure’ i.e. ‘hole’?
  - Can the degree of distortion of f and its relation to subcritical and/or nonlinear growth be established or at least bounded?

  So far use self trapping condition:

  $\delta f \sim \frac{\Delta v}{\epsilon}$ Better?