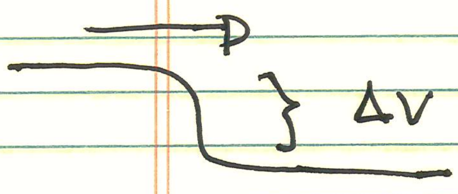
Supplement: Gas Dynamic Shocks
Lecture III

a.) Scale

Consider Burgers Eqn.

$$\partial_t V + V \partial_x V - \nu \partial_x^2 V = 0$$



for shock width

$$\frac{\Delta V \, \Delta V}{W} \sim \nu \frac{\Delta V}{W^2}$$

$$\boxed{W \sim \nu / \Delta V}$$

dissipation sets shock thickness

for $\nu \sim l_{mfp} c_s$

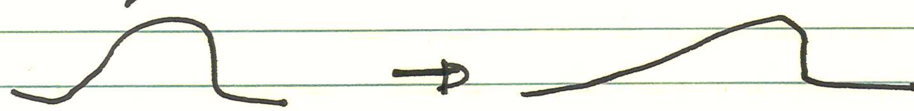
$\Delta V \sim c_s$

$$\boxed{W \sim l_{mfp}}$$

characteristic thickness of shock layer.

b.) Entropy Production

In gas-dynamics
- initially ideal dynamics
- entropy constant

but 

pulse steepens and shocks

- sharp gradients produced in shock
  - ⇒ couple to diffusive dissipation
  - ⇒ drive collisional transport
  - ⇒ produce entropy.

N.B. : - Entropy production required in shock
  - sets arrow of time

– to calculate:

$$\frac{dS}{dt} = \overset{\text{factor \#}}{(\,)}\left(-\Gamma_x \nabla X\right) \rightarrow$$

$\nabla X \rightarrow$ thermodynamic force

$\Gamma_x \rightarrow$ flux

i.e. $\Gamma_x = -D_x \nabla X$

$$dS/dt \sim (\,)\, \underbrace{D_x (\nabla X)^2}_{\text{entropy production rate density}}$$

then, for Burgers shock:

$$dS/dt \sim \nu (\partial_x v)^2$$

$$\sim \nu \frac{(\Delta v)^2}{w^2} \sim \nu (\Delta v)^2 \frac{(\Delta v)^2}{\nu^2}$$

$$\sim (\Delta v)^4/\nu$$

but, total entropy production is integrated over shock thickness

$$\int dx \frac{dS}{dt} \sim \frac{d\mathcal{S}}{dt} \sim w \frac{dS}{dt}$$

$$\sim \frac{\cancel{\nu}}{\Delta V} \frac{\cancel{\nu}}{\cancel{\nu^2}} (\Delta V)^2 (\Delta V)^2$$

$$\sim (\Delta V)^3$$

so total entropy production:

$$d\mathcal{S}/dt \sim (\Delta V)^3$$

- independent ν !
- entropy / heating produced by collisions but total $d\mathcal{S}/dt$ independent of ν.