54. For four heads: $W = 1 \rightarrow \Delta S = k\ln W = \left(1.38\times10^{-23}\text{ J/K}\right)\ln 1 = \boxed{0}$

For 3 heads, 1 tail:

$W = 4 \rightarrow \Delta S = k\ln W = \left(1.38\times10^{-23}\text{ J/K}\right)\ln 4 = \boxed{1.91\times10^{-23}\text{ J/K}}$

For 2 heads, 2 tails: $W = 6 \rightarrow \Delta S = k\ln W = \left(1.38\times10^{-23}\text{ J/K}\right)\ln 6 = \boxed{2.47\times10^{-23}\text{ J/K}}$

For 1 head, 3 tails:

$W = 4 \rightarrow \Delta S = k\ln W = \left(1.38\times10^{-23}\text{ J/K}\right)\ln 4 = \boxed{1.91\times10^{-23}\text{ J/K}}$

For four tails: $W = 1 \rightarrow \Delta S = k\ln W = \left(1.38\times10^{-23}\text{ J/K}\right)\ln 1 = \boxed{0}$

55. From the table below, we see that there are a total of $2^6 = 64$ microstates.

Macrostate	Possible Microstates (H = heads, T = tails)						Number of Microstates
6 heads, 0 tails	H H H H H H						1
5 heads, 1 tails	H H H H H T	H H H H T H	H H H T H H	H H T H H H	H T H H H H	T H H H H H	6
4 heads, 2 tails	H H H H T T	H H H T H T	H H T H H T	H T H H H T	T H H H H T		15
	H H H T T H	H H T H T H	H T H H T H	T H H H T H	H H T T H H		
	H T H T H H	T H H T H H	H T T H H H	T H T H H H	T T H H H H		
3 heads, 3 tails	H H H T T T	H H T H T T	H T H H T T	T H H H T T	H H T T H T		20
	H T H T H T	T H H T H T	H T T H H T	T H T H H T	T T H H H T		
	T T T H H H	T T H T H H	T H T T H H	H T T T H H	T T H H T H		
	T H T H T H	H T T H T H	T H H T T H	H T H T T H	H H T T T H		
2 heads, 4 tails	T T T T H H	T T T H T H	T T H T T H	T H T T T H	H T T T T H		15
	T T T H H T	T T H T H T	T H T T H T	H T T T H T	T T H H T T		
	T H T H T T	H T T H T T	T H H T T T	H T H T T T	H H T T T T		
1 heads, 5 tails	T T T T T H	T T T T H T	T T T H T T	T T H T T T	T H T T T T	H T T T T T	6
0 heads, 6 tails	T T T T T T						1

(*a*) The probability of obtaining three heads and three tails is $\boxed{20/64}$ or $\boxed{5/16}$.

(*b*) The probability of obtaining six heads is $\boxed{1/64}$.

56. When throwing two dice, there are 36 possible microstates.

(*a*) The possible microstates that give a total of 7 are: (1)(6), (2)(5), (3)(4), (4)(3), (5)(2), and (6)(1). Thus the probability of getting a 7 is $6/36 = \boxed{1/6}$.

(*b*) The possible microstates that give a total of 11 are: (5)(6) and (6)(5). Thus the probability of getting an 11 is $2/36 = \boxed{1/18}$.

(*c*) The possible microstates that give a total of 4 are: (1)(3), (2)(2), and (3)(1). Thus the probability of getting a 5 is $3/36 = \boxed{1/12}$.

57. (*a*) There is only one microstate for 4 tails: TTTT. There are 6 microstates with 2 heads and 2 tails: HHTT, HTHT, HTTH, THHT, THTH, and TTHH. Use Eq. 20-14 to calculate the entropy change.

$$\Delta S = k\ln W_2 - k\ln W_1 = k\ln\frac{W_2}{W_1} = \left(1.38\times10^{-23}\ \text{J/K}\right)\ln 6 = \boxed{2.47\times10^{-23}\ \text{J/K}}$$

(*b*) Apply Eq. 20-14 again. There is only 1 final microstate, and about 1.0×10^{29} initial microstates.

$$\Delta S = k\ln W_2 - k\ln W_1 = k\ln\frac{W_2}{W_1} = \left(1.38\times10^{-23}\ \text{J/K}\right)\ln\left(\frac{1}{1.0\times10^{29}}\right) = \boxed{-9.2\times10^{-22}\ \text{J/K}}$$

(*c*) These changes are much smaller than those for ordinary thermodynamic entropy changes.

For ordinary processes, there are many orders of magnitude more particles than we have considered in this problem. That leads to many more microstates and larger entropy values.

58. The number of microstates for macrostate A is $W_{\text{A}} = \frac{10!}{10!0!} = 1.$ The number of microstates for

macrostate B is $W_{\text{B}} = \frac{10!}{5!5!} = 252.$

(*a*) $\Delta S = k\ln W_B - k\ln W_A = k\ln\frac{W_B}{W_A} = \left(1.38\times10^{-23}\ \text{J/K}\right)\ln 252 = \boxed{7.63\times10^{-23}\ \text{J/K}}$

Since $\Delta S > 0,$ this $\boxed{\text{can occur naturally}}$.

(*b*) $\Delta S = k\ln W_A - k\ln W_B = -k\ln\frac{W_B}{W_A} = -\left(1.38\times10^{-23}\ \text{J/K}\right)\ln 252 = \boxed{-7.63\times10^{-23}\ \text{J/K}}$

Since $\Delta S < 0,$ this $\boxed{\text{cannot occur naturally}}$.