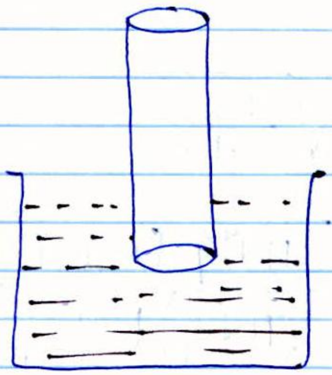
3.4



submerged length $= l$

Total length $= L$

Since the cylinder floats

$$\frac{\text{submerged volume}}{\text{Total volume}} = \frac{\rho_{cyl.}}{\rho_{liquid}}$$

$$\therefore \rho_{cyl}/\rho_{liquid} = \frac{\pi d^2/4\, l}{\pi d^2/4\, L} = (l/L).$$

If the cylinder is displaced/pushed down by a distance x.

it would displace $(\pi d^2/4)\, \rho_{liquid}\, x$ amount of liquid.

Thus the upward force would be

$$F = -\pi d^2/4\, \rho_{liquid}\, g\, x$$

$$= -\left(\pi d^2/4\, \rho_{liquid}\, g\right) x$$

Compare with $F = -kx$ to obtain

$$k = (\pi d^2/4)\, \rho_{liquid}\, g$$

$\therefore$ frequency $= \dfrac{\omega}{2\pi} = \dfrac{1}{2\pi}\sqrt{\dfrac{k}{m}}$.

$$= \left(\frac{1}{2\pi}\right)\sqrt{\frac{\pi d^2/4 \; \rho_{liquid}\, g}{(\pi d^2/4)\, L \, \rho_{cyl}}}$$

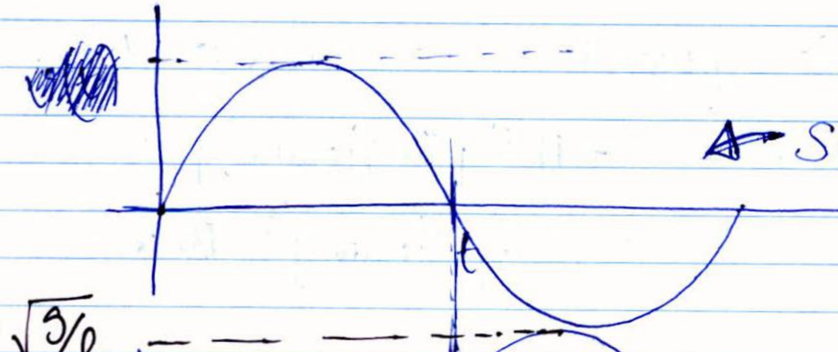
$$= \frac{1}{2\pi}\sqrt{\left(\frac{g}{L}\right)\frac{\rho_{liqud}}{\rho_{cyl}}}$$

$$= \frac{1}{2\pi}\sqrt{\frac{g}{L}\cdot\frac{L}{l}} = \left(\frac{1}{2\pi}\right)\sqrt{g/l}$$

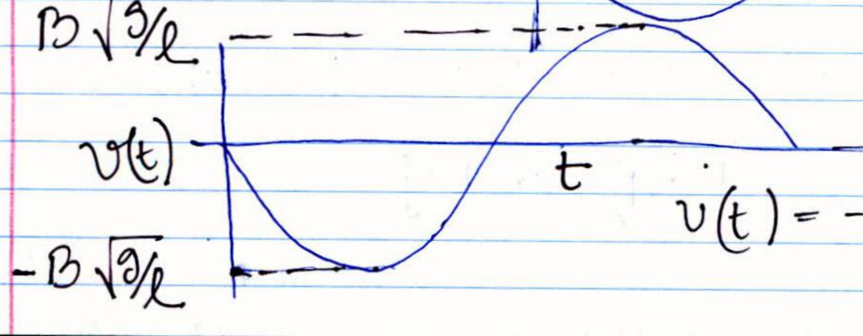
if the displacement from equilibrium is x,

$x(t) = B\cos\left(\sqrt{g/l}\; t\right)$. (I'm taking downward direction to be positive)

$v(t) = -B\sqrt{g/l}\,\sin\left(\sqrt{g/l}\; t\right)$.



← $\sin(\omega t)$ looks like this.



$v(t) = -B\sqrt{g/l}\,\sin\left(\sqrt{g/l}\, t\right)$.

3.11.

Suppose, the piston is moved by distance x to compress the gas

if the initial volume is V_1, after compression, it would become

$$V_2 = (V_1 - xA)$$ where A is cross-sectional area!!

Now, $$P_2 V_2^{\gamma} = P_1 V_1^{\gamma}$$

$$P_2 = P_1 \left(1 - \frac{xA}{V_1}\right)^{-\gamma}$$

$$\simeq P_1 \left(1 + \frac{\gamma x A}{V_1}\right)$$ (assume small x).

$$\therefore (P_2 - P_1) = \left(\frac{P_1}{V_1}\right) \gamma A x.$$

Hence, the extra force would be (in upward direction)

$$(P_2 - P_1) A = \left(\frac{P_1}{V_1}\right) \gamma A^2 x. \Rightarrow K = \left(\frac{P_1}{V_1} \gamma A^2\right)$$

@ equilibrium $mg = P_1 A$. (weight of piston is balanced by the force due to pressure).

$$\therefore \text{frequency } (\omega) = \cancel{\tfrac{1}{2\pi}} \sqrt{K/m} = \sqrt{\frac{P_1/V_1 \, \gamma A^2}{P_1 A/g}} = \sqrt{\frac{\gamma A g}{V_1}}$$

$$= \sqrt{\gamma g / l}.$$

where l is length of column.

3.13 $x = Ae^{-\alpha t}\cos\omega t$

$\frac{dx}{dt} = -A\alpha e^{-\alpha t}\cos\omega t - Ae^{-\alpha t}\omega\sin\omega t$

$\frac{d^2x}{dt^2} = A\alpha^2 e^{-\alpha t}\cos\omega t + 2A\alpha e^{-\alpha t}\omega\sin\omega t - \omega^2 A e^{-\alpha t}\cos\omega t$

$\frac{d^2x}{dt^2} + \gamma\frac{dx}{dt} + \omega_0^2 x = 0$

$\Rightarrow \left(\alpha^2 - \omega^2 - \gamma\alpha + \omega_0^2\right)\cos(\omega t)$

$+ \left(2\alpha\omega - \gamma\omega\right)\sin(\omega t) = 0$

co-efficients of $\sin(\omega t)$ & $\cos(\omega t)$ have to vanish separately.

$\therefore\ 2\alpha\omega - \gamma\omega = 0 \Rightarrow \gamma = 2\alpha \Rightarrow \alpha = \gamma/2$

$\alpha^2 - \gamma\alpha = -\omega_0^2 + \omega^2$

$\Rightarrow \omega^2 = \gamma^2/4 - \gamma^2/2 + \omega_0^2 = \left(\omega_0^2 - \gamma^2/4\right)$

$\therefore\ \alpha = \gamma/2\ ;\quad \omega = \sqrt{\omega_0^2 - \gamma^2/4}$

4.6

(a) The key point is to realize that the earth's surface is accelerating, so if one chooses to work in the reference frame of earth, one has to consider pseudo force.

pseudo acceleration is precisely given by

$$\left(-\frac{d^2\eta}{dt^2}\right).$$

Thus we have

$$\underbrace{\frac{d^2y}{dt^2} + \gamma\frac{dy}{dt} + \omega_0^2 y}_{\text{as if it is in rest frame}} = \underbrace{-\frac{d^2\eta}{dt^2}}_{\text{effect of non-inertial frame}}$$

(b) $\eta = C\cos\omega t$, $\quad -\frac{d^2\eta}{dt^2} = C\omega^2\cos(\omega t)$.

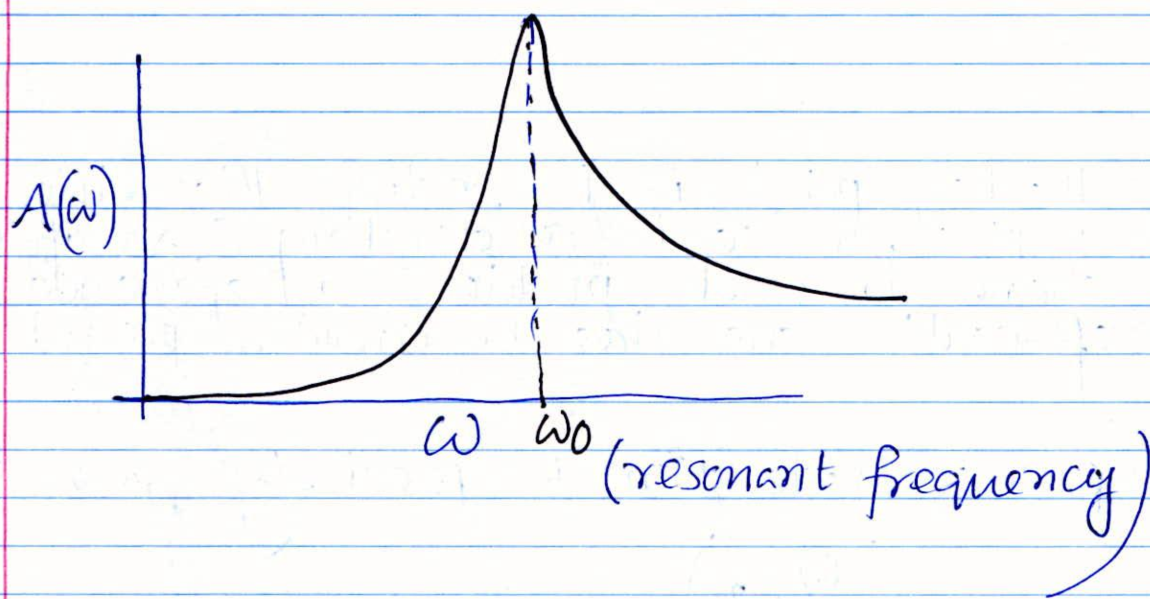
$$\therefore \frac{d^2y}{dt^2} + \gamma\frac{dy}{dt} + \omega_0^2 y = C\omega^2\cos(\omega t).$$

$$y = A(\omega)\cos(\omega t - \delta).$$

where $A(\omega) = \dfrac{C\omega^2}{\left[(\omega_0^2-\omega^2)^2 + \gamma^2\omega^2\right]^{1/2}}$

$$\tan\delta(\omega) = \left(\gamma\omega/\omega_0^2-\omega^2\right).$$

(c)



(d) $\gamma = \omega_0/Q$.

$$A(\omega) = \frac{C\omega^2}{\left[(\omega_0^2-\omega^2)^2 + \left(\frac{\omega\omega_0}{Q}\right)^2\right]^{1/2}}$$

$\omega_0 = 2\pi/30 = \pi/15$, $Q = 2$

$$A(\omega) = \frac{C\omega^2}{\left[\left(\pi^2/225 - \omega^2\right)^2 + \left(\frac{\pi\omega}{30}\right)^2\right]^{1/2}}$$

$C\omega^2 = 10^{-9}\ ms^{-2}$ $\qquad \omega = 2\pi/1200 = \pi/600$

Plug in the values to figure out $A(\omega)$.

4.10.

(a) $F = -bv$

work done against this force

$dW = -F\,dx = bv\,dx$

$= bv\left(\frac{dx}{dt}\right)dt$

$\therefore \left(\frac{dW}{dt}\right) = bv^2.$

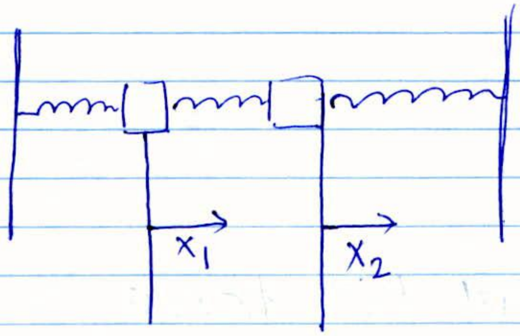
(b) $\frac{dW}{dt} = b\left(\frac{dx}{dt}\right)^2 = bA^2\omega^2 \sin^2(\omega t - \delta)$

$\left\langle\frac{dW}{dt}\right\rangle = bA^2\omega^2 \left\langle \sin^2(\omega t - \delta)\right\rangle$

$= \frac{1}{2} bA^2\omega^2$

(c) it's a straight forward substitution!!

5.4.



Let's say block 1 gets displaced by $x_1(t)$.
block 2 gets displaced by $x_2(t)$.

$$m\frac{d^2x_1}{dt^2} = -k_A x_1 - k_C(x_1 - x_2).$$

$$m\frac{d^2x_2}{dt^2} = -k_B x_2 - k_C(x_2 - x_1).$$

$\therefore$ normal modes will be given by eigenvalues of following matrix

$$\therefore m\omega_i^2 = \text{Eig.}\begin{pmatrix} (k_A + k_C) & -k_C \\ -k_C & k_C + k_B \end{pmatrix}.$$

$$\therefore \omega_i^2 = \frac{1}{2m}\left(k_A + k_B + 2k_C \pm \sqrt{(k_A - k_B)^2 + 4k_C^2}\right)$$

when $k_C^2 = k_A k_B$.

$$\omega_i^2 = \frac{1}{2m}\left(k_A + k_B + 2k_C \pm (k_A + k_B)\right)$$

$$= \begin{cases} \frac{1}{m}\cancel{2}(k_A + k_B + k_C). \\ \frac{1}{m}\cancel{2}k_C. \end{cases}$$

(Answer)

5.2.

(a) $\omega_1 = \sqrt{g/l}$

$\omega_2 = \left(g/l + \frac{2k}{m}\right)^{1/2}.$

it is given that if one pendulum is clamped, the period of other is 1.25 s.

i.e. $\left(g/l + \frac{k}{m}\right)^{1/2} = \frac{2\pi}{1.25}$

$g/l = \frac{9.8}{0.4}\ s^{-2}$

$= 24.5\ s^{-2}$

$\therefore \left(24.5 + k/m\right)^{1/2} = \frac{2\pi}{1.25} = 5.02.$

$\therefore k/m = \left[(5.02)^2 - 24.5\right]\ s^{-2}$

$= 0.766\ s^{-2}.$

$\therefore \omega_1 = \sqrt{g/l} = 4.94\ s^{-1} \Rightarrow T_1 = 1.27\ s$

$\omega_2 = \left(24.5 + 1.532\right)^{1/2}\ s^{-1}$

$= 5.10\ s^{-1}. \Rightarrow T_2 = 1.23\ s$

b) The pendulum will oscillate

$$x(t) = A\cos\omega_1 t + A\cos\omega_2 t$$
$$= 2A\cos\left[\left(\frac{\omega_1+\omega_2}{2}\right)t\right]\cos\left[\left(\frac{\omega_1-\omega_2}{2}\right)t\right]$$

Time period for such a case would be

$$\frac{2\pi}{\left[(\omega_1-\omega_2)\right]/2} = \frac{4\pi}{\omega_2-\omega_1}$$

$$= \frac{4\pi}{\left(\frac{2k}{m}+g/l\right)^{1/2} - \sqrt{g/l}}$$