5.14 $A_{pn} = C_n \sin\left(\frac{pn\pi}{N+1}\right).$

Here $N = 3$.

$$A_{pn} = C_n \sin\left(\frac{pn\pi}{4}\right).$$

$$A_{11} : A_{21} : A_{31} :: 1/\sqrt{2} : 1 : 1/\sqrt{2}$$

$$A_{12} : A_{22} : A_{32} :: 1 : 0 : -1$$

$$A_{13} : A_{23} : A_{33} :: 1/\sqrt{2} : -1 : 1/\sqrt{2}.$$

(Answer)

5.17 The displacement of the first & second oscillator are :

$$y_{1N} = C_N \sin\left(\frac{N\pi}{N+1}\right)\cos(\omega_N t).$$

$$y_{2N} = C_N \sin\left(\frac{2N\pi}{N+1}\right)\cos(\omega_N t).$$

$$y_{1N} = C_N \sin\left(\pi - \pi/N+1\right)\cos(\omega_N t) = C_N \sin\left(\pi/N+1\right)\cos\omega_N t$$

$$y_{2N} = C_N \sin\left(2\pi - \frac{2\pi}{N+1}\right)\cos(\omega_N t) = -C_N \sin\left(2\pi/N+1\right)\cos\omega_N t$$

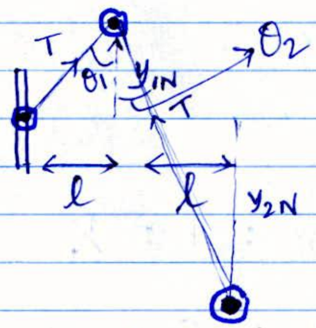
In large N limit

$$y_{1N} \approx C_N \left(\pi/N+1\right)\cos\omega_N t$$

$$y_{2N} \approx -2C_N \left(\pi/N+1\right)\cos\omega_N t$$

$\therefore\ y_{2N} \simeq -2y_{1N}$

Let us look at picture



$y_{1N} = y \Rightarrow \cos\theta_1 = y/l$

$y_{2N} = -2y$

$\therefore \cos\theta_2 = \left(3y/l\right)$.

$$\text{Force} = T\cos\theta_1 + T\cos\theta_2$$
$$= T\left(y/l\right) + T\left(\frac{3y}{l}\right).$$
$$= \left(\frac{4T}{l}\right)y.$$

(It requires us to assume $(90-\theta_1), (90-\theta_2)$ are small angles which is true in large N limit).

$$\therefore m\frac{d^2y}{dt^2} = -\frac{4T}{l}y$$

$$\frac{d^2y}{dt^2} = -4\left(\frac{T}{lm}\right)y = -4\omega_0^2 y$$

$$\therefore \boxed{\omega_N = 2\omega_0}$$ (Proved)

6.1. a) $f = \frac{1}{2L}\left(\frac{T}{\mu}\right)^{1/2}$

$$= \frac{1}{5}\left(\frac{10}{\frac{0.01}{2.5}}\right)^{1/2} \text{ Hz}$$

$$= \frac{1}{5} \times 50 \text{ Hz}$$

$$= 10 \text{ Hz}$$

b) - It's plucked and touched at $0.5\,m = \frac{2.5}{5}\,m$.

thus $f = \left(\frac{5n}{2L}\right)\left(\frac{T}{\mu}\right)^{1/2}$ where $n = 1, 2, \ldots$

$$= n\left(\frac{10}{0.01/2.5}\right)^{1/2} \text{ Hz}.$$

$$= 50\,n \text{ Hz}.$$ where $n = 1, 2, \ldots$

6.2

$$\left.\begin{aligned} f_1 &= \frac{1}{2L}\left(\frac{TL}{M}\right)^{1/2} \\ f_2 &= \frac{1}{L}\left(\frac{TL}{M}\right)^{1/2} \\ f_3 &= \frac{3}{2L}\left(\frac{TL}{M}\right)^{1/2} \end{aligned}\right\} \text{ frequency of string}$$

Normal modes of 3 coupled oscillator

$$\omega_n = 2\left(\frac{T}{\frac{M}{3}\,L/4}\right)^{1/2} \sin\left(\frac{n\pi}{2\cdot 4}\right).$$

$$\omega_n = \frac{4\sqrt{3}}{L}\left(\frac{TL}{M}\right)^{1/2} \sin\left(\frac{n\pi}{8}\right).$$

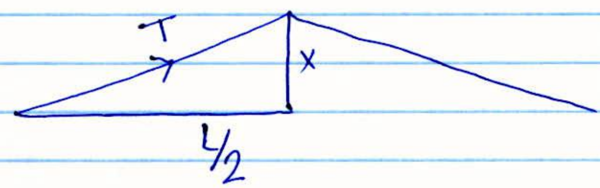
$$f_n^c = \left(\frac{\omega_n}{2\pi}\right) = \left(\frac{4\sqrt{3}}{\pi}\sin\left(\frac{n\pi}{8}\right)\right)\left(\frac{1}{2L}\right)\left(\frac{TL}{M}\right)^{1/2}$$

$$f_1^c/f_1 = \frac{4\sqrt{3}}{\pi}\sin\left(\pi/8\right) = 0.84$$

$$f_2^c/f_2 = \frac{2\sqrt{3}}{\pi}\sin\left(\pi/4\right) = 0.78$$

$$f_3^c/f_3 = \frac{4}{\sqrt{3}\pi}\sin\left(3\pi/8\right) = 0.68$$

6.12. a)



Force in transverse direction (following the hint).

$= 2T\cos\theta$

$= 2T\, x/\sqrt{x^2 + L^2/4}$

$\therefore$ Work done $= \int_0^h 2T\cos\theta \, dx$

$= T\int_0^h \frac{2x}{(x^2 + L^2/4)^{1/2}} dx$

$= T\int_0^{h^2} \frac{dy}{(y + L^2/4)^{1/2}} = 2T\left[\sqrt{h^2 + L^2/4} - L/2\right]$

* Energy of subsequent oscillation *

(Answer)