# Solutions to problems from kinetic theory of gases

## 1 Problem 1

We assume the gas is an ideal gas whose velocities are distributed according to the Maxwell-Boltzmann distribution for the speeds $f(v)$. For a certain gas at a specific temperature,

$$f(v)dv \propto v^2 e^{-\frac{mv^2}{2k_B T}} dv$$

i.e., for the same $dv$

$$\frac{f(v_1)}{f(v_2)} = \Big(\frac{v_1}{v_2}\Big)^2 e^{-\frac{m(v_1^2 - v_2^2)}{2k_B T}}$$

$$\Rightarrow \ln \frac{f(v_1)}{f(v_2)} = 2 \ln \frac{v_1}{v_2} - \frac{m(v_1^2 - v_2^2)}{2k_B T}$$

$$\Rightarrow T = \frac{v_1^2 - v_2^2}{\frac{2k_B}{m}\Big(2 \ln \frac{v_1}{v_2} - \ln \frac{f(v_1)}{f(v_2)}\Big)}$$

In this problem, $v_1 = 2000$ m/s, $v_2 = 1000$ m/s, $f(v_1) = 2\ f(v_2)$, $k_B/m = 8311/28\ m^2/s^2$ K. Plugging in the values, we get $\underline{T = 7290\text{K}}$.

## 2 Problem 2

The components of velocities, e.g. $v_x$, of an ideal gas under Maxwell-Boltzmann statistics are isotropic and have a Gaussian probability distribution e.g. $g(v_x)$. Note that there is no $v_x^2$ prefactor as there is for $f(v)$. Probability distributions of velocities in all three directions have the same prefactors and functional form. The number of particles with velocities between $v_x$ and $v_x + dv_x$ is

$$g(v_x)dv_x \propto e^{-\frac{mv_x^2}{2k_B T}} dv_x$$

$$\Rightarrow \frac{g(v_x)}{g(v_y)} = e^{-\frac{m(v_x^2 - v_y^2)}{2k_B T}}$$

$$\Rightarrow \ln \frac{g(v_x)}{g(v_y)} = -\frac{m(v_x^2 - v_y^2)}{2k_B T}$$

$$\Rightarrow T = -\frac{v_x^2 - v_y^2}{\frac{2k_B}{m}\Big(\ln \frac{g(v_x)}{g(v_y)}\Big)}$$

In this problem, $v_x = 500$ m/s, $v_y = 1000$ m/s, $g(v_x) = 2\ g(v_y)$, $k_B/m = 8311/32\ m^2/s^2$ K. Plugging in the values, we get $\underline{T = 2083\text{K}}$.

# 3 Problem 3

We will use a subscript of '1x' for $He$ and '2x' for $O_2$. The probability distribution for the x component of velocity for a gas with molecular mass $m$ at temperature $T$ is given by,

$$g(v_x)dv_x = \left(\frac{m}{2\pi k_B T}\right)^{1/2} e^{-\frac{mv_x^2}{2k_BT}} dv_x$$

Due to isotropicity, the probability distribution for the y and z components are similar. Now, we have

$$\begin{aligned} g(v_{1x})dv_{1x} &= g(v_{2x})dv_{2x} \\ \Rightarrow m_1^{1/2} e^{-\frac{m_1 v_{1x}^2}{2k_BT}} dv_{1z} &= m_2^{1/2} e^{-\frac{m_2 v_{2x}^2}{2k_BT}} dv_{2x} \\ \Rightarrow \frac{1}{2}\ln\frac{m_1}{m_2} + \ln\frac{dv_{1x}}{dv_{2x}} &= \frac{1}{2k_BT}(m_1 v_{1x}^2 - m_2 v_{2x}^2) \\ \Rightarrow T &= \frac{v_{1z}^2 - \frac{m_2}{m_1} v_{2x}^2}{\frac{2k_B}{m_1}\left(\frac{1}{2}\ln\frac{m_1}{m_2} + \ln\frac{dv_1}{dv_{2x}}\right)} \end{aligned}$$

Plugging in the numbers, $v_{1x} = 900$ m/s, $v_{2x} =$ -450 m/s, $dv_{1x} = 5$ m/s, $dv_{2x} = 10$ m/s, $m_1 = 4$u, $m_2 = 32$u, $k_B/m_1 = 8311/4\ m^2/s^2K$, we get $\underline{T = 112.48\text{K}}$.

# 4 Problem 4

We can use the same expression for the temperature found in Problem 1 and solve for $m$ instead of $T$ while setting $f(v_1) = f(v_2)$ i.e.,

$$m = \frac{4\ln\frac{v_1}{v_2} k_B T}{v_1^2 - v_2^2}$$

Plugging in $v_1 = 300$ m/s, $v_2 = 600$ m/s, $T = (20 + 273)$ K $= 293$ K, $k_B = 8311$ u $m^2/s^2K$ which gives $\underline{m = 25\text{ u}}$. For the root-mean-square velocity, we use the formula

$$v_{rms} = \sqrt{\frac{3k_BT}{m}}$$

to get $\underline{v_{rms} = 540\text{ m/s}}$.