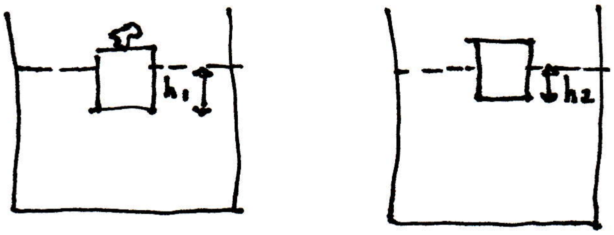
Physics 4B — Final Exam Solutions — Spring 2018

Problem 1



A = cross-sectional area of ice cube

$\rho_{ice} = 0.917 \frac{g}{cm^3}$, $\rho_{H_2O} = 1 \frac{g}{cm^3}$

with fly: $m_{tot} = m_{ice} + m_{fly} = 21 g$

$$\rho_{H_2O} \cdot A \cdot h_1 = m_{tot} = m_{ice} + m_{fly}$$
$$\rho_{H_2O} \cdot A \cdot h_2 = m_{ice} \quad \Rightarrow \rho_{H_2O} \cdot A \cdot (h_1 - h_2) = m_{fly}$$

$$A = V_{ice}^{2/3} = \left(\frac{m_{ice}}{\rho_{ice}}\right)^{2/3} \Rightarrow h_1 - h_2 = \frac{m_{fly}}{\rho_{H_2O}} \left(\frac{\rho_{ice}}{m_{ice}}\right)^{2/3} = 0.128 cm$$

so amplitude is ~~$A =$~~ $\boxed{h_1 - h_2 = 0.128 cm}$ (a)

Force is $m_{fly} \cdot g = k(h_1 - h_2)$ where k is the "force constant"

$$\Rightarrow k = \frac{m_{fly} \cdot g}{h_1 - h_2} = \rho_{H_2O} \left(\frac{m_{ice}}{\rho_{ice}}\right)^{2/3} \cdot g \Rightarrow$$

$$\omega^2 = \frac{k}{m_{ice}} = \rho_{H_2O} \cdot \frac{1}{m_{ice}^{1/3} \rho_{ice}^{2/3}} \cdot g = \frac{g}{h_2} = 382.9$$

$\omega = 19.57 rad/s = 2\pi f \Rightarrow \boxed{f = 3.11 Hz}$ (b)

(c) We can find energy from potential or kinetic:

$h = (h_1 - h_2)\cos\omega t \Rightarrow \dot{h} = \omega(h_1 - h_2)\sin\omega t \Rightarrow$

$\Rightarrow E_{kin} = \frac{1}{2} m_{ice} \cdot \omega^2 (h_1 - h_2)^2 = \boxed{62.7 erg}$ (c)

or $E_{pot} = E_{kin}$

Note that it is also $\frac{1}{2} \times m_{fly} \times (h_1 - h_2) \cdot g$

Problem 2

Ideal gas: $P = \frac{\rho}{m} kT$, ρ = density, m = molecular mass

Since P is same, density of air at 100°C = 373 K is, with $\rho_0 = 1.225 \frac{kg}{m^3}$

$$\rho_{hot} = \frac{293}{373} \rho_0 = 0.962 \ kg/m^3 \qquad V = 2800 m^3$$

The mass that can be lifted is

$$\Delta m = m_{cold\ air} - m_{hot\ air} = (\rho_0 - \rho_{hot}) V = 736.4 \ kg$$

subtracting 400 kg = 336.4 kg = 65 kg × 5 + 11.4 kg

$\boxed{\text{it can lift 5 people of 65 kg each}}$ (a)

(b) $$\frac{\Delta Q}{\Delta t} = k \cdot A \cdot \frac{(T_1 - T_2)}{l} = 2 \cdot 10^6 W \ , \quad T_1 - T_2 = 80 K,$$

$k = 0.25 W/m.K$. To find A = area, assume it is a sphere.

$$V = \frac{4\pi}{3} R^3 \Rightarrow R = \left(\frac{3V}{4\pi}\right)^{1/3} \Rightarrow A = 4\pi R^2 = 4\pi \left(\frac{3V}{4\pi}\right)^{2/3} \Rightarrow$$

$$\Rightarrow \boxed{A = 960.7 m^2} \Rightarrow l = \frac{k A (T_1 - T_2)}{\Delta Q / \Delta t} = 0.0096 m \approx \boxed{1 cm} \text{ (b)}$$

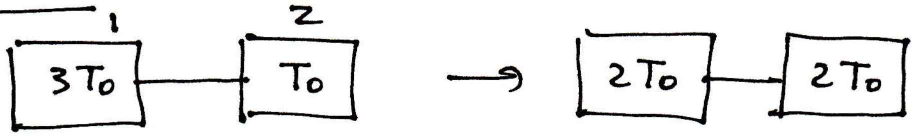
(c) Assume weight to be lifted = 736.4 kg . $\rho(h) = \rho_0 e^{-h/h_0}$

$$\Delta m = m_{cold} - m_{hot} = V\left(\rho(h) - \rho(h) \cdot \frac{293}{383}\right) = 736.4 \Rightarrow$$

with $\rho_0 = 1.225 \ kg/m^3$ $\Rightarrow e^{-h/h_0} = \frac{736.4}{2800 \times 1.225 \times \left(1 - \frac{293}{383}\right)} = 0.914$

$$\Rightarrow \boxed{h = h_0 \ln(1/0.914) = 722 m} \text{ (c)}$$

Problem 3



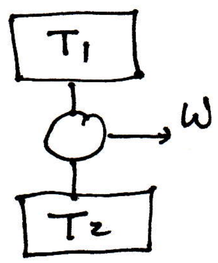
$Q_0 = C \cdot T_0$, with C = heat capacity of one body.

$\Delta S = \Delta S_1 + \Delta S_2$, 1 = hot, 2 = cold

$$\Delta S_1 = \int_{3T_0}^{2T_0} \frac{dQ}{T} = C \ln \frac{2}{3} \; ; \; \Delta S_2 = \int_{T_0}^{2T_0} \frac{dQ}{T} = C \ln 2 \Rightarrow$$

$$\Rightarrow \boxed{\Delta S = C \ln \frac{4}{3} = \frac{Q_0}{T_0} \ln \frac{4}{3} = 0.288 \frac{Q_0}{T_0}} \text{ (a)}$$

(b)



operating a reversible engine in-between the 2 bodies: assume final temperature reached is T_f, with no change in total entropy.

$$\Delta S_1 = \int_{3T_0}^{T_f} \frac{dQ}{T} = C \ln \frac{T_f}{3T_0} \; ; \; \Delta S_2 = \int_{T_0}^{T_f} \frac{dQ}{T} = C \ln \frac{T_f}{T_0} \Rightarrow$$

$$\Delta S = \Delta S_1 + \Delta S_2 = C \ln \frac{T_f^2}{3T_0^2} = 0 \Rightarrow T_f^2 = 3T_0^2 \Rightarrow$$

$$\Rightarrow \boxed{T_f = \sqrt{3}\, T_0 = 1.732\, T_0} \text{ (c)}$$

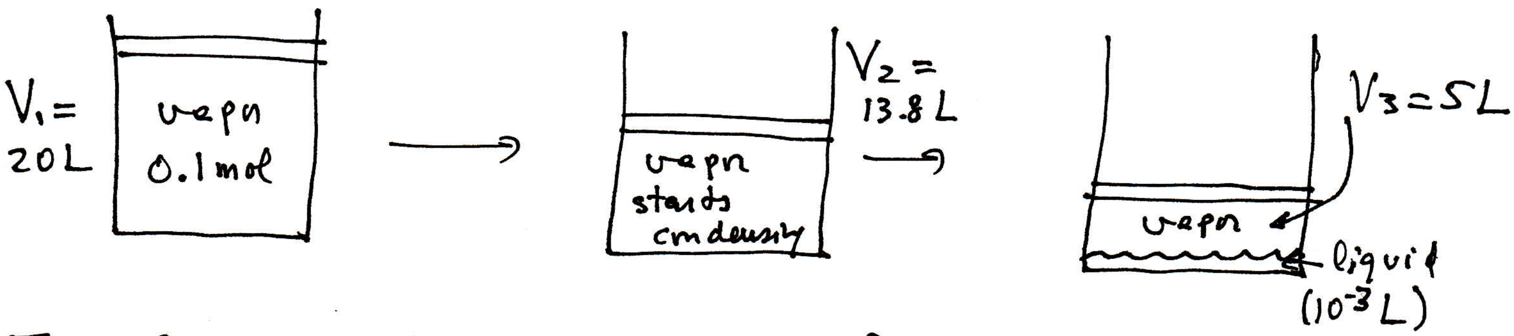
hot body released $Q_1 = (3T_0 - T_f)C$, cold body absorbed $Q_2 = (T_f - T_0)C$

$\Rightarrow$ work done was $W = Q_1 - Q_2 = C(4T_0 - 2T_f) = Q_0(4 - 2\sqrt{3})$

$$\Rightarrow \boxed{W = (4 - 2\sqrt{3})\, Q_0 = 0.536\, Q_0} \text{ (b)}$$

(d) $\boxed{\text{Change in entropy in environment is zero}}$

Problem 4



$T = 60°C = 333\,K$ $\qquad P_v = 2\times 10^4\,Pa$

Ignoring the volume occupied by water which is very small

$$P_v V_3 = n_{vapor}\cdot R\cdot T \Rightarrow n_{vapor} = \frac{2\times10^4 \times 5\times10^{-3}}{8.314\times 333} = 0.036$$

So we have $\boxed{0.036 \text{ mol vapor}, \ 0.064 \text{ mol liquid}}$ (a)

Check: liquid $= 1.15g = 1.15\,cm^3 = 10^{-3}L$

(b) First find volume V_2:

$$V_2 = \frac{nRT}{P_v} = \frac{0.1\times 8.314\times 333}{2\times10^4}\,m^3 = 0.0138\,m^3 = \boxed{13.8\,L}$$

Work done in compressing from V_1 to V_2:

$$W_{12} = \int_{V_2}^{V_1} P\,dV = nRT\ln\frac{V_1}{V_2} = 0.1\times8.314\times333\times\ln\frac{20}{13.8}\,J$$

$\Rightarrow \boxed{W_{12} = 102.7\,J}$. Then, in compressing from V_2 to V_3:

$$W_{23} = P_v(V_2 - V_3) = 2\times10^4\times(20-13.8)\times10^{-3}\,J = 124\,J$$

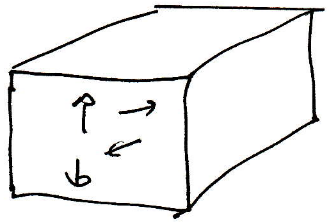
$\Rightarrow \boxed{W_{23} = 124J} \Rightarrow$ total work (b) $\boxed{W = W_{12} + W_{23} = 226.7J}$

(c) latent heat is $L_v = 130\,J/mol$, 0.064 mol condenses, $L_v\cdot n_{cond} = 8.32J$

Total heat absorbed by heat reservoir $= W + L_v\cdot n_{cond} = \boxed{235.02\,J}$ (c)

because internal energy of ideal gas only depends on T, so doesn't change.

Problem 5



$v_x^{\circ} = 350\ \text{m/s} = v_y^{\circ} = \dots$

Total kinetic energy doesn't change.

$K_{tot} = N \cdot \frac{m}{2}(v_x^{\circ})^2$. In thermal equilibrium, kin. energy per molecule is $\frac{3}{2}kT \Rightarrow \frac{m}{2}(v_x^{\circ})^2 = \frac{3}{2}kT \Rightarrow \boxed{T = \frac{(v_x^{\circ})^2 m}{3k} = 98.3\,K}$

(b) $g(v) = Cv^2 e^{-mv^2/2kT}$, most probable v is $g'(v_m) = 0$

$$\Rightarrow 2v_m - v_m^3 \cdot \frac{2m}{2kT} = 0 \Rightarrow v_m^2 = \frac{2kT}{m} = \frac{2}{3}(v_x^{\circ})^2 \Rightarrow$$

$$\Rightarrow \boxed{v_m = \sqrt{\frac{2}{3}}\, v_x^{\circ} = 286\ \text{m/s}}$$

(c) Adiabatic compression: $TV^{\gamma-1} = \text{const}$, $\gamma = \frac{5}{3} \Rightarrow$

$$T_2 = T_1\left(\frac{V_1}{V_2}\right)^{\gamma-1} = T_1 \cdot 2^{2/3} \Rightarrow \boxed{T_2 = 156\,K}$$

So most probable speed is now $\boxed{v_m' = 2^{1/3} v_m = 360\ \text{m/s}}$

(d) For diatomic gas, energy is shared by 5 degrees of freedom

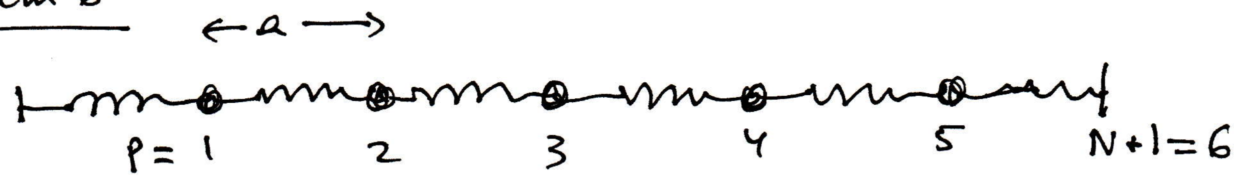
$$\frac{5}{2}kT' = \frac{1}{2}m(v_x^{\circ})^2 \Rightarrow \boxed{T' = \frac{2}{5} \cdot \frac{\frac{1}{2}m(v_x^{\circ})^2}{k} = 59\,K} = \frac{3}{5}T$$

$$\boxed{v_m'' = \sqrt{\frac{3}{5}}\, v_m = 222\ \text{m/s}}$$

(e) Now, $\gamma = \frac{7}{5} \Rightarrow \gamma - 1 = \frac{2}{5} \Rightarrow \boxed{T_2 = T' \cdot 2^{\frac{2}{5}} = 78\,K}$

$$\boxed{v_m''' = 2^{1/5} \cdot v_m'' = 255\,K}$$

Problem 6



$$\xi_{pn} = C_n \sin\left(\frac{\pi}{N+1} p n\right) \cos(\omega_n t)$$

For $\xi_{p=3} = 0$: $\sin\left(\frac{3\pi}{6} n\right) = \sin\left(\frac{\pi}{2} n\right) = 0 \Rightarrow \boxed{n = 2 \text{ or } n = 4}$

For $n=2$: $\xi_{1,2} = \sin\left(\frac{\pi}{3}\right)$, $\xi_{2,2} = \sin\left(\frac{2\pi}{3}\right)$ same sign

For $n=4$: $\xi_{1,4} = \sin\left(\frac{2\pi}{3}\right)$, $\xi_{2,4} = \sin\left(\frac{4\pi}{3}\right)$ opposite sign

Period: $\frac{T_2}{T_1} = \frac{\omega_1}{\omega_2}$; $\omega_n = 2\omega_0 \sin\left(\frac{\pi}{2(N+1)} n\right)$

$\Rightarrow \omega_n = 2\omega_0 \sin\left(\frac{\pi}{12} n\right) \Rightarrow \boxed{T_2 = \frac{\sin\left(\frac{\pi}{12}\right)}{\sin\left(\frac{\pi}{6}\right)} T_1 = 5.2\,s}$ (a)

$\boxed{T_4 = \frac{\sin\left(\frac{\pi}{12}\right)}{\sin\left(\frac{\pi}{3}\right)} T_1 = 2.99\,s}$ (b)

Wavelengths: $\sin\left(\frac{\pi}{N+1} p n\right) = \sin\left(\frac{2\pi}{2(N+1)a} n\, pa\right) = \sin\left(\frac{2\pi}{\lambda_n} pa\right) \Rightarrow$

$\Rightarrow \lambda_n = \frac{2(N+1)a}{n} = \frac{12a}{n} \Rightarrow \boxed{\lambda_2 = 6a, \quad \lambda_4 = 3a}$ (c)

Speed of traveling waves: $v_n = \lambda_n / T_n \Rightarrow$

$\boxed{v_2 = \frac{6}{5.2s} a = 1.15 a/s}$ $\boxed{v_4 = \frac{3a}{2.99s} = 1.00 a/s}$ (d)

They are not identical because we don't have $n \ll N$ so there is some dispersion. They are close because $n_1, n_2 < N$ and $n_1 - n_2 \ll N$

Problem 7

Fundemental frequency: $f_1 = \frac{v}{2L} \Rightarrow 2Lf_1 = v$

$$v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{TL}{m}}$$

Note that when we put finger to make L shorter, $\mu = m/L$ should not change (i.e. m also changes). v doesn't change. So

$$2Lf_1 = 2(L - 0.02m) f_2 \Rightarrow L(f_2 - f_1) = 0.02 f_2 m \Rightarrow$$

$$\Rightarrow L = \frac{0.02 f_2}{f_2 - f_1} m = \frac{2 \times 513}{13} \Rightarrow \boxed{L = 78.9 cm}$$

$$v = 2Lf_1 = 0.789 \times 2 \times 500 \, m/s \Rightarrow \boxed{v = 789 \, m/s}$$

$$\frac{T}{m} L = v^2 \Rightarrow m = \frac{TL}{v^2} = \frac{800 \times 0.789}{789^2} kg = 0.0010 kg$$

$$\Rightarrow \boxed{m = 1g}$$

Problem 8

Sound wave: $\xi = A\sin(kx - \omega t)$

$\lambda = 10\,cm$, $v = 500\,m/s$, $A = 10^{-8}\,m$, $\rho = 700\,kg/m^3$

(a) $v = \frac{\omega}{k}$, $k = \frac{2\pi}{\lambda} \Rightarrow \omega = \frac{2\pi}{\lambda} v = \frac{500 \times 2\pi}{0.1} \frac{rad}{s} \Rightarrow$

$\boxed{\omega = 31{,}416\ rad/s}$ (a)

(b) $\dot{\xi} = -\omega A \cos(kx - \omega t)$ max speed is for cos = −1

$|\dot{\xi}_{max}| = \omega A = 31{,}416 \times 10^{-8}\ \frac{m}{s} = \boxed{0.3\ mm/s}$ (b)

(c) Pressure variation:

$\Delta P = B \frac{d\xi}{dx}$, B = bulk modulus

$v = \sqrt{\frac{B}{\rho}} \Rightarrow B = v^2 \rho = 500^2 \times 700 \frac{N}{m^2} = \boxed{1.75 \times 10^8 \frac{N}{m^2} = B}$

$\frac{d\xi}{dx} = kA\cos(kx - \omega t) \Rightarrow$ maximum pressure variation is

$\Delta P_M = B \cdot \frac{2\pi}{\lambda} \cdot A = 1.75 \times 10^8 \cdot \frac{2\pi}{0.1} \times 10^{-8}\ Pa = \boxed{110\ Pa = \Delta P_M}$ (c)

(d) $I = 2\pi^2 v \rho f^2 A^2 = 2\pi^2 \cdot 500 \times 700 \times 5000^2 \times 10^{-16}\ W/m^2$

$\Rightarrow \boxed{I = 0.017\ W/m^2}$ (d) used that $\boxed{f = \frac{v}{\lambda} = \frac{500\,m/s}{0.1\,m} = 5000\,Hz}$

(e) Bulk modulus $B = \gamma P$, $\gamma = \frac{5}{3}$ for monatomic $\Rightarrow P = \frac{3}{5} B$

$\Rightarrow \boxed{P = 1.05 \times 10^8\ Pa}$ (e)