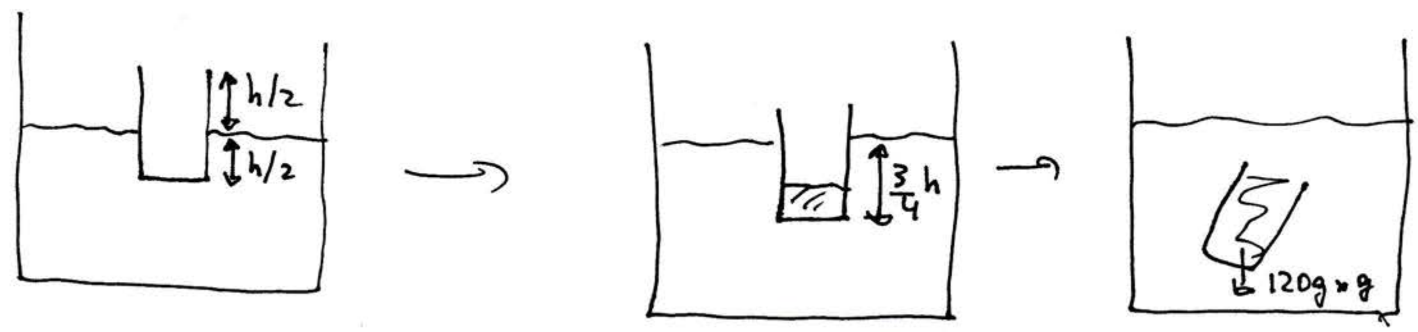
Physics 4B — Quiz 1 Solutions — Spring 2018

Problem 1 $h = 15\text{ cm}$ = height of glass



(a) Let A = cross-section of the glass, m = its mass

Initially, $mg = \rho_{H_2O} \cdot A \cdot \frac{h}{2} g \Rightarrow \frac{3}{2} m = \frac{3}{2} \rho_{H_2O} A \cdot \frac{3}{4} h$

then, $m + 100g = \rho_{H_2O} \cdot A \cdot \frac{3}{4} h$

$\Rightarrow m + 100g = \frac{3}{2} m \Rightarrow \boxed{m = 200g}$ mass of the glass

(b) $A = \frac{2m}{h \cdot \rho_{H_2O}} = \frac{400g}{15 g/cm^2} = \frac{400}{15} cm^2 = \pi r^2 \Rightarrow \boxed{r = 2.9 cm}$

(c) The total volume of the glass including the interior is

$$V = A \cdot h = \frac{2m}{\rho_{H_2O}} = 400 cm^3$$

If V_{glass} is the volume occupied by the glass walls of the glass, the buoyant force when the glass is full of water is

$F_B = \rho_{H_2O} \cdot V_{glass} \cdot g = (200g - 120g) g \Rightarrow$

$\Rightarrow V_{glass} = \frac{80g}{\rho_{H_2O}} = \boxed{80 cm^3}$ So the volume of water to fill the glass is $\boxed{V_{H_2O} = 400 cm^3 - 80 cm^3 = 320 cm^3}$

(d) The density of the glass is

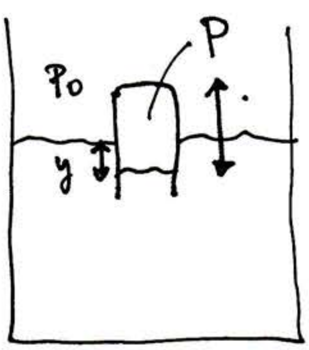
$$\rho_{glass} = \frac{200\,g}{80\,cm^3} = \boxed{2.5\,g/cm^3}$$

(e) If d is the average thickness of the glass:

$$V_{glass} = 2\pi r \cdot h \cdot d + \pi r^2 \cdot d = d\,(2\pi r h + \pi r^2)$$

$$\Rightarrow\quad d = \frac{V_{glass}}{\pi r(2h+r)} = \frac{80\,cm^3}{\pi \cdot 2.9\,cm\,(20.8\,cm)} = \boxed{4mm}$$

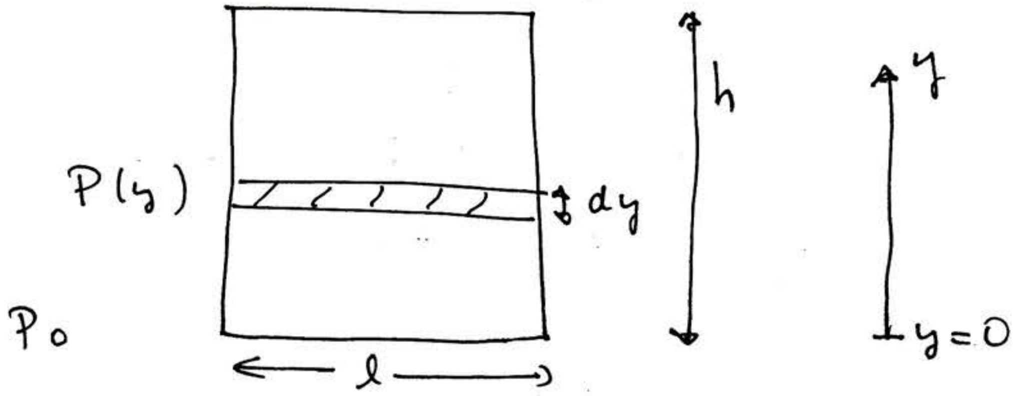
(f) If we put glass upside down:



some water gets in, air gets compressed,
$P = P_0 + \rho g y > P_0 \Longrightarrow$

buoyant force is smaller, so the glass is $\boxed{\text{more submerged}}$ than when it was upright.

Problem 2



(a) $F_0 = P_0 l h = 1.013\times10^5 \times 10 \times 100\, N = \boxed{1.013\times10^8 N}$

so $\boxed{F_0 = P_0 l h = 1.013\times10^8 N}$

(b) $P(y) = P_0 e^{-(\rho_0 g/P_0)y} = P_0 e^{-y/y_0}$

$y_0 = \frac{P_0}{\rho_0 g} = \frac{1.013\times10^5}{1.29\times 9.80}\, m = \boxed{8013 m}$

$dF = P(y)\, l\, dy = P_0 l e^{-y/y_0} \Rightarrow$

$\Rightarrow F = l \int_0^h P(y) dy = P_0 l \int_0^h dy\, e^{-y/y_0} = P_0 l y_0 (1 - e^{-h/y_0})$

$1 - e^{-h/y_0} = 1 - e^{-100/8013} = 0.0124$

$F = P_0 l y_0 \times 0.0124 = P_0 l h \times \frac{y_0}{h} \times 0.0124 = 1.00653\times10^8 N$

$\boxed{F - F_0 = -647{,}104 N}$

(c) Using Taylor: $1 - e^{-h/y_0} = 1 - (1 - \frac{h}{y_0} + \frac{1}{2}\frac{h^2}{y_0^2}) = \frac{h}{y_0} - \frac{1}{2}\frac{h^2}{y_0^2}$

$F = P_0 l y_0 \left(\frac{h}{y_0} - \frac{1}{2}\frac{h^2}{y_0^2}\right) = P_0 l h \left(1 - \frac{h}{2y_0}\right) \Rightarrow \boxed{F - F_0 = -F_0 \cdot \frac{h}{2y_0} = -632{,}097 N}$

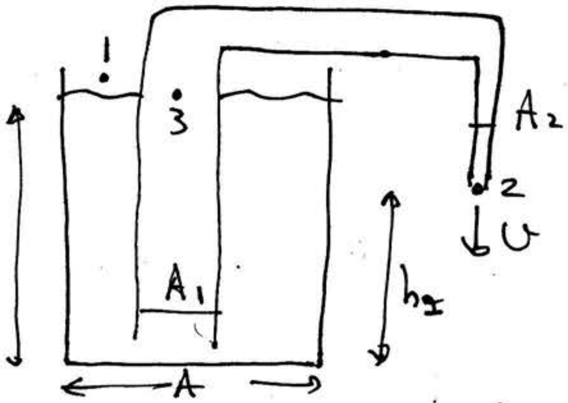
Alternatively, expanding $P(y)$ using Taylor

$$P(y) = P_0 e^{-y/y_0} = P_0 \left(1 - \frac{y}{y_0}\right)$$

$$F = l \int_0^h P(y)\, dy = l P_0 h - \frac{l P_0}{y_0} \int_0^h y\, dy =$$

$$= l P_0 h - \frac{l P_0}{y_0} \frac{h^2}{2} = \boxed{P_0 l h \left(1 - \frac{h}{2 y_0}\right)}$$ same answer

Problem 3



$A = 3m^2$

$h = 10m$

$h_2 = 6m$

$A_2 = 5cm^2$, $A_1 = 10cm^2$

$P + \rho g y + \frac{1}{2}\rho v^2 = \text{const.}$ Bernoulli-eq.

(a) Using B. eq. in points 1 and 2

$$P_0 + \rho \cdot g \cdot 10m = P_0 + \rho \cdot g \cdot 6m + \frac{1}{2}\rho v^2 \Rightarrow$$

$$\Rightarrow \boxed{v = \sqrt{2g \times 4m} = 8.85 m/s}$$

(b) Using B. eq. in points 1 and 3

$$P_0 = P_3 + \frac{1}{2}\rho v_3^2 \Rightarrow P_3 = P_0 - \frac{1}{2}\rho v_3^2$$

By continuity: $A_1 v_3 = A_2 \cdot v \Rightarrow \boxed{v_3 = \frac{A_2}{A_1} v = 4.43 m/s}$

$$P_3 = 1.013 \times 10^5 Pa - \frac{1}{2} \times 1000 \times 4.43^2 Pa = \boxed{9.15 \times 10^4 Pa}$$

(c) The mass flow rate through 2 is:

$$\frac{dm}{dt} = \rho \cdot A_2 \cdot v \Rightarrow m = \rho \cdot A_2 \cdot v \cdot t \quad \text{(ignoring changes in } v\text{)}$$

Total mass that flows out before stopping: $m = A \cdot 4m \cdot \rho = 12{,}000\ kg$

$$t = \frac{m}{\rho A_2 v} = \frac{12{,}000}{1000 \times 5 \times 10^{-4} \times 8.85} s = 2712 s = \boxed{45 \text{ minutes}}$$

Exact answer: take into account that v gets smaller as height drops:

$$v = \sqrt{2g(4-y)} \qquad y = \text{how much height drops}$$

$$dt = \frac{dm}{\rho A_2 \sqrt{2g(4-y)}} \quad ; \quad dm = \rho . A . dy \ , \ A = 3m^2$$

$$\Rightarrow \quad dt = \frac{A}{A_2} \frac{dy}{\sqrt{2g(4-y)}} = \frac{A}{A_2\sqrt{2g}} \frac{dy}{\sqrt{4-y}} \Rightarrow$$

$$t = \frac{A}{A_2\sqrt{2g}} \left(-2\sqrt{(4-y)} \right]_0^4 = \frac{4A}{A_2\sqrt{2g}} = 5424s = \boxed{90 \text{ minutes}}$$