Physics 4B — Quiz 2 Solutions — Spring 2018

Problem 1

(a) The 10 g of ice occupy volume $10\,cm^3 = 0.01\,L$ ⇒ we can ignore it compared with 1.5 L.

1 mol of air occupies 22.4 L ⇒ we have $n = \frac{1.5}{22.4} = 0.067$ mols of air

~~atomic~~ molecular weight = 29 ⇒ **1.94 g of air**

(b) Relative humidity is **100%** because lid is sealing it

(c) Heating up to 100°C = 373 K we have

$$P_{air} = P_{air}(0°C) \times \frac{373}{273} = 1.37\,atm$$

The water vapor has saturated vapor pressure = 1 atm ⇒

$$\boxed{P_{total} = 2.37\,atm}$$

(d) We have water vapor at pressure 1 atm, volume 1.5 l, T = 373 K

$$\Rightarrow n_{vapor} = \frac{1.5}{22.4} \times \frac{273}{373} = 0.049\,mol$$

molecular weight = 18 ⇒ **0.88 g of water vapor**

⇒ **9.12 g of liquid water**

Problem 2

$v_{rms} = 600\, m/s \quad ; \quad \overline{v^2} = v_{rms}^2$

$E_{kin} = \frac{1}{2} m \overline{v^2} = \frac{3}{2} kT \Rightarrow T = \frac{m\overline{v^2}}{3k}$

$\Rightarrow T = \frac{20 \times 1.66 \times 10^{-27}}{3 \cdot 1.38 \times 10^{-23}} \times 600^2\, K \Rightarrow \boxed{T = 289\, K}$ (a)

(b) $g(v) = C v^2 e^{-\frac{1}{2}\frac{mv^2}{kT}}$ use that $\boxed{\frac{m}{kT} = \frac{3}{v_{rms}^2}}$

$\frac{g(v_1)}{g(v_2)} = \frac{v_1^2}{v_2^2} e^{-\frac{3}{2 v_{rms}^2}(v_1^2 - v_2^2)}$

$v_1 = 1200\, m/s = 2 v_{rms}$

$v_2 = 600\, m/s = v_{rms}$

$\Rightarrow \frac{3}{2 v_{rms}^2}(v_1^2 - v_2^2) = \frac{3}{2}(4-1) = \frac{9}{2} \Rightarrow$

$\frac{g(v_1)}{g(v_2)} = 4 e^{-9/2} = 0.0444$. Now the interval $dv_1 = 10\, m/s$,

$dv_2 = 5\, m/s \Rightarrow$ **89 molecules with v between 1200 m/s and 1210 m/s**

(c) $f(v_x) = C e^{-\frac{m}{2kT} v_x^2}$

$\frac{f(v_{1x})}{f(v_{2x})} = \frac{f(0)}{f(v_{rms})} = e^{\frac{m}{2kT} v_{rms}^2} = e^{1.5} = 4.482$

$\Rightarrow$ **4,482 molecules with v_x between −1 m/s and 1 m/s**

(d) The force exerted in 1 collision is

$$F_1 = \frac{\Delta(mv)}{\Delta t} = \frac{2 m v_x}{\Delta t} \quad ; \quad \Delta t = \frac{2l}{v_x}$$, with l the length

of the container in that direction $\Rightarrow$ $F_1 = \frac{m \overline{v_x^2}}{l}$

We have 1 mol = 6.02×10^{23} molecules, force = 9000N $\Rightarrow$

$$l = \frac{N_A\, m v_x^2}{9000N} \qquad m v_x^2 = kT = 1.38 \times 10^{-23} \times 289\, J \Rightarrow$$

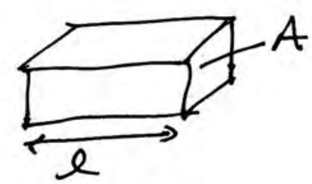
$$l = \frac{6.02 \times 10^{23} \times 1.38 \times 10^{-23} \times 289}{9000}\, m = 0.267\, m = \boxed{27\, cm}$$

The time between 2 collisions is $\Delta t = \frac{2l}{v_x}$.

estimating $v_x \sim v_{rms}$, $\boxed{\Delta t = \frac{0.267}{600}\, s = 0.0004\, s = 0.4\, ms}$

Alternative derivation:

$$F = P \cdot A = \frac{N_A k T}{V} \cdot A = \frac{N_A k T}{l}$$



$$\Rightarrow l = \frac{N_A k T}{F} = 27\, cm$$

Problem 3

$$\rho = \frac{m}{V} \quad , \quad \frac{\Delta \rho}{\rho} = - \frac{\Delta V}{V}$$

$$\frac{\Delta V}{V} = 3\alpha \Delta T = 3 \times 12 \times 10^{-6} \times 40 = 1.44 \times 10^{-3}$$

$$\Rightarrow \Delta \rho = -1.44 \times 10^{-3} \times 2{,}300 \frac{kg}{m^3} = -3.3 \frac{kg}{m^3}$$

$$\Rightarrow \boxed{\rho(50^\circ C) = 2{,}297\ kg/m^3}$$

(b) $\frac{F}{A} = E \cdot \frac{\Delta l}{l}$; $\frac{\Delta l}{l} = \alpha \Delta T = 4.8 \times 10^{-4}$

$$\Rightarrow \frac{F}{A} = E \alpha \Delta T = 20 \times 10^{9} \times 12 \times 10^{-6} \times 40 \frac{N}{m^2} = \boxed{9.6 \times 10^{6} N}$$

(c) For the F/A to be $3 \times 10^{6}\ N/m^2$, we need

$$\frac{\Delta l}{l} = 12 \times 10^{-6} \times 40 \times \frac{3}{9.6} = 1.5 \times 10^{-4}$$

so we need a spacer with $\frac{\Delta l}{l} = (4.8 - 1.5) \times 10^{-4} = 3.3 \times 10^{-4}$

with $l = 3\,m$ $\Rightarrow$ $\boxed{\Delta l = 0.99\,mm \sim 1\,mm \text{ space between blocks}}$