Physics 4B — Quiz 3 Solutions — Spring 2018

Problem 1 $C_{H_2O} = 1\,\text{cal/g°C}$, $C_{ice} = 0.5\,\text{cal/g°C}$, $L_F = 79.7\,\text{cal/g}$

Heat released in cooling 10g of H_2O from 10°C to 0°C:

$$Q_{H_2O} = 1\,\frac{\text{cal}}{\text{g°C}} \times 10\text{g} \times 10\text{°C} = 100\,\text{cal}$$

Heat released in freezing 10g of H_2O

$$Q_{freeze} = mL_F = 79.7 \times 10\,\text{cal} = 797\,\text{cal}$$

Heat to raise T of ice from −10°C to 0°C

$$Q_{ice} = M_{ice}\,C_{ice} \cdot \Delta T = 100\text{g} \times 0.5\,\frac{\text{cal}}{\text{g°C}} \times 10\text{°C} = \boxed{500\,\text{cal}}$$

Conclusion: in the final state, water and ice will coexist

$$\Rightarrow \boxed{T_{final} = 0\text{°C}} \quad \text{(a)}$$

(b) x grams of water will freeze, so that

$$x \cdot L_F + Q_{H_2O} = Q_{ice} \Rightarrow x \times 79.7 + 100 = 500 \Rightarrow$$

$$\Rightarrow \boxed{x = \frac{400}{79.7}\,\text{g} = 5\text{g}}$$

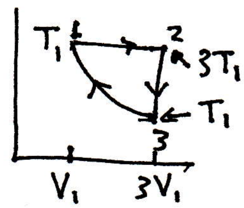
$\Rightarrow$ total ice in final state: $\boxed{105\,\text{g}}$

Problem 2

(a) The work is $W = \int P\,dV$; $PV = RT$ for 1 mol

$$W_{12} = P_1(V_2 - V_1) = P_1(3V_1 - V_1) = 2P_1V_1 = 2RT_1$$

$$W_{23} = 0$$



$$W_{31} = \int_{V_3}^{V_1} \frac{RT_1}{V}\,dV = RT_1 \ln\frac{V_1}{V_3} = -RT_1 \ln 3$$

$\Rightarrow$ total net work in cycle: $\boxed{W = RT_1(2 - \ln 3) = 0.90\,RT_1}$

(b) The system absorbs heat in 12 because: it is performing positive work, its temperature is increasing, hence internal energy is increasing, from $\Delta E_{int} = Q - W$ we conclude Q_{12} is positive.

The system releases heat in 23 because it is doing no work, its temperature is decreasing, hence $E_{int} < 0$, hence $Q_{23} < 0$

The system releases heat in 31 because its internal energy is not changing since T is constant, so $Q_{31} = W_{31}$, and $W_{31} < 0$ so $Q_{31} < 0$.

(c) Heat absorbed in cycle is Q_{12}, process at constant pressure

$$Q_{12} = C_p(T_2 - T_1)\ ;\ T_2 = 3T_1\ ;\ C_p = \frac{5}{2}R \text{ for monatomic gas}$$

$\Rightarrow$ $\boxed{Q_{12} = 5RT_1}$

(d) $\boxed{e = W/Q_{13} = 0.9/5 = 0.18}$

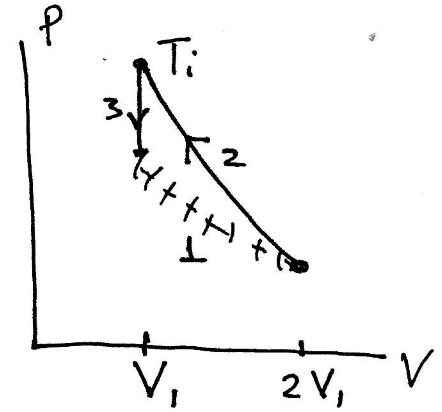
(e) Lowest T is T_1, highest is $3T_1$. $\boxed{e_{carnot} = 1 - \frac{T_1}{3T_1} = 0.66 > e = 0.18}$

(f) Heat in 23: $Q_{23} = -C_v(T_3 - T_2) = -2T_1 C_v = \boxed{-3RT_1 = Q_{23}}$

$\boxed{Q_{31} = W_{31} = -RT_1 \ln 3}$ $\boxed{Q_{tot} = Q_{12} + Q_{23} + Q_{31} = (2 - \ln 3)RT_1 = W}$

since $\Delta E_{int} = 0$ in cycle

Problem 3



(a) Process 1: free expansion.

T doesn't change. So for gas, $dQ = dW$,

$$\Delta S_1 = \int \frac{dQ}{T} = \int_{V_i}^{V_f} \frac{RT}{V \cdot T} dV = R \ln \frac{V_f}{V_i}$$

$\Rightarrow$ $\boxed{\Delta S_1 = R \ln 2}$

No heat is exchanged with environment $\Rightarrow$ $\boxed{\Delta S_1^{env} = 0}$

(b) Process 2: adiabatic compression

$\boxed{\Delta S_2 = 0}$ for gas, since no heat is exchanged

$\boxed{\Delta S_2^{env} = 0}$ for environment, " " " " " "

(c) At constant volume we have for the gas, $W = 0$ $\Rightarrow$

$dE_{int} = C_V \, dT = dQ$ $\Rightarrow$

$$\Delta S_3 = \int_{T_i}^{T_f} C_V \frac{dT}{T} = C_V \ln \frac{T_f}{T_i}$$

. $T_f = T_1$. To find the T_i,

use $TV^{\gamma-1} =$ constant along adiabat $\Rightarrow$ $T_1 (2V_1)^{\gamma-1} = T_i V_1^{\gamma-1}$ $\Rightarrow$

$\Rightarrow$ $\boxed{T_i = 2^{\gamma-1} T_1}$ $\Rightarrow$ $\Delta S_3 = C_V \ln 2^{\gamma-1}$; $C_V = \frac{3}{2} R$, $\gamma - 1 = \frac{2}{3}$

$\Rightarrow$ $\Delta S_3 = -\frac{3}{2} R \cdot \frac{2}{3} \cdot \ln 2 = \boxed{-R \ln 2 = -\Delta S_3}$

Or, we could have found this from $\Delta S_1 + \Delta S_2 + \Delta S_3 = 0$ for cycle.

For environment, $\Delta S_3^{env} = \frac{Q}{T_1}$, $Q = C_V (T_i - T_1) = \frac{3}{2} R (2^{\gamma-1} - 1) T_1$

$\Rightarrow$ $\boxed{\Delta S_3^{env} = \frac{3}{2} (2^{2/3} - 1) R = 0.88 R}$

(d) $\boxed{\Delta S_{univ}(1,2,3) = \Delta S_3^{env} = 0.88 R}$ $\boxed{\text{since } \Delta S^{gas} = 0 \text{ for a cycle, whether reversible or not}}$