

Problem 1 (10 pts)

Consider a system of 10 dice, all of different colors. Each dice can show 1 to 6 dots on its upper surface. Define the macrostate of the system by the variable:

N = sum of the dots on the upper surface of all the dice.

Consider the entropy $S(N)$ of this system. Assume the numerical value of Boltzmann's constant is $k=1$.

- What is the numerical value of $S(10)$?
- What is the numerical value of $S(11)$?
- What is the numerical value of $S(12)$?
- For which value of N is $S(N)$ maximum?
- Assume the system is in the macrostate $N=12$ initially and you shake it for a long time. Find the probability that the final macrostate will be $N=11$.

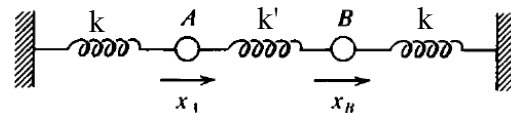
Problem 2 (10 pts + 3 pts extra credit)

A child that weighs 25kg is using a swing of chain length 3m. It takes 10 oscillations for the swing amplitude to decay to half its initial value.

- What is approximately the period of oscillation, in seconds? (you may assume no damping, since it is small).
- What is the Q of this oscillator? ($Q=\omega_0/\gamma$, $\gamma=b/m$, bv =damping force, m =mass).
- Verify that the approximation you used in (a) is good by comparing the frequencies with and without damping.
- How much energy (in J) is dissipated in 10 oscillations if the initial angle was 30° ?
- Assume the child is pumping with her legs to keep the swinging at 30° angle. What is approximately the power she has to supply? Give your answer in Watts.

Hints: (i) assume pumping frequency=natural frequency; (ii) there is more than one way to solve (e); (iii) the instantaneous power supplied is (damping force) \times velocity; (iv) assuming 30° is a "small" angle is ok.

Problem 3 (10 pts)



Consider two harmonic oscillators of mass m and spring constant k . When they are uncoupled, their period is 6 seconds. When they are coupled with a spring of spring constant k' , the ratio of the two normal mode frequencies is 3.

- Find k' in terms of k .
- If the system is started with the two masses displaced equally in opposite directions from their equilibrium positions and zero velocities, what is the earliest time (in seconds) for which their displacement is the same?
- If the system is started with the two masses displaced equally in the same direction and zero velocities, what is the earliest time (in seconds) for which one of the masses has zero displacement relative to the equilibrium position?