Physics 4B | Quiz 4 Solutions | Spring 2018

Problem 1

$S = k \ln W$, W = # of microstates for a given macrostate

take $k = 1$. 10 dice. $N = \sum$ dots on upper surface

(a) $N = 10$. The only microstate with $N = 10$ is with all dice showing 1 dot $\Rightarrow$ $W(N=10) = 1$ $\Rightarrow$

$\Rightarrow$ $\boxed{S(10) = 0}$

(b) $N = 11$ $\Rightarrow$ 9 dice shows 1, one dice shows 2. There are 10 microstates $\Rightarrow$ $\boxed{S(11) = \ln 10 = 2.30}$

(c) $N = 12$: we can have $N = 12$ as follows:

(i) 9 dice show 1, one dice shows 3: 10 microstates

(ii) 8 dice show 1, two show 2: $\frac{10 \times 9}{2} = 45$ microstates

total $= 55 = W(N=12)$ $\Rightarrow$ $\boxed{S(N=12) = \ln 55 = 4.01}$

(d) Maximum $N = 60$, minimum $N = 10$, average is 35.

Clearly for $N = 35$ there will be more microstates than for any other N $\Rightarrow$ $S(N=35)$ is maximum.

(e) The final state after shaking can be any microstate that is possible. There are 6^{10} possible microstates. Out of those, 10 microstates have $N = 11$. So the probability that the final macrostate is $N = 11$ is $\boxed{\frac{10}{6^{10}} = 0.000000165}$

Problem 2

For pendulum, $w_0 = \sqrt{\frac{g}{l}} = \sqrt{\frac{9.8}{3}}$ rad/s $= 1.81$ rad/s

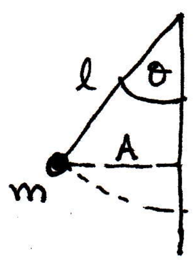
Period $T = \frac{2\pi}{w_0} \Rightarrow \boxed{T = 3.48\,s}$ (a)

(b) The amplitude as function of time is $A(t) = A_0 e^{-\frac{\gamma}{2}t}$.

So $e^{-\frac{\gamma}{2}\cdot 10T} = \frac{1}{2} \Rightarrow 5\gamma T = \ln 2 \Rightarrow \boxed{\gamma = 0.04\,s^{-1}}$

(c) $w = \sqrt{w_0^2 - \left(\frac{\gamma}{2}\right)^2} \cong w_0 = 1.81$ rad/s

(d)



The potential energy of the pendulum at angle θ is $U = -mgl(\cos\theta - 1) = mgl(1 - \cos\theta)$

After 10 oscillations, amplitude decreased from $\theta_0 = 30°$ to $\theta_1 = 15°$. The difference in potential energy is dissipated.

$$\Delta E = mgl(\cos 15° - \cos 30°) = 0.1\,mgl$$

$\Rightarrow \Delta E = 0.1 \times 25 \times 9.8 \times 3$ J $\Rightarrow \boxed{\Delta E = 73.5\,J}$

Or, approximately $U(\theta) = \frac{1}{2} mgl\theta^2$ gives $\boxed{\Delta E = 75.6\,J}$

(e) For a <u>very rough</u> answer one could say: it is dissipated 75 J in 10 oscillations, that gives 7.5 J per oscillation, so pumping has to supply 7.5 J of energy in T = 3.48 s $\Rightarrow$ 7.5/3.48 Watts = 2.2 W. But it requires more energy when the amplitude is larger, to keep it oscillating at 30° takes 4 times more energy than at 15°. So on average, about twice as much to keep it at 30°, i.e. ~4 Watts.

More accurately:

In 1 period, amplitude decreases from A_0 to $A_0 e^{-\frac{\gamma}{2}T}$

So $\delta(A^2) = A_0^2(1 - e^{-\gamma T}) \cong A_0^2 \gamma T$

energy is $E = \frac{1}{2} m \omega_0^2 A^2 \Rightarrow$

$\Rightarrow \delta E = \frac{1}{2} m \omega_0^2 \delta A^2 = \frac{1}{2} m \omega_0^2 A_0^2 \gamma T$

To keep it oscillating at initial amplitude, pumping has to supply δE energy per oscillation (time T), hence

$$\boxed{\text{power} = \frac{\delta E}{T} = \frac{1}{2} m \omega_0^2 A_0^2 \gamma}$$

Alternative argument: P = damping force × velocity $\Rightarrow$

$\Rightarrow P = b v^2 = \gamma m v^2$

$x(t) = A\cos(\omega t)$, $v = \omega A \sin(\omega t)$, $\overline{v^2} = \frac{1}{2}\omega^2 A^2 \Rightarrow$

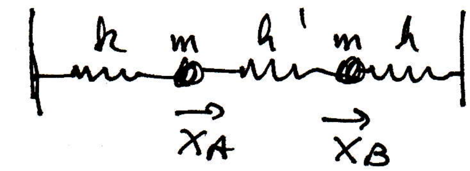
$P = \gamma m \overline{v^2} = \frac{1}{2}\gamma m \omega^2 A^2$, as above $(\omega = \omega_0)$.

Quantitatively:

initial energy is $E_0 = \frac{1}{2} m \omega_0^2 A_0^2 = mg\ell(1 - \cos\theta_0)$

$\Rightarrow E_0 = 98.5\,\text{J}$, $\boxed{P = E_0\gamma = 3.94\,\text{W}}$

Problem 3



Fn 2 coupled oscillators,

$$\ddot{x}_A = -\frac{k}{m}x_A - \frac{k'}{m}(x_A - x_B)$$

$$\ddot{x}_B = -\frac{k}{m}x_B - \frac{k'}{m}(x_B - x_A)$$

$\omega_0 = \sqrt{\frac{k}{m}}$, $\omega_c = \sqrt{\frac{k'}{m}}$. Normal $\overbrace{\text{frequencies}}^{\text{modes}}$ are

$$\omega_1^2 = \omega_0^2, \quad \omega_2^2 = \omega_0^2 + 2\omega_c^2$$

$$\frac{\omega_2^2}{\omega_1^2} = 1 + 2\frac{\omega_c^2}{\omega_0^2}; \text{ if } \omega_2/\omega_1 = 3 \Rightarrow$$

$$1 + 2\frac{\omega_c^2}{\omega_0^2} = 9 \Rightarrow \frac{\omega_c^2}{\omega_0^2} = 4 \Rightarrow \frac{\omega_c}{\omega_0} = 2.$$

$$k'/k = \omega_c^2/\omega_0^2 \Rightarrow \boxed{k' = 4k}$$

(b) ~~Fo~~ $x_A(t) = C_1\cos(\omega_1 t) + C_2\cos(\omega_2 t)$

$x_B(t) = C_1\cos(\omega_1 t) - C_2\cos(\omega_2 t)$

If $x_A(0) = -x_B(0) \Rightarrow C_1 = 0$. Fn $x_A(t) = x_B(t) \Rightarrow$

$C_2\cos(\omega_2 t) = -C_2\cos(\omega_2 t) \Rightarrow \omega_2 t = \frac{\pi}{2} \Rightarrow$

$\Rightarrow t = \frac{\pi}{2\omega_2} = \frac{\pi}{6\omega_0} = \frac{1}{12} \times \frac{2\pi}{\omega_0} = \frac{1}{12}T = \boxed{0.5\,s}$ since $T = 6s$

(c) If $x_A(0) = x_B(0) \Rightarrow C_2 = 0$. ~~Fo~~ $x_A(t) = C_1\cos\omega_1 t = 0$

$\Rightarrow \omega_1 t = \frac{\pi}{2} \Rightarrow t = \frac{\pi}{2\omega_1} = \frac{\pi}{2\omega_0} = \frac{1}{4}\frac{2\pi}{\omega_0} = \frac{1}{4}T = \boxed{1.5\,s}$