— An Introduction

Cosmic Rays — TKC Memorial I, II

see: Kulsrud, Chapt. 12; and refs. therein,
also Parker, "Cosmic Magnetic Fields"

→ Cosmic Rays - OV
(Short intro to BIG subject)

→ single nucleon
— well hit baseball
→ 10^9 GeV+

- high energy particles exhibiting power law spectrum/distribution in energy

i.e. $N(E) = \epsilon^{-2.7}$ 'knee' 'ankle' ($1 \to 10^6$ GeV)

- galactic, extra-galactic in origin { — origin? — propagation?

— *really collisionless — coupling ?! → $\underline{B}$!

- constitute additional component:

i.e. plasma = thermals + CRs

CR's → { finite energy, pressure / negligible inertia

also MFE: plasma = thermals + { EP / α's

'thermal' plasma → MHD (where $\nabla n f_{hot}$ is energy source)

CR → kinetic theory

- FAQ's re: CR's:

$\Rightarrow$ origin, spatial distribution

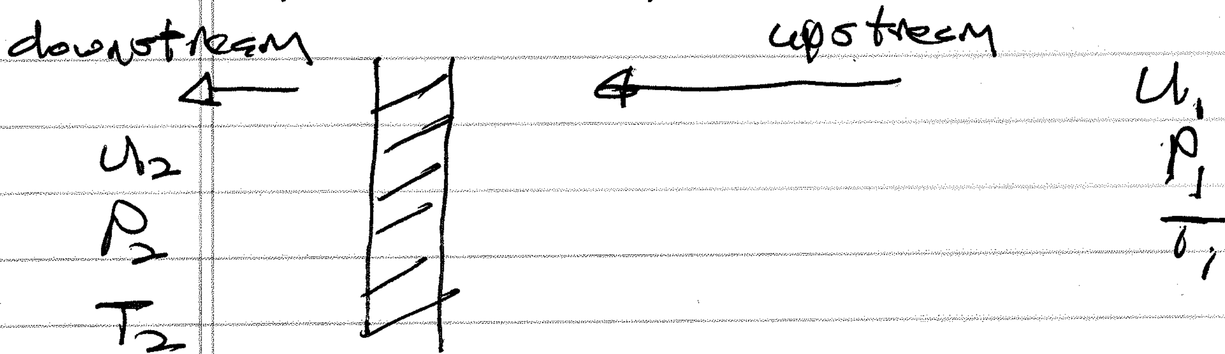
$\Rightarrow$ production / acceleration
$\Leftrightarrow$ confinement to accelerator ?!

$\Rightarrow$ interaction with / role in galactic structure.

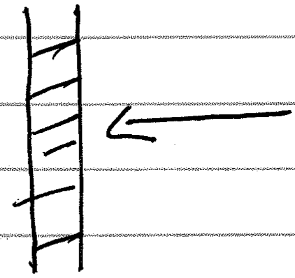
$\Rightarrow$ One origin: Shocks, especially SNR

$\Leftrightarrow$ DSA: Diffusive shock acceleration, 'First Order Fermi' process

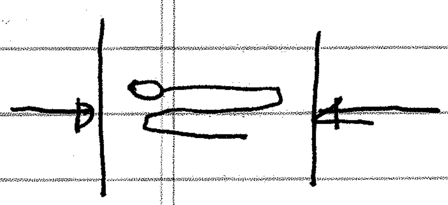
- shock $\Rightarrow$ compression



$u_1 > u_2$, $\rho_2 > \rho_1$, $T_2 > T_1$ $\Big\}$ RH conditions + EOS (can be modified by CR)
$\Rightarrow$ ratios



energy gain by compression, i.e. akin

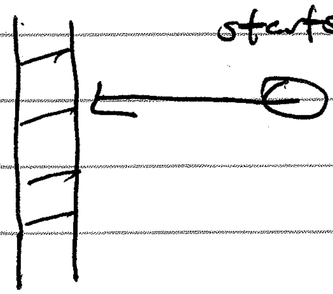


What are the "walls"?

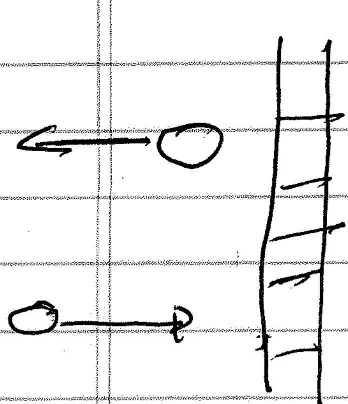
→ Alfven wave-induced pitch angle scattering! pitch $\measuredangle = \Theta = v_{\parallel}/v_{\perp}$

i.e.

starts upstream



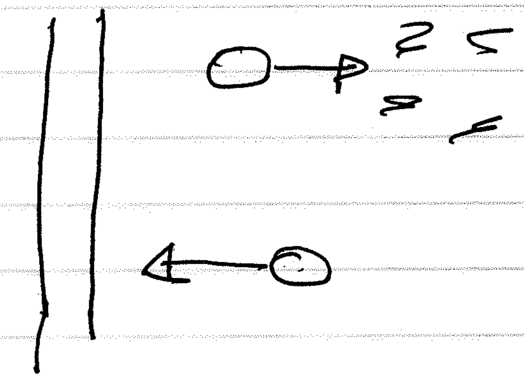
→



— downstream, with higher energy

— AW's scatter pitch $\measuredangle$ of CR

→



— particle returns upstream,

— particle reverses ready to re-cross the shock

⇒ multiple crossings boost energy of the particle

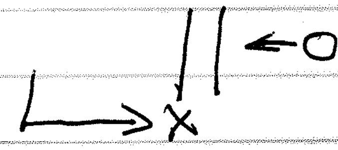
how? → along stream diffusion!

i.e. general CR evolution equation:
$f \equiv$ dist CR.

$$\partial_t f + \underline{v}\cdot\underline{\nabla} f = \underline{\nabla}\cdot\big(\underbrace{\hat{n} D \hat{n}\cdot\nabla f}_{\text{diffusion}}\big) + \frac{1}{3}\underbrace{(\underline{\nabla}\cdot\underline{v})}_{\text{compression (free energy source)}}\, p\,\frac{\partial f}{\partial p}$$

~ "convection-diffusion eqn."

~ simplified form:



$$v\frac{\partial f}{\partial x} - \frac{\partial}{\partial x} D \frac{\partial f}{\partial x} = \frac{1}{3}(u_+ - u_-)\, p\,\frac{\partial f}{\partial p}$$

- D ensures particle bounces thru shock many times
- D impedes streaming away from the shock.

What is D?

$$D \simeq \frac{v_{Th}^2}{\nu} \to \frac{c^2}{\nu}$$

(along B)

where $\nu = \frac{\pi}{2} \Omega_c \left(\frac{\delta B}{B}\right)^2 \Rightarrow$ mock-up of QL pitch $\measuredangle$ scattering

cyclotron frequency

turbulent pitch $\measuredangle$ scattering

AW spectrum.

N.B. Convection-Diffusion equation has bizarre appearance:

$$v \frac{\partial f}{\partial x} - \frac{\partial}{\partial x} D \frac{\partial f}{\partial x} = \frac{1}{3}(u_+ - u_-) \, p \frac{\partial f}{\partial p}$$

$$D \cong c^2 \Big/ \frac{\pi}{2} \Omega_c \left(\frac{\delta B}{B}\right)^2$$

$\Rightarrow$ Begs the question: where does the Alfven wave come from?

$\Leftrightarrow$

Beam instability driven by CR's.

$\Rightarrow$ CR's drive Alfven wave via beam.

Story will be: — bulk plasma supports wave
— For $V_D > V_A$ CR's drive wave
→ high frequency resonance

N.B. — Consequence of pitch ∡ scattering induced by CR-driven Alfvenic turbulence is ≈ CR isotropy.

— Pitch ∡ scattering ⟺ self-confinement (to shock)

→ Another Origin — {Lower Energies

2nd Order Fermi — ≈ Q.L.T.

i.e. — Consider $\underline{\underline{D}}_{\perp v_\perp v_\perp}$, $D_{\parallel v_\parallel v_\parallel}$ produced by Alfvenic turbulence.

— Cyclotron resonance possible

so

$$\partial_t \langle f \rangle = \frac{\partial}{\partial \underline{v}_\perp} \cdot \underline{\underline{D}}_\perp \cdot \frac{\partial}{\partial \underline{v}_\perp} \langle f \rangle + \frac{\partial}{\partial v_\parallel} D_\parallel \frac{\partial \langle f \rangle}{\partial v_\parallel}$$

$+ \ldots$

taking energy moment:

$$\frac{\partial \mathcal{E}_{Part}}{\partial t} = \int d^3v \; \hat{n}_\perp \cdot \underline{\underline{D}}_\perp \cdot \hat{n}_\perp \langle f \rangle + \int d^3v \; D_{\parallel} \langle f \rangle + \cdots$$

⇒ AW turbulence heats tail distribution
⟺ and CR acceleration!

i.e. AW cascade → dissipation ν, η
AW cascade → [CR acceleration / Collisionless heating]

branching ratio

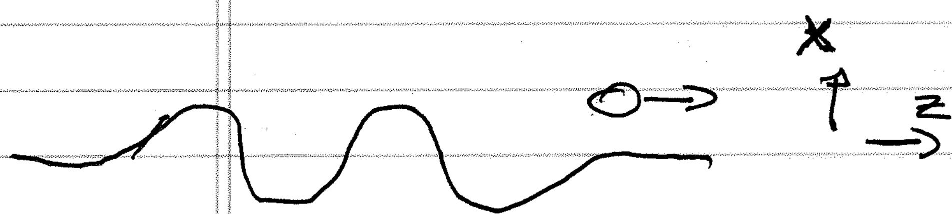
so Many key aspects of story depend upon:

→ AW-induced pitch angle scattering
→ CR-induced AW instability

⇒ together define CR self-confinement.
(Turbulence as good guy!)

⟺ cyclotron resonance is key!

→ Pitch ∡ Scattering by CRs



– AW $\quad \delta \underline{B}_\perp = \hat{x}\, \delta B \sin(kz - \omega t)$

seek $\quad \Delta p_z \sim q \int_{\text{period}} dt \left(\frac{\underline{v} \times \delta \underline{B}}{c}\right)$

↓
change in
z-momentum
(to reverse)

Now, $\quad \delta \underline{B}_\perp = (\ )\ \hat{x}$

so $\quad v_y = v_\perp \sin(\Omega t + \phi)$

↓ cyclotron freq → phase

$z = z_0 + v_z t$

$$q(\underline{v} \times \delta \underline{B})_z(t) = q\, v_\perp\, \delta B_x \sin(kz_0 + kv_z t)\sin(\Omega t + \phi)$$

$$= \frac{1}{2} q\, v_\perp\, \delta B_x \Big\{ \cos\left[(kv_z - \omega + \Omega)t + (kz_0 + \phi)\right] - \cos\left[(kv_z - \omega - \Omega)t + (kz_0 - \phi)\right] \Big\}$$

Obviously: DC / resonant interaction

$$k_z v_z - \omega \mp \Omega \simeq 0$$

here $k_z v_A \sim \omega$ $\ll$ others

↳ cyclotron resonance.

choose ⊖, then:

$$\Delta p_z = q \int_{\text{period}} dt \left(\frac{\underline{v} \times \underline{B}}{c} \right)_z \qquad \begin{cases} \tilde{\tau} = 2\pi / (k_z v_z - \cancel{\omega}) \\ \simeq 2\pi / (k v_z - \cancel{k v_A}) \\ \simeq 2\pi / k v_z \end{cases}$$

$$= \frac{1}{2} q \frac{\hat{v}_\perp \delta B_x}{c} \left(\frac{2\pi}{k v_z} \right) \cos(k z_0 - \phi)$$

but $k_z v_z - \Omega \simeq 0$ i.e. wave frequency small. ($v_z > v_A$)

$$\Delta p_z = \frac{\pi q \hat{v}_\perp \delta B}{c \Omega} \cos\phi'$$

$$= \pi \frac{\cancel{q} v_\perp}{\cancel{c} \cancel{q} B} \delta B \, m \cancel{c} \cos\phi' \, \hat{n}_\perp$$

$$= \pi p_\perp \left(\frac{\delta B}{B} \right) \cos\phi' \sin\theta$$

$p_\perp$: magn

$\sin\theta$: pitch ∡ factor.

so $$\boxed{\Delta p_z \cong \pi p_\perp \sin\theta \frac{\delta B}{B} \cos\phi}$$

p_z scatter in 1 period.

Then,
$$p_z = p\cos\theta$$
$$\delta p_z = -p\sin\theta\,\delta\theta$$
$$= \pi p \sin\theta \left(\frac{\delta B}{B}\right)\cos\phi$$

$$\boxed{\delta\theta = -\pi \frac{\delta B}{B}\cos\phi}$$ pitch $\measuredangle$ scatter.

For multiple, uncorrelated scatterings, each with duration τ:

$$1/\tau = {}^{2\pi}/_{|k \cdot v_z|}$$

$$\langle \Delta\theta^2 \rangle \cong \frac{\pi^2}{2}\left(\frac{\delta B}{B}\right)^2 \frac{t}{\tau}$$

and again taking $k_z v_z \cong \Omega$ resonance

$\Rightarrow$

pitch ∡ diffusivity

$$\langle \Delta\Theta^2 \rangle = 2 D t$$

$$\boxed{D = \frac{\pi}{8}\,\Omega\left(\frac{\delta B}{B}\right)^2}$$

→ pitch ∡ diffusivity.

N.B.:

- good example of cyclotron-resonant process:
  $k_z v_z - \Omega + \omega = 0$
- need only small $\delta B/B$ for fast pitch ∡ scattering
  ① Resonance
- $\delta\Theta \sim (\delta B/B) \sim \alpha$

∡ perturbed field makes with mean

$$k v_z \sim \Omega \quad \text{and} \quad v_\perp \sim v_\parallel$$

$$\frac{\lambda}{2\pi} \sim \frac{v}{\Omega} \sim \rho_L$$

→ tied to resonance condition.

larmor.

so, $k_z V_z \gg \Omega$

$\rho_L \gg \lambda$ $\Rightarrow$ cosmic ray "~adiabatic", i.e. zips thru wave.
$\rightarrow$ no scattering

$k_z V_z \ll \Omega$ $\Rightarrow$ cosmic ray follows field (perturbed)

$\rho_L \ll \lambda$ $\rightarrow$ no scattering

Need resonant interaction!

(2) If n wavelengths long packet,

$$\langle (\Delta\theta)^2 \rangle = \left(n \frac{\pi}{8} \Omega \left(\frac{\delta B}{B}\right)^2 \right) t$$

but:

$$(\delta B/B)^2 = \Delta k I(k)$$

but $\Delta k = k/n$ $\rightarrow$ intensity

so

$$\boxed{D_\theta = \frac{\pi}{8} \Omega I}$$

③ The 90° Problem

$k_z V_z \sim \Omega$

$k \cos\theta \, V_z \sim \Omega \Rightarrow \frac{\lambda}{2\pi} = \rho_L \cos\theta$

⇒ $\theta \to 90^\circ$, CR A.W scattering too weak to scatter past 90°.

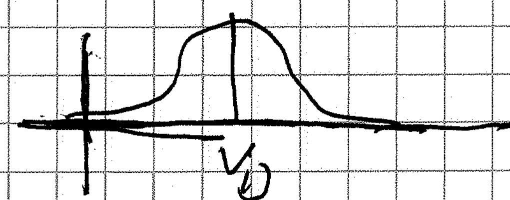
⇒ saved by secondary mechanism — or resonance broadening.

Now

How CRs generate AWs.

CR Alfven Wave Instability

- CR distribution isotropic → why?
- drift anisotropy i.e.



$V_D > V_A$ → instability → TBD

- instability will act to drive $V_D \to V_A$ ⇒ isotropy; i.e. anisotropy dissipated rapidly.

"The nail that sticks up will be beaten down".

so

$$\frac{d\underline{p}}{dt} = q\left(\underline{E} + \frac{\underline{v}\times\underline{B}}{c}\right)$$

$$\frac{\partial f}{\partial t} + \underline{\nabla}\cdot(\underline{v}f) + \underline{\nabla}_p\cdot\left[q\left(\underline{E} + \frac{\underline{v}\times\underline{B}}{c}\right)f\right] = 0$$

so

$$\boxed{\frac{\partial f}{\partial t} + \underline{v}\cdot\underline{\nabla} f + q\left(\underline{E} + \frac{\underline{v}\times\underline{B}}{c}\right)\cdot\underline{\nabla}_p f = 0.}$$

<u>Full</u> Vlasov Eqn.

here $\left\{\begin{array}{l}\tilde{E}\\ \tilde{B}.\end{array}\right.$

Now, $\underline{B} = B_0\hat{z}$.

<u>Recall</u>:

$$\underline{\nabla}\times\underline{B} = \frac{4\pi}{c}\underline{J} + \frac{1}{c}\frac{\partial\underline{E}}{\partial t}$$

$$\underline{\nabla}\times\underline{E} = -\frac{1}{c}\frac{\partial\underline{B}}{\partial t}$$

$$\Rightarrow \quad -\frac{i\omega}{c}\underline{\underline{\epsilon}}\cdot\underline{E}_\omega = -\frac{i\omega}{c}\underline{E} + \frac{4\pi}{c}\underline{J}_\omega$$

and $\quad \tilde{\underline{B}} = \underline{k}c\times\tilde{\underline{E}}/\omega$

so

$$\frac{\partial \tilde{f}}{\partial t} + \underline{v}\cdot\nabla\tilde{f} + q\left(\frac{\underline{v}\times\underline{B}_0}{c}\right)\cdot\nabla_p\tilde{f}$$

$$= -q\left[\underline{\tilde{E}} + \frac{\underline{v}\times\underline{\tilde{B}}}{c}\right]\cdot\nabla_p\langle f\rangle$$

$$= -q\left[\underline{E} + \frac{\underline{v}\times(\underline{k}\times\underline{E})}{\omega}\right]\cdot\nabla_p\langle f\rangle$$

$$= -q\left[\left(1-\frac{k_z v_z}{\omega}\right)\underline{E} + \frac{\underline{v}\cdot\underline{E}\,\underline{k}}{\omega}\right]\cdot\nabla_p\langle f\rangle$$

For LHS: For $\underline{k} = k\hat{z}$

$$-i\omega\tilde{f} + ikv_z\tilde{f} + \frac{q}{c}(\underline{v}\times\underline{B}_0)\cdot\nabla_p\tilde{f}$$

$$= -q\left[\left(1-\frac{k v_z}{\omega}\right)\underline{E} + \frac{\underline{v}\cdot\underline{E}\,\underline{k}}{\omega}\right]\cdot\nabla_p\langle f\rangle$$

Now, useful to convert to cylindrical polar in velocity space

$$\frac{q}{c}\underline{v}\times\underline{B}_0\cdot\nabla_p\tilde{f} = -\frac{q v_\perp B_0}{c p_\perp}\frac{\partial}{\partial\phi}\tilde{f}$$

$$= -\Omega\frac{\partial\tilde{f}}{\partial\phi}$$

So at last,

$$\boxed{-i(\omega - k_z v_z)\tilde{f}_1 - \Omega \frac{\partial \tilde{f}}{\partial \phi} = -q A E \cos\phi}$$

where:

$$\boxed{A = \left(1 - \frac{k_z v_z}{\omega}\right)\frac{\partial \langle f \rangle}{\partial p_\perp} + \frac{k v_\perp}{\omega}\frac{\partial \langle f \rangle}{\partial p_z}}$$

RHS factor

Trick will be: $\tilde{J} = \tilde{J}_{plasma} + \tilde{J}_{CR}$

$\tilde{J}_{plasma}$: simple (AW)

$\tilde{J}_{CR}$: resonant ~~CR~~ CR → need f_1

Proceeding: $\tilde{f}_1 = \sum_n f_{1,n} e^{-in\phi}$

$$-\Omega \, \partial f_1/\partial\phi = \pm i n \Omega f_1$$

↑ cyclotron harmonic. $n = 1$ here.

$$f_1 = -qE\frac{1}{2}\left[\underbrace{\frac{iAe^{i\phi}}{\omega - k_z v_z + \Omega}}_{Rt} + \underbrace{\frac{iAe^{-i\phi}}{\omega - k_z v_z - \Omega}}_{Left}\right]$$

Rt ⟶   ⟵ Left

then,

$$f_1 = \overbrace{-\frac{qE}{2}\,\frac{iAe^{i\phi}}{\omega_z - k_z v_z + \Omega}}^{f_+} \;\overbrace{-\,\frac{qE}{2}\,\frac{iAe^{-i\phi}}{\omega - k_z v_z - \Omega}}^{f_-}$$

Rt $\rightarrow$ $\leftarrow$ Left

Resonance:

$$-\omega + k_z v_z = \Omega = \frac{\Omega_0}{\gamma}$$

$\gamma \rightarrow$ rel factor $1/\left(1-\frac{v^2}{c^2}\right)^{1/2}$

$$\Omega_0 = \frac{qB_0}{mc}$$

as ω irrelevant:

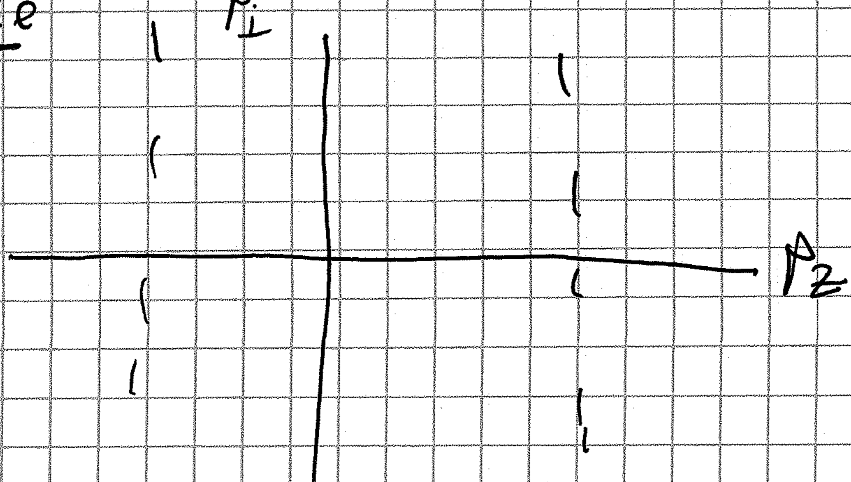
$$\underbrace{k_z v_z \gamma = k_z p_z = \Omega_0}_{\text{resonance}}$$

$$\underline{E} = E_x \hat{x}$$

Now, perturbed resonant current

$$\tilde{J}_{1,\,cr} = q\int (f_+ + f_-)\, v_\perp\, (\cos\phi\, \hat{x} + \sin\phi\, \hat{y})\, d^3p$$

i.e. resonance lines



$$\boxed{\tilde{J}_{cR} = -\frac{i q^2}{2} \int \frac{v_\perp A}{\omega - k_z v_z + \Omega} (\hat{x} + i\hat{y}) E \, d^3p - \frac{i q^2}{2} \int \frac{v_\perp A \quad d^3p (\hat{x} - i\hat{y})}{\omega - k_z v_z - \Omega} E}$$

{ purely shifted dist.
{ $\hat{y}$ bits cancel

[$k_\perp \to 0$
[$A\,\omega$
[wave $\hat{z}$
[$E \to \hat{x}$

Now: $\underline{k} \times \underline{k} \times \underline{E} = -\frac{\omega^2}{c^2} \underline{E} + 4\pi \tilde{J}(\omega)$

$$\underline{k} \times \underline{k} \times \underline{E} = -\epsilon \frac{\omega^2}{c^2} \underline{E}$$

$$\boxed{\frac{k^2 c^2}{\omega^2} = \epsilon_0 + \epsilon_{xx}^{cr}}$$

$J_{pl} \qquad J_{cR}.$

$$\boxed{\epsilon_0 = \frac{c^2}{V_A^2}}$$

$$\boxed{\epsilon_{xx}^{cr} = \frac{i 4\pi \tilde{J}_{cR}}{\omega E} = \frac{q^2}{2} \frac{4\pi}{\omega} \int \frac{A v_\perp \, d^3p}{\omega - k_z v_z + \Omega}}$$

$+_{,}$ - summed

Now,

$$\frac{1}{\omega - kv_z + \Omega} = \frac{P}{(\ )} - i\pi\,\delta(\omega - kv_z + \Omega)$$

$$= \frac{P}{(\ )} - i\pi\,\delta(v_z - \Omega_0/k)$$

$$\omega = \omega_0 + \underbrace{\delta\omega}_{\to CR}$$

$$= kV_A + \delta\omega$$

so

$$\frac{k^2c^2}{\omega^2} = \frac{c^2}{V_A^2} + \epsilon_{xx}^{CR}$$

$\omega = \omega_0 + \delta\omega$

$$-2\delta\omega\,\frac{k^2c^2}{\omega_0^3} = \epsilon_{xx}^{CR}$$

$$\Rightarrow\ -2\frac{\delta\omega}{\omega_0}\frac{c^2}{V_A^2} = \epsilon^{CR}$$

$$\delta\omega = -\frac{V_A^2}{c^2}\frac{\omega_0}{2}\epsilon^{CR}.$$

$$\to -\frac{\omega_0}{2}\epsilon^{CR}$$

$\delta\omega = i\gamma$

then

$$\boxed{\gamma = -\frac{\omega_0}{2}\, \mathrm{Im}\, \epsilon^{CR}}$$

and

$$\mathrm{Im}\, \epsilon^{CR} =$$

$$\mathrm{Im}\left[-\frac{g^2}{4}\, 4\pi\, \frac{V_A^2}{c^2}\, \epsilon(\pi i) \int V_\perp A\, \delta(k_z v_z - \Omega - \omega)\, d^3p\right]$$

$\therefore$

$$\boxed{\gamma_{\parallel} = \pi^2 g^2 \frac{V_A^2}{c^2} \int d^3p\; v_\perp \left[\left(1 - \frac{k_z v_z}{\omega}\right)\frac{\partial f_0}{\partial p_\perp} + \frac{k v_\perp}{\omega}\frac{\partial f_0}{\partial p_z}\right] \delta(k v_z - \Omega)}$$

At last

and can say:

$$\gamma = 0 \quad \text{for:} \; -\left(1 - \frac{k_z v_z}{\omega}\right)\frac{\partial f_0}{\partial p_\perp} = \frac{k v_\perp}{\omega}\frac{\partial f_0}{\partial p_z}$$

$$\boxed{\frac{\partial f_0/\partial p_z}{\partial f_0/\partial p_\perp} = \frac{-(\omega - k_z v_z)}{k_\perp v_\perp}}$$

Marginal contours.

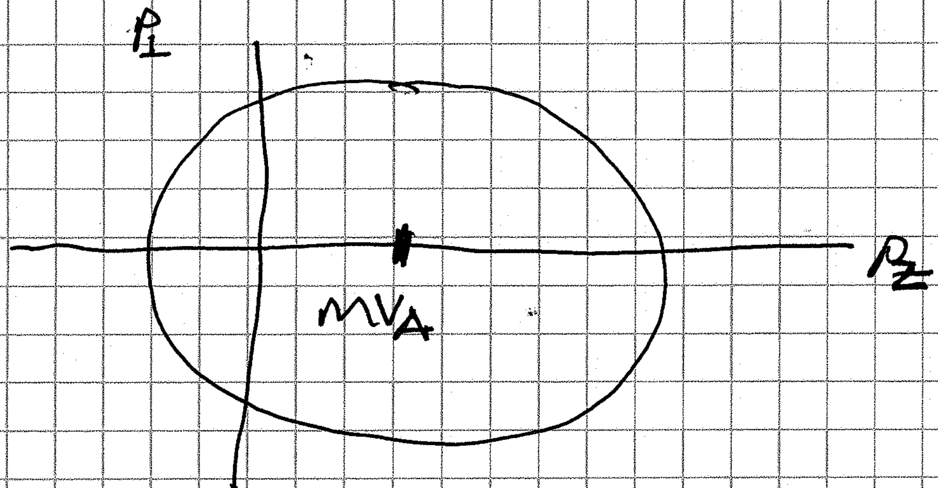
on

$$\gamma = (\ )\int \Delta p_\perp \frac{\partial f_0}{\partial p_\perp} + \Delta p_\parallel \frac{\partial f_0}{\partial p_\parallel}$$

$$= (\ )\int (\ )\,\underline{\Delta p}\cdot \underline{\nabla}_p f_0$$

i.e. grad f along curve

Where $\dfrac{\Delta p_\perp}{\Delta p_\parallel} = \dfrac{\omega - k_z v_z}{k_\perp v_\perp}$



→ In wave frame, this is circle along which pure pitch $\measuredangle$ scattering occurs.

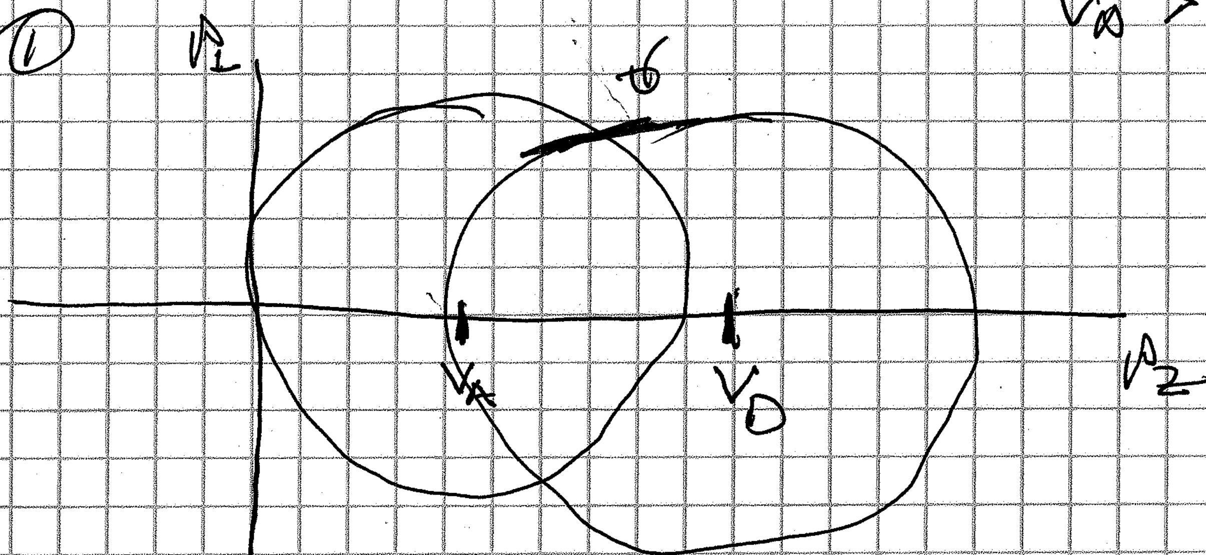
i.e. $v_z = v_A$, $\omega = k v_A$

$$\frac{\Delta p_\perp}{\Delta p_\parallel} = \frac{0}{k_\perp v_\perp} \rightarrow 0$$

no change

Consider 2 cases:

(1) $V_D >> V_A$

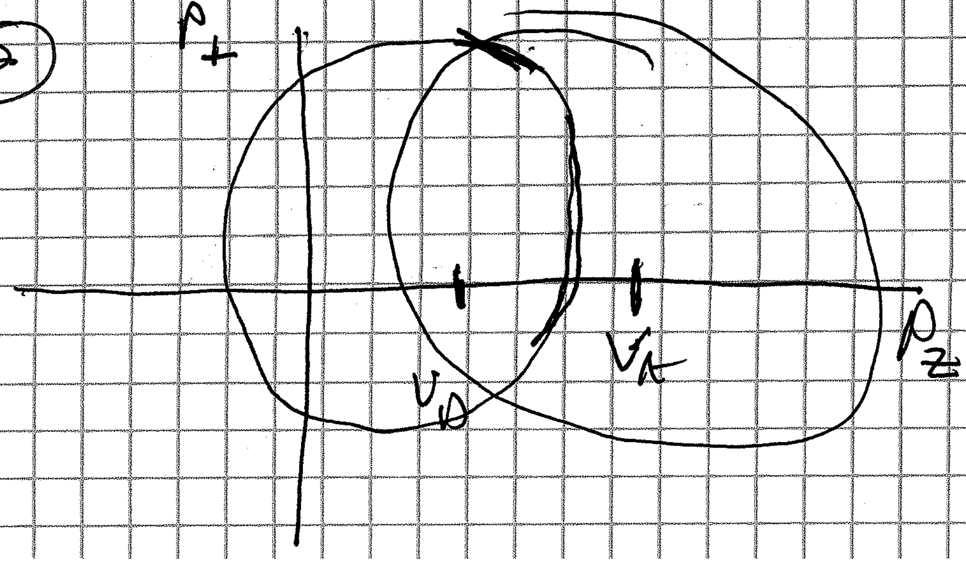


i.e. $\frac{\Delta \rho_\perp}{\Delta \rho_\parallel} = \frac{V_A - V_D}{V_\perp} < 0$

and: $\frac{\partial f_0}{\partial \rho_\perp} < 0$ $\underline{\gamma > 0}$

$\Rightarrow$ f_0 increases along circle along which ρ_z increases $\rightarrow$ instability.
i.e. net + slope

(2)



f_0 decreases on circle along which ρ_z increases

$\rightarrow$ stability

i.e.

$$\frac{\Delta p_\perp}{\Delta p_\parallel} = \frac{v_A - v_D}{v_\perp} > 0$$

net $\ominus$ $\frac{\partial f_0}{\partial p}$ ✓ → stability

$\boxed{\text{Need } v_D > v_A \text{ instability.}}$

The crank:

- v_D bulk velocity
- isotropic CR f_0 in co-moving frame

∴ $p_\perp' = p_\perp$

$$p_z' = p_z - \frac{v_D \epsilon}{c^2}$$

drop $O(v_D^2/c^2)$

$$f(p_\perp, p_z) = F(p') = F\left(\sqrt{p_\perp'^2 + p_z'^2}\right)$$

relativistic invariant

$$\left(1 - \frac{v_z}{v_A}\right)\frac{\partial f_0}{\partial p_\perp} + \frac{v_\perp}{v_A}\frac{\partial f_0}{\partial p_z}$$

$$= \frac{dF}{dp'}\frac{p_\perp}{p'}\left(1 - \frac{v_D}{v_A}\right) + h.o.$$

so

$$\boxed{\gamma^{\omega}_{A} = -i\,\omega_{1,I} = \pi^2 g^2 \frac{v_A^2}{c^2}\left(1 - \frac{v_0}{v_A}\right) * \int \frac{dF}{dp}\,\frac{p_\perp^2}{p}\,\delta(k_\parallel p_\parallel - \Omega_0)\,d^3p}$$

instability for $v_0 > v_A$

as $dF/dp < 0$

Full:

$$\boxed{\gamma = \frac{\pi}{4}\,\Omega_0\,\frac{\overset{\#\,>\,p_1}{N_{cr}(p > p_1)}}{n}\;\overset{\#}{c_n}\left(\frac{v_0 - v_A}{v_A}\right)}$$

uses power law spectrum.

→ stability story. ✓

→ Back to Pitch ∠ Scattering

– Pitch ∠ scattering → $\frac{q}{mc}\underline{V}\times\tilde{B}$

⇒ no work

⇒ no change in energy

– can crank from QL
(high ω):

$$\frac{\partial f}{\partial t}+v_z\frac{\partial f}{\partial z}=\frac{\partial}{\partial \mu}\left[(1-\mu^2)D_\mu\frac{\partial f}{\partial \mu}\right]$$

$$\mu=\cos\theta_p$$

$$D_\mu=\frac{\pi}{4}\,\Omega\,k\,I(k_r)$$

$$k_r=\frac{\Omega}{v_z}=\frac{\Omega}{\mu v}=\frac{1}{\mu\rho_L}$$

resonant k

and

$$\left(\frac{\delta B}{B}\right)^2=\int dk\, I(k)$$

→ CR Pressure and Energy

$\underline{B} = B_0 \hat{z}$

$\underline{P_{cr}} = P_{cr}(z) \quad \Rightarrow V_D, \; J_{CR}$

i.e.

$$\rho \frac{d\underline{v}}{dt} = -\nabla p + \frac{q}{c} \underline{v} \times \underline{B}$$

→ diamagnetic flow, current

⇒

$$J_{CR} = n_{cr} q V_D = \frac{\nabla P_{cr}}{B} \quad \rightarrow \text{CR diamagnetic current}$$

Now, $V_D > V_A \rightarrow$ waves

i.e. CRs $\xrightarrow{\text{waves}}$ Plasma

[CR's accelerate mean plasma

Force that CR exert on mean plasma.

see:

pitch ∠ scatt. $\Omega_i \left(\frac{\delta B}{B}\right)^2$

$$\cancel{\frac{\partial f}{\partial t}} + v_z \frac{\partial f}{\partial z} = \nu \frac{\partial}{\partial \mu} \left(\frac{1-\mu^2}{2}\right) \frac{\partial F}{\partial \mu}$$

$$-\mu v \frac{F}{L} \rightarrow \text{stopping length}$$

$$-\frac{v\mu}{L}F = \nu\frac{\partial}{\partial\mu}\left(\frac{1-\mu^2}{2}\right)\frac{\partial F}{\partial\mu}$$

$\int d\mu$ Chap. Enskog: $F_0 + F_1$

[vanishes $\mu = 1$

$$-\frac{v}{L}\left(\frac{\mu^2-1}{2}\right)F_0 = \nu\left(\frac{1-\mu^2}{2}\right)\frac{\partial F_1}{\partial\mu}$$

$$\frac{\partial F_1}{\partial\mu} = \frac{v}{\nu L}F_0$$

$\nu \sim$ const.

v moment

$$V_D - V_A = \frac{\int f_1 \mu v\, d^3p}{\int F d^3p} = \frac{v^2}{3\nu L} \sim \frac{c^2}{3\nu L}$$

So

$$\boxed{V_D - V_A = \frac{c^2}{3\nu L}}$$

but

$$\gamma_{CR\text{ crit}} = \frac{\pi}{4}\Omega_0\left(\frac{V_D - V_A}{V_A}\right)\frac{N_{CR}}{n}$$

But γ ~~ΔD~~ change in WMD

$$\frac{dP_w}{dt} = 2\gamma\, k N$$

$$= 2\gamma\, k \frac{\mathcal{E}_w}{\omega}$$

$$= 2\gamma \left(\frac{\delta B}{B}\right)^2 \frac{1}{V_A}$$

$\underline{\mathrm{So}}$

$$\frac{dP_w}{dt} = 2\frac{\pi}{4}\Omega_0 \frac{(V_D - V_A)}{V_A}\frac{(\delta B)^2}{4\pi}\frac{N_{cr}}{n}\frac{1}{V_A}$$

$$= \frac{\Omega_0}{8}\frac{c^2}{3\gamma L}\frac{1}{V_A^2}(\delta B)^2 \frac{N_{CR}}{n}$$

Now,
$$\nu = \frac{\pi}{2}\Omega\left(\frac{\delta B}{B}\right)^2$$

$$V_A^2 = B_0^2 / 4\pi\rho_{PL}$$

$\overline{\gamma}$ normal.

i.e.

$$\frac{dP_w}{dt} = \frac{\Gamma_{CR}}{3} N_{cr} \frac{Mc^2}{L}$$

$$\gamma N_{CR} (Mc^2) = \mathcal{E}_{CR}$$

CR energy density

80

$$\boxed{\frac{dP_w}{dt} = \frac{P_{CR}}{L} \cong -\nabla P_{CR}.}$$

CR pressure gradient $\rightarrow dP_w/dt$.
($L \geq$ ~~...~~)

Leads to convection - diffusion:

$$\frac{\partial f}{\partial t} + v_z \frac{\partial f}{\partial z} = \frac{\partial}{\partial \mu} (1-\mu^2) D \frac{\partial f}{\partial \mu} + ?$$

creation

$\Rightarrow$ compression

i.e.

$$\frac{\partial P_{CR}}{\partial t} + \underline{V} \cdot \underline{\nabla} P_{CR} = -\frac{4}{3} (\underline{\nabla} \cdot \underline{V}) P_{CR}$$

as

$$\frac{d \mathcal{E}_{pl}}{dt} = -V_A \cdot P_{CR}$$

$$+ \frac{1}{3} (\underline{\nabla} \cdot \underline{V}) p \frac{\partial f}{\partial p}$$

Changes CR energy.