

3. **THINK** We treat the oxygen gas in this problem as ideal and apply the ideal-gas law.

**EXPRESS** In solving the ideal-gas law equation  $pV = nRT$  for  $n$ , we first convert the temperature to the Kelvin scale:  $T_i = (40.0 + 273.15)\text{K} = 313.15\text{K}$ , and the volume to SI units:  $V_i = 1000\text{cm}^3 = 10^{-3}\text{m}^3$ .

**ANALYZE** (a) The number of moles of oxygen present is

$$n = \frac{pV_i}{RT_i} = \frac{(1.01 \times 10^5 \text{ Pa})(1.000 \times 10^{-3} \text{ m}^3)}{(8.31 \text{ J/mol} \cdot \text{K})(313.15 \text{ K})} = 3.88 \times 10^{-2} \text{ mol}.$$

(b) Similarly, the ideal gas law  $pV = nRT$  leads to

$$T_f = \frac{pV_f}{nR} = \frac{(1.06 \times 10^5 \text{ Pa})(1.500 \times 10^{-3} \text{ m}^3)}{(3.88 \times 10^{-2} \text{ mol})(8.31 \text{ J/mol} \cdot \text{K})} = 493 \text{ K}.$$

We note that the final temperature may be expressed in degrees Celsius as  $220^\circ\text{C}$ .

**LEARN** The final temperature can also be calculated by noting that  $\frac{p_i V_i}{T_i} = \frac{p_f V_f}{T_f}$ , or

$$T_f = \left(\frac{p_f}{p_i}\right)\left(\frac{V_f}{V_i}\right)T_i = \left(\frac{1.06 \times 10^5 \text{ Pa}}{1.01 \times 10^5 \text{ Pa}}\right)\left(\frac{1500 \text{ cm}^3}{1000 \text{ cm}^3}\right)(313.15 \text{ K}) = 493 \text{ K}.$$

7. (a) Equation 19-45 (which gives 0) implies  $Q = W$ . Then Eq. 19-14, with  $T = (273 + 30.0)\text{K}$  leads to gives  $Q = -3.14 \times 10^3 \text{ J}$ , or  $|Q| = 3.14 \times 10^3 \text{ J}$ .

(b) That negative sign in the result of part (a) implies the transfer of heat is *from* the gas.

11. **THINK** The process consists of two steps: isothermal expansion, followed by isobaric (constant-pressure) compression. The total work done by the air is the sum of the works done for the two steps.

**EXPRESS** Suppose the gas expands from volume  $V_i$  to volume  $V_f$  during the isothermal portion of the process. The work it does is

$$W_1 = \int_{V_i}^{V_f} p dV = nRT \int_{V_i}^{V_f} \frac{dV}{V} = nRT \ln \frac{V_f}{V_i},$$

where the ideal gas law  $pV = nRT$  was used to replace  $p$  with  $nRT/V$ . Now  $V_i = nRT/p_i$  and  $V_f = nRT/p_f$ , so  $V_f/V_i = p_i/p_f$ . Also replace  $nRT$  with  $p_i V_i$  to obtain

$$W_1 = p_i V_i \ln \frac{p_i}{p_f}.$$

During the constant-pressure portion of the process the work done by the gas is  $W_2 = p_f(V_i - V_f)$ . The gas starts in a state with pressure  $p_f$ , so this is the pressure throughout this portion of the process. We also note that the volume decreases from  $V_f$  to  $V_i$ . Now  $V_f = p_i V_i / p_f$ , so

$$W_2 = p_f \left( V_i - \frac{p_i V_i}{p_f} \right) = (p_f - p_i) V_i.$$

**ANALYZE** For the first portion, since the initial gauge pressure is  $1.03 \times 10^5$  Pa,

$$p_i = 1.03 \times 10^5 \text{ Pa} + 1.013 \times 10^5 \text{ Pa} = 2.04 \times 10^5 \text{ Pa}.$$

The final pressure is atmospheric pressure:  $p_f = 1.013 \times 10^5$  Pa. Thus,

$$W_1 = (2.04 \times 10^5 \text{ Pa})(0.14 \text{ m}^3) \ln \left( \frac{2.04 \times 10^5 \text{ Pa}}{1.013 \times 10^5 \text{ Pa}} \right) = 2.00 \times 10^4 \text{ J}.$$

Similarly, for the second portion, we have

$$W_2 = (p_f - p_i) V_i = (1.013 \times 10^5 \text{ Pa} - 2.04 \times 10^5 \text{ Pa})(0.14 \text{ m}^3) = -1.44 \times 10^4 \text{ J}.$$

The total work done by the gas over the entire process is

$$W = W_1 + W_2 = 2.00 \times 10^4 \text{ J} + (-1.44 \times 10^4 \text{ J}) = 5.60 \times 10^3 \text{ J}.$$

**LEARN** The work done by the gas is positive when it expands, and negative when it contracts.

13. (a) At point  $a$ , we know enough information to compute  $n$ :

$$n = \frac{pV}{RT} = \frac{(2500 \text{ Pa})(1.0 \text{ m}^3)}{(8.31 \text{ J/mol}\cdot\text{K})(200 \text{ K})} = 1.5 \text{ mol.}$$

(b) We can use the answer to part (a) with the new values of pressure and volume, and solve the ideal gas law for the new temperature, or we could set up the gas law in terms of ratios (note:  $n_a = n_b$  and cancels out):

$$\frac{p_b V_b}{p_a V_a} = \frac{T_b}{T_a} \Rightarrow T_b = (200 \text{ K}) \left( \frac{7.5 \text{ kPa}}{2.5 \text{ kPa}} \right) \left( \frac{3.0 \text{ m}^3}{1.0 \text{ m}^3} \right)$$

which yields an absolute temperature at  $b$  of  $T_b = 1.8 \times 10^3 \text{ K}$ .

(c) As in the previous part, we choose to approach this using the gas law in ratio form:

$$\frac{p_c V_c}{p_a V_a} = \frac{T_c}{T_a} \Rightarrow T_c = (200 \text{ K}) \left( \frac{2.5 \text{ kPa}}{2.5 \text{ kPa}} \right) \left( \frac{3.0 \text{ m}^3}{1.0 \text{ m}^3} \right)$$

which yields an absolute temperature at  $c$  of  $T_c = 6.0 \times 10^2 \text{ K}$ .

(d) The net energy added to the gas (as heat) is equal to the net work that is done as it progresses through the cycle (represented as a right triangle in the  $pV$  diagram shown in Fig. 19-20). This, in turn, is related to  $\pm$  “area” inside that triangle (with area =  $\frac{1}{2}$ (base)(height)), where we choose the plus sign because the volume change at the largest pressure is an *increase*. Thus,

$$Q_{\text{net}} = W_{\text{net}} = \frac{1}{2} (2.0 \text{ m}^3) (5.0 \times 10^3 \text{ Pa}) = 5.0 \times 10^3 \text{ J.}$$

14. Since the pressure is constant the work is given by  $W = p(V_2 - V_1)$ . The initial volume is  $V_1 = (AT_1 - BT_1^2)/p$ , where  $T_1 = 315$  K is the initial temperature,  $A = 24.9$  J/K and  $B = 0.00662$  J/K<sup>2</sup>. The final volume is  $V_2 = (AT_2 - BT_2^2)/p$ , where  $T_2 = 325$  K. Thus

$$\begin{aligned}W &= A(T_2 - T_1) - B(T_2^2 - T_1^2) \\ &= (24.9 \text{ J/K})(325 \text{ K} - 315 \text{ K}) - (0.00662 \text{ J/K}^2)[(325 \text{ K})^2 - (315 \text{ K})^2] = 207 \text{ J}.\end{aligned}$$

42. The internal energy is

$$E_{\text{int}} = \frac{3}{2}nRT = \frac{3}{2}(1.0 \text{ mol})(8.31 \text{ J/mol}\cdot\text{K})(273 \text{ K}) = 3.4 \times 10^3 \text{ J}.$$

44. Two formulas (other than the first law of thermodynamics) will be of use to us. It is straightforward to show, from Eq. 19-11, that for any process that is depicted as a *straight line* on the  $pV$  diagram, the work is

$$W_{\text{straight}} = \left( \frac{p_i + p_f}{2} \right) \Delta V$$

which includes, as special cases,  $W = p\Delta V$  for constant-pressure processes and  $W = 0$  for constant-volume processes. Further, Eq. 19-44 with Eq. 19-51 gives

$$E_{\text{int}} = n \left( \frac{f}{2} \right) RT = \left( \frac{f}{2} \right) pV$$

where we have used the ideal gas law in the last step. We emphasize that, in order to obtain work and energy in joules, pressure should be in pascals ( $\text{N/m}^2$ ) and volume should be in cubic meters. The degrees of freedom for a diatomic gas is  $f = 5$ .

(a) The internal energy change is

$$\begin{aligned} E_{\text{int } c} - E_{\text{int } a} &= \frac{5}{2} (p_c V_c - p_a V_a) = \frac{5}{2} \left( (2.0 \times 10^3 \text{ Pa})(4.0 \text{ m}^3) - (5.0 \times 10^3 \text{ Pa})(2.0 \text{ m}^3) \right) \\ &= -5.0 \times 10^3 \text{ J.} \end{aligned}$$

(b) The work done during the process represented by the diagonal path is



$$W_{\text{diag}} = \left( \frac{p_a + p_c}{2} \right) (V_c - V_a) = (3.5 \times 10^3 \text{ Pa})(2.0 \text{ m}^3)$$

which yields  $W_{\text{diag}} = 7.0 \times 10^3 \text{ J}$ . Consequently, the first law of thermodynamics gives

$$Q_{\text{diag}} = \Delta E_{\text{int}} + W_{\text{diag}} = (-5.0 \times 10^3 + 7.0 \times 10^3) \text{ J} = 2.0 \times 10^3 \text{ J}.$$

(c) The fact that  $\Delta E_{\text{int}}$  only depends on the initial and final states, and not on the details of the “path” between them, means we can write  $\Delta E_{\text{int}} = E_{\text{int } c} - E_{\text{int } a} = -5.0 \times 10^3 \text{ J}$  for the indirect path, too. In this case, the work done consists of that done during the constant pressure part (the horizontal line in the graph) plus that done during the constant volume part (the vertical line):

$$W_{\text{indirect}} = (5.0 \times 10^3 \text{ Pa})(2.0 \text{ m}^3) + 0 = 1.0 \times 10^4 \text{ J}.$$

Now, the first law of thermodynamics leads to

$$Q_{\text{indirect}} = \Delta E_{\text{int}} + W_{\text{indirect}} = (-5.0 \times 10^3 + 1.0 \times 10^4) \text{ J} = 5.0 \times 10^3 \text{ J}.$$

47. (a) The work is zero in this process since volume is kept fixed.

(b) Since  $C_V = \frac{3}{2}R$  for an ideal monatomic gas, then Eq. 19-39 gives  $Q = +374 \text{ J}$ .

(c)  $\Delta E_{\text{int}} = Q - W = +374 \text{ J}$ .

(d) Two moles are equivalent to  $N = 12 \times 10^{23}$  particles. Dividing the result of part (c) by  $N$  gives the average translational kinetic energy change per atom:  $3.11 \times 10^{-22} \text{ J}$ .

54. The fact that they rotate but do not oscillate means that the value of  $f$  given in Table 19-3 is relevant. In Section 19-11, it is noted that  $\gamma = C_p/C_V$  so that we find  $\gamma = 7/5$  in this case. In the state described in the problem, the volume is

$$V = \frac{nRT}{p} = \frac{(2.0 \text{ mol})(8.31 \text{ J/mol}\cdot\text{K})(300 \text{ K})}{1.01 \times 10^5 \text{ N/m}^2} = 0.049 \text{ m}^3.$$

Consequently,

$$pV^\gamma = (1.01 \times 10^5 \text{ N/m}^2)(0.049 \text{ m}^3)^{1.4} = 1.5 \times 10^3 \text{ N}\cdot\text{m}^{2.2}.$$

55. (a) Let  $p_i$ ,  $V_i$ , and  $T_i$  represent the pressure, volume, and temperature of the initial state of the gas. Let  $p_f$ ,  $V_f$ , and  $T_f$  represent the pressure, volume, and temperature of the final state. Since the process is adiabatic  $p_i V_i^\gamma = p_f V_f^\gamma$ , so

$$p_f = \left( \frac{V_i}{V_f} \right)^\gamma p_i = \left( \frac{4.3 \text{ L}}{0.76 \text{ L}} \right)^{1.4} (1.2 \text{ atm}) = 13.6 \text{ atm} \approx 14 \text{ atm}.$$

We note that since  $V_i$  and  $V_f$  have the same units, their units cancel and  $p_f$  has the same units as  $p_i$ .

59. Since  $\Delta E_{\text{int}}$  does not depend on the type of process,

$$(\Delta E_{\text{int}})_{\text{path 2}} = (\Delta E_{\text{int}})_{\text{path 1}}.$$

Also, since (for an ideal gas) it only depends on the temperature variable (so  $\Delta E_{\text{int}} = 0$  for isotherms), then

$$(\Delta E_{\text{int}})_{\text{path 1}} = \sum (\Delta E_{\text{int}})_{\text{adiabat}}.$$

Finally, since  $Q = 0$  for adiabatic processes, then (for path 1)

$$(\Delta E_{\text{int}})_{\text{adiabatic expansion}} = -W = -40 \text{ J}$$

$$(\Delta E_{\text{int}})_{\text{adiabatic compression}} = -W = -(-25) \text{ J} = 25 \text{ J}.$$

Therefore,  $(\Delta E_{\text{int}})_{\text{path 2}} = -40 \text{ J} + 25 \text{ J} = -15 \text{ J}.$

63. In the following,  $C_V = \frac{3}{2}R$  is the molar specific heat at constant volume,  $C_p = \frac{5}{2}R$  is the molar specific heat at constant pressure,  $\Delta T$  is the temperature change, and  $n$  is the number of moles.

The process  $1 \rightarrow 2$  takes place at constant volume.

(a) The heat added is

$$Q = nC_V \Delta T = \frac{3}{2}nR \Delta T = \frac{3}{2}(1.00 \text{ mol})(8.31 \text{ J/mol} \cdot \text{K})(600 \text{ K} - 300 \text{ K}) = 3.74 \times 10^3 \text{ J}.$$

(b) Since the process takes place at constant volume, the work  $W$  done by the gas is zero, and the first law of thermodynamics tells us that the change in the internal energy is

$$\Delta E_{\text{int}} = Q = 3.74 \times 10^3 \text{ J}.$$

(c) The work  $W$  done by the gas is zero.

The process  $2 \rightarrow 3$  is adiabatic.

(d) The heat added is zero.

(e) The change in the internal energy is

$$\Delta E_{\text{int}} = nC_V \Delta T = \frac{3}{2} nR \Delta T = \frac{3}{2} (1.00 \text{ mol})(8.31 \text{ J/mol} \cdot \text{K})(455 \text{ K} - 600 \text{ K}) = -1.81 \times 10^3 \text{ J}.$$

(f) According to the first law of thermodynamics the work done by the gas is

$$W = Q - \Delta E_{\text{int}} = +1.81 \times 10^3 \text{ J}.$$

The process  $3 \rightarrow 1$  takes place at constant pressure.

(g) The heat added is

$$Q = nC_p \Delta T = \frac{5}{2} nR \Delta T = \frac{5}{2} (1.00 \text{ mol})(8.31 \text{ J/mol} \cdot \text{K})(300 \text{ K} - 455 \text{ K}) = -3.22 \times 10^3 \text{ J}.$$

(h) The change in the internal energy is

$$\Delta E_{\text{int}} = nC_V \Delta T = \frac{3}{2} nR \Delta T = \frac{3}{2} (1.00 \text{ mol})(8.31 \text{ J/mol} \cdot \text{K})(300 \text{ K} - 455 \text{ K}) = -1.93 \times 10^3 \text{ J}.$$

(i) According to the first law of thermodynamics the work done by the gas is

$$W = Q - \Delta E_{\text{int}} = -3.22 \times 10^3 \text{ J} + 1.93 \times 10^3 \text{ J} = -1.29 \times 10^3 \text{ J}.$$

(j) For the entire process the heat added is

$$Q = 3.74 \times 10^3 \text{ J} + 0 - 3.22 \times 10^3 \text{ J} = 520 \text{ J}.$$

(k) The change in the internal energy is

$$\Delta E_{\text{int}} = 3.74 \times 10^3 \text{ J} - 1.81 \times 10^3 \text{ J} - 1.93 \times 10^3 \text{ J} = 0.$$

(l) The work done by the gas is

$$W = 0 + 1.81 \times 10^3 \text{ J} - 1.29 \times 10^3 \text{ J} = 520 \text{ J}.$$

(m) We first find the initial volume. Use the ideal gas law  $p_1 V_1 = nRT_1$  to obtain

$$V_1 = \frac{nRT_1}{p_1} = \frac{(1.00 \text{ mol})(8.31 \text{ J/mol} \cdot \text{K})(300 \text{ K})}{(1.013 \times 10^5 \text{ Pa})} = 2.46 \times 10^{-2} \text{ m}^3.$$

(n) Since  $1 \rightarrow 2$  is a constant volume process,  $V_2 = V_1 = 2.46 \times 10^{-2} \text{ m}^3$ . The pressure for state 2 is

$$p_2 = \frac{nRT_2}{V_2} = \frac{(1.00 \text{ mol})(8.31 \text{ J/mol} \cdot \text{K})(600 \text{ K})}{2.46 \times 10^{-2} \text{ m}^3} = 2.02 \times 10^5 \text{ Pa}.$$

This is approximately equal to 2.00 atm.

(o)  $3 \rightarrow 1$  is a constant pressure process. The volume for state 3 is

$$V_3 = \frac{nRT_3}{p_3} = \frac{(1.00 \text{ mol})(8.31 \text{ J/mol} \cdot \text{K})(455 \text{ K})}{1.013 \times 10^5 \text{ Pa}} = 3.73 \times 10^{-2} \text{ m}^3.$$

(p) The pressure for state 3 is the same as the pressure for state 1:  $p_3 = p_1 = 1.013 \times 10^5 \text{ Pa}$  (1.00 atm)