5. **THINK** The pressure difference between two sides of the window results in a net force acting on the window.

EXPRESS The air inside pushes outward with a force given by $p_i A$, where p_i is the pressure inside the room and A is the area of the window. Similarly, the air on the outside pushes inward with a force given by $p_o A$, where p_o is the pressure outside. The magnitude of the net force is $F = (p_i - p_o)A$.

ANALYZE Since 1 atm = 1.013×10^5 Pa, the net force is

$$F = (p_i - p_o)A = (1.0 \text{ atm} - 0.96 \text{ atm})(1.013\times10^5 \text{ Pa/atm})(3.4 \text{ m})(2.1 \text{ m})$$
$$= 2.9 \times 10^4 \text{ N}.$$

7. (a) The pressure difference results in forces applied as shown in the figure. We consider a team of horses pulling to the right. To pull the sphere apart, the team must exert a force at least as great as the horizontal component of the total force determined by "summing" (actually, integrating) these force vectors.

We consider a force vector at angle θ. Its leftward component is $\Delta p \cos \theta dA$, where dA is the area element for where the force is applied. We make use of the symmetry of the problem and let dA be that of a ring of constant θ on the surface. The radius of the ring is $r = R \sin \theta$, where R is the radius of the sphere. If the angular width of the ring is $d\theta$, in radians, then its width is $R\, d\theta$ and its area is $dA = 2\pi R^2 \sin \theta\, d\theta$. Thus the net horizontal component of the force of the air is given by

$$F_h = 2\pi R^2 \Delta p \int_0^{\pi/2} \sin\theta \cos\theta d\theta = \pi R^2 \Delta p \sin^2\theta \Big|_0^{\pi/2} = \pi R^2 \Delta p.$$

(b) We use 1 atm = 1.01×10^5 Pa to show that Δp = 0.90 atm = 9.09×10^4 Pa. The sphere radius is R = 0.30 m, so

$$F_h = \pi(0.30 \text{ m})^2(9.09 \times 10^4 \text{ Pa}) = 2.6 \times 10^4 \text{ N}.$$

(c) One team of horses could be used if one half of the sphere is attached to a sturdy wall. The force of the wall on the sphere would balance the force of the horses.

9. The hydrostatic blood pressure is the gauge pressure in the column of blood between feet and brain. We calculate the gauge pressure using Eq. 14-7.

(a) The gauge pressure at the heart of the *Argentinosaurus* is

$$p_{\text{heart}} = p_{\text{brain}} + \rho g h = 80\ \text{torr} + (1.06\times10^3\ \text{kg/m}^3)(9.8\ \text{m/s}^2)(21\ \text{m} - 9.0\ \text{m})\left(\frac{1\ \text{torr}}{133.33\ \text{Pa}}\right)$$

$$= 1.0\times10^3\ \text{torr}.$$

(b) The gauge pressure at the feet of the *Argentinosaurus* is

$$p_{\text{feet}} = p_{\text{brain}} + \rho g h' = 80\ \text{torr} + (1.06\times10^3\ \text{kg/m}^3)(9.8\ \text{m/s}^2)(21\ \text{m})\left(\frac{1\ \text{torr}}{133.33\ \text{Pa}}\right)$$

$$= 80\ \text{torr} + 1642\ \text{torr} = 1722\ \text{torr} \approx 1.7\times10^3\ \text{torr}.$$

14. We estimate the pressure difference (specifically due to hydrostatic effects) as follows:

$$\Delta p = \rho g h = (1.06 \times 10^3 \text{ kg/m}^3)(9.8 \text{ m/s}^2)(1.83 \text{ m}) = 1.90 \times 10^4 \text{ Pa.}$$

20. (a) The force on face A of area A_A due to the water pressure alone is

$$F_A = p_A A_A = \rho_w g h_A A_A = \rho_w g (2d) d^2 = 2\left(1.0\times10^3 \text{ kg/m}^3\right)\left(9.8 \text{ m/s}^2\right)\left(5.0 \text{ m}\right)^3$$
$$= 2.5\times10^6 \text{ N.}$$

Adding the contribution from the atmospheric pressure,

$$F_0 = (1.0 \times 10^5 \text{ Pa})(5.0 \text{ m})^2 = 2.5 \times 10^6 \text{ N,}$$

we have

$$F_A' = F_0 + F_A = 2.5\times10^6 \text{ N} + 2.5\times10^6 \text{ N} = 5.0\times10^6 \text{ N.}$$

(b) The force on face B due to water pressure alone is

$$F_B = p_{\text{avg}B} A_B = \rho_\omega g \left(\frac{5d}{2}\right) d^2 = \frac{5}{2}\rho_w g d^3 = \frac{5}{2}\left(1.0\times10^3 \text{ kg/m}^3\right)\left(9.8 \text{ m/s}^2\right)\left(5.0 \text{ m}\right)^3$$
$$= 3.1\times10^6 \text{ N.}$$

Adding the contribution from the atmospheric pressure,

$$F_0 = (1.0 \times 10^5 \text{ Pa})(5.0 \text{ m})^2 = 2.5 \times 10^6 \text{ N,}$$

we obtain

$$F_B' = F_0 + F_B = 2.5\times10^6 \text{ N} + 3.1\times10^6 \text{ N} = 5.6\times10^6 \text{ N.}$$

26. The gauge pressure you can produce is

$$p = -\rho g h = -\frac{\left(1000\,\mathrm{kg/m^3}\right)\left(9.8\,\mathrm{m/s^2}\right)\left(4.0\times10^{-2}\,\mathrm{m}\right)}{1.01\times10^{5}\ \mathrm{Pa/atm}} = -3.9\times10^{-3}\,\mathrm{atm}$$

where the minus sign indicates that the pressure inside your lung is less than the outside pressure.

27. **THINK** The atmospheric pressure at a given height depends on the density distribution of air.

EXPRESS If the air density were uniform, ρ =const., then the variation of pressure with height may be written as: $p_2 = p_1 - \rho g(y_2 - y_1)$. We take y_1 to be at the surface of Earth, where the pressure is $p_1 = 1.01 \times 10^5$ Pa, and y_2 to be at the top of the atmosphere, where the pressure is $p_2 = 0$. On the other hand, if the density varies with altitude, then

$$p_2 = p_1 - \int_0^h \rho g \, dy \, .$$

For the case where the density decreases linearly with height, $\rho = \rho_0 (1 - y/h)$, where ρ_0 is the density at Earth's surface and $g = 9.8 \text{ m/s}^2$ for $0 \le y \le h$, the integral becomes

$$p_2 = p_1 - \int_0^h \rho_0 g \left(1 - \frac{y}{h}\right) dy = p_1 - \frac{1}{2}\rho_0 g h.$$

ANALYZE (a) For uniform density with $\rho = 1.3 \text{ kg/m}^3$, we find the height of the atmosphere to be

$$y_2 - y_1 = \frac{p_1}{\rho g} = \frac{1.01 \times 10^5 \text{ Pa}}{(1.3 \text{ kg/m}^3)(9.8 \text{m/s}^2)} = 7.9 \times 10^3 \text{ m} = 7.9 \text{ km}.$$

(b) With density decreasing linearly with height, $p_2 = p_1 - \rho_0 g h / 2$. The condition $p_2 = 0$ implies

$$h = \frac{2p_1}{\rho_0 g} = \frac{2(1.01 \times 10^5 \text{ Pa})}{(1.3 \text{ kg/m}^3)(9.8 \text{ m/s}^2)} = 16 \times 10^3 \text{ m} = 16 \text{ km}.$$

LEARN Actually the decrease in air density is approximately exponential, with pressure halved at a height of about 5.6 km.

28. (a) According to Pascal's principle, $F/A = f/a \rightarrow F = (A/a)f$.

(b) We obtain

$$f = \frac{a}{A}F = \frac{(3.80\text{ cm})^2}{(53.0\text{ cm})^2}(20.0\times 10^3\text{ N}) = 103\text{ N}.$$

The ratio of the squares of diameters is equivalent to the ratio of the areas. We also note that the area units cancel.

31. **THINK** The block floats in both water and oil. We apply Archimedes' principle to analyze the problem.

EXPRESS Let V be the volume of the block. Then, the submerged volume in water is $V_s = 2V/3$. Since the block is floating, by Archimedes' principle the weight of the displaced water is equal to the weight of the block, i.e., $\rho_w V_s = \rho_b V$, where ρ_w is the density of water, and ρ_b is the density of the block.

ANALYZE (a) We substitute $V_s = 2V/3$ to obtain the density of the block:

$$\rho_b = 2\rho_w/3 = 2(1000 \text{ kg/m}^3)/3 \approx 6.7 \times 10^2 \text{ kg/m}^3.$$

(b) Now, if ρ_o is the density of the oil, then Archimedes' principle yields $\rho_o V_s' = \rho_b V$. Since the volume submerged in oil is $V_s' = 0.90V$, the density of the oil is

$$\rho_o = \rho_b \left(\frac{V}{V'}\right) = (6.7\times10^2 \text{ kg/m}^3)\frac{V}{0.90V} = 7.4\times10^2 \text{ kg/m}^3.$$

LEARN Another way to calculate the density of the oil is to note that the mass of the block can be written as

$$m = \rho_b V = \rho_o V_s' = \rho_w V_s.$$

Therefore,

$$\rho_o = \rho_w \left(\frac{V_s}{V_s'}\right) = (1000 \text{ kg/m}^3)\frac{2V/3}{0.90V} = 7.4\times10^2 \text{ kg/m}^3.$$

That is, by comparing the fraction submerged with that in water (or another liquid with known density), the density of the oil can be deduced.

33. **THINK** The iron anchor is submerged in water, so we apply Archimedes' principle to calculate its volume and weight in air.

EXPRESS The anchor is completely submerged in water of density ρ_w. Its apparent weight is $W_{app} = W - F_b$, where $W = mg$ is its actual weight and $F_b = \rho_w gV$ is the buoyant force.

ANALYZE (a) Substituting the values given, we find the volume of the anchor to be

$$V = \frac{W - W_{app}}{\rho_w g} = \frac{F_b}{\rho_w g} = \frac{200\ \text{N}}{\left(1000\ \text{kg/m}^3\right)\left(9.8\ \text{m/s}^2\right)} = 2.04 \times 10^{-2}\ \text{m}^3.$$

(b) The mass of the anchor is $m = \rho_{Fe} g$, where ρ_{Fe} is the density of iron (found in Table 14-1). Therefore, its weight in air is

$$W = mg = \rho_{Fe} Vg = (7870\ \text{kg/m}^3)(2.04 \times 10^{-2}\ \text{m}^3)(9.80\ \text{m/s}^2) = 1.57 \times 10^3\ \text{N}.$$

LEARN In general, the apparent weight of an object of density ρ that is completely submerged in a fluid of density ρ_f can be written as $W_{app} = (\rho - \rho_f)Vg$.

35. The problem intends for the children to be completely above water. The total downward pull of gravity on the system is

$$3(356\,\text{N}) + N\rho_{\text{wood}}gV$$

where N is the (minimum) number of logs needed to keep them afloat and V is the volume of each log:

$$V = \pi(0.15\text{ m})^2\,(1.80\text{ m}) = 0.13\text{ m}^3.$$

The buoyant force is $F_b = \rho_{\text{water}}gV_{\text{submerged}}$, where we require $V_{\text{submerged}} \le NV$. The density of water is 1000 kg/m^3. To obtain the minimum value of N, we set $V_{\text{submerged}} = NV$ and then round our "answer" for N up to the nearest integer:

$$3(356\,\text{N}) + N\rho_{\text{wood}}gV = \rho_{\text{water}}gNV \;\Rightarrow\; N = \frac{3(356\,\text{N})}{gV(\rho_{\text{water}} - \rho_{\text{wood}})}$$

which yields $N = 4.28 \to 5$ logs.

37. For our estimate of $V_{\text{submerged}}$ we interpret "almost completely submerged" to mean

$$V_{\text{submerged}} \approx \frac{4}{3}\pi r_o^3 \quad \text{where } r_o = 60 \text{ cm}.$$

Thus, equilibrium of forces (on the iron sphere) leads to

$$F_b = m_{\text{iron}} g \;\Rightarrow\; \rho_{\text{water}} g V_{\text{submerged}} = \rho_{\text{iron}} g \left(\frac{4}{3}\pi r_o^3 - \frac{4}{3}\pi r_i^3\right)$$

where r_i is the inner radius (half the inner diameter). Plugging in our estimate for $V_{\text{submerged}}$ as well as the densities of water (1.0 g/cm^3) and iron (7.87 g/cm^3), we obtain the inner diameter:

$$2r_i = 2r_o\left(1 - \frac{1.0\ \text{g/cm}^3}{7.87\ \text{g/cm}^3}\right)^{1/3} = 57.3 \text{ cm}.$$

41. Let V_i be the total volume of the iceberg. The non-visible portion is below water, and thus the volume of this portion is equal to the volume V_f of the fluid displaced by the iceberg. The fraction of the iceberg that is visible is

$$\text{frac} = \frac{V_i - V_f}{V_i} = 1 - \frac{V_f}{V_i}.$$

Since iceberg is floating, Eq. 14-18 applies:

$$F_g = m_i g = m_f g \;\Rightarrow\; m_i = m_f.$$

Since $m = \rho V$, the above equation implies

$$\rho_i V_i = \rho_f V_f \;\Rightarrow\; \frac{V_f}{V_i} = \frac{\rho_i}{\rho_f}.$$

Thus, the visible fraction is

$$\text{frac} = 1 - \frac{V_f}{V_i} = 1 - \frac{\rho_i}{\rho_f}.$$

(a) If the iceberg ($\rho_i = 917\ \text{kg/m}^3$) floats in salt water with $\rho_f = 1024\ \text{kg/m}^3$, then the fraction would be

$$\text{frac} = 1 - \frac{\rho_i}{\rho_f} = 1 - \frac{917\ \text{kg/m}^3}{1024\ \text{kg/m}^3} = 0.10 = 10\%.$$

(b) On the other hand, if the iceberg floats in fresh water ($\rho_f = 1000\ \text{kg/m}^3$), then the fraction would be

$$\text{frac} = 1 - \frac{\rho_i}{\rho_f} = 1 - \frac{917\ \text{kg/m}^3}{1000\ \text{kg/m}^3} = 0.083 = 8.3\%.$$

51. **THINK** We use the equation of continuity to solve for the speed of water as it leaves the sprinkler hole.

EXPRESS Let v_1 be the speed of the water in the hose and v_2 be its speed as it leaves one of the holes. The cross-sectional area of the hose is $A_1 = \pi R^2$. If there are N holes and A_2 is the area of a single hole, then the equation of continuity becomes

$$v_1 A_1 = v_2 (N A_2) \quad \Rightarrow \quad v_2 = \frac{A_1}{N A_2} v_1 = \frac{R^2}{N r^2} v_1$$

where R is the radius of the hose and r is the radius of a hole.

ANALYZE Noting that $R/r = D/d$ (the ratio of diameters) we find the speed to be

$$v_2 = \frac{D^2}{N d^2} v_1 = \frac{(1.9\,\text{cm})^2}{24(0.13\,\text{cm})^2}(0.91\,\text{m/s}) = 8.1\ \text{m/s}.$$

LEARN The equation of continuity implies that the smaller the cross-sectional area of the sprinkler hole, the greater the speed of water as it emerges from the hole.

54. (a) The equation of continuity provides (26 + 19 + 11) L/min = 56 L/min for the flow rate in the main (1.9 cm diameter) pipe.

(b) Using $v = R/A$ and $A = \pi d^2/4$, we set up ratios:

$$\frac{v_{56}}{v_{26}} = \frac{56/\pi(1.9)^2/4}{26/\pi(1.3)^2/4} \approx 1.0.$$

57. **THINK** We use the Bernoulli equation to solve for the flow rate, and the continuity equation to relate cross-sectional area to the vertical distance from the hole.

EXPRESS According to the Bernoulli equation:

$$p_1 + \tfrac{1}{2}\rho v_1^2 + \rho g h_1 = p_2 + \tfrac{1}{2}\rho v_2^2 + \rho g h_2 ,$$

where ρ is the density of water, h_1 is the height of the water in the tank, p_1 is the pressure there, and v_1 is the speed of the water there; h_2 is the altitude of the hole, p_2 is the pressure there, and v_2 is the speed of the water there. The pressure at the top of the tank and at the hole is atmospheric, so $p_1 = p_2$. Since the tank is large we may neglect the water speed at the top; it is much smaller than the speed at the hole. The Bernoulli equation then simplifies to $\rho g h_1 = \tfrac{1}{2}\rho v_2^2 + \rho g h_2$.

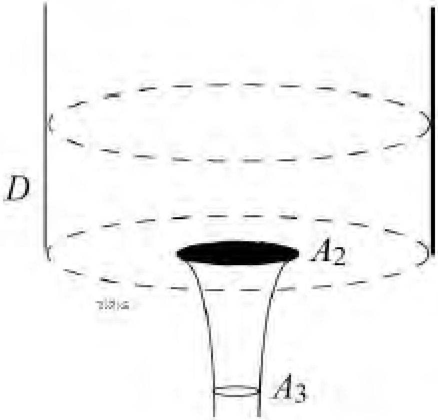
ANALYZE (a) With $D = h_1 - h_2 = 0.30\ \text{m}$, the speed of water as it emerges from the hole is

$$v_2 = \sqrt{2g(h_1 - h_2)} = \sqrt{2\left(9.8\,\text{m/s}^2\right)(0.30\,\text{m})} = 2.42\,\text{m/s}.$$

Thus, the flow rate is

$$A_2v_2 = (6.5\times10^{-4}\ \text{m}^2)(2.42\ \text{m/s}) = 1.6\times10^{-3}\ \text{m}^3/\text{s}.$$

(b) We use the equation of continuity: $A_2v_2 = A_3v_3$, where $A_3 = \frac{1}{2}A_2$ and v_3 is the water speed where the area of the stream is half its area at the hole (see diagram below).



Thus,

$$v_3 = (A_2/A_3)v_2 = 2v_2 = 4.84\ \text{m/s}.$$

The water is in free fall and we wish to know how far it has fallen when its speed is doubled to 4.84 m/s. Since the pressure is the same throughout the fall, $\frac{1}{2}\rho v_2^2 + \rho g h_2 = \frac{1}{2}\rho v_3^2 + \rho g h_3$. Thus,

$$h_2 - h_3 = \frac{v_3^2 - v_2^2}{2g} = \frac{(4.84\,\text{m/s})^2 - (2.42\,\text{m/s})^2}{2(9.8\,\text{m/s}^2)} = 0.90\ \text{m}.$$

LEARN By combing the two expressions obtained from Bernoulli's equation and equation of continuity, the cross-sectional area of the stream may be related to the vertical height fallen as

$$h_2 - h_3 = \frac{v_3^2 - v_2^2}{2g} = \frac{v_2^2}{2g}\left[\left(\frac{A_2}{A_3}\right)^2 - 1\right] = \frac{v_3^2}{2g}\left[1 - \left(\frac{A_3}{A_2}\right)^2\right].$$

59. **THINK** The elevation and cross-sectional area of the pipe are changing, so we apply the Bernoulli equation and continuity equation to analyze the flow of water through the pipe.

EXPRESS To calculate the flow speed at the lower level, we use the equation of continuity: $A_1v_1 = A_2v_2$. Here A_1 is the area of the pipe at the top and v_1 is the speed of the water there; A_2 is the area of the pipe at the bottom and v_2 is the speed of the water there. As for the pressure at the lower level, we use the Bernoulli equation:

$$p_1 + \tfrac{1}{2}\rho v_1^2 + \rho g h_1 = p_2 + \tfrac{1}{2}\rho v_2^2 + \rho g h_2 ,$$

where ρ is the density of water, h_1 is its initial altitude, and h_2 is its final altitude.

ANALYZE (a) From the continuity equation, we find the speed at the lower level to be

$$v_2 = (A_1/A_2)v_1 = [(4.0\text{ cm}^2)/(8.0\text{ cm}^2)]\,(5.0\text{ m/s}) = 2.5\text{m/s}.$$

(b) Similarly, from the Bernoulli equation, the pressure at the lower level is

$$\begin{aligned} p_2 &= p_1 + \frac{1}{2}\rho\left(v_1^2 - v_2^2\right) + \rho g\left(h_1 - h_2\right) \\ &= 1.5\times10^5\text{ Pa} + \frac{1}{2}(1000\text{kg/m}^3)\left[(5.0\text{m/s})^2 - (2.5\text{m/s})^2\right] + (1000\text{kg/m}^3)(9.8\text{m/s}^2)(10\text{ m}) \\ &= 2.6\times10^5\text{ Pa}. \end{aligned}$$

LEARN The water at the lower level has a smaller speed ($v_2 < v_1$) but higher pressure ($p_2 > p_1$).

64. (a) The volume of water (during 10 minutes) is

$$V = (v_1 t) A_1 = (15\,\text{m/s})(10\,\text{min})(60\,\text{s/min})\left(\frac{\pi}{4}\right)(0.03\,\text{m})^2 = 6.4\,\text{m}^3.$$

(b) The speed in the left section of pipe is

$$v_2 = v_1\left(\frac{A_1}{A_2}\right) = v_1\left(\frac{d_1}{d_2}\right)^2 = (15\,\text{m/s})\left(\frac{3.0\,\text{cm}}{5.0\,\text{cm}}\right)^2 = 5.4\,\text{m/s}.$$

(c) Since

$$p_1 + \tfrac{1}{2}\rho v_1^2 + \rho g h_1 = p_2 + \tfrac{1}{2}\rho v_2^2 + \rho g h_2$$

and $h_1 = h_2, p_1 = p_0$, which is the atmospheric pressure,

$$p_2 = p_0 + \frac{1}{2}\rho\left(v_1^2 - v_2^2\right) = 1.01\times10^5\,\text{Pa} + \frac{1}{2}\left(1.0\times10^3\,\text{kg/m}^3\right)\left[(15\,\text{m/s})^2 - (5.4\,\text{m/s})^2\right]$$
$$= 1.99\times10^5\,\text{Pa} = 1.97\,\text{atm}.$$

Thus, the gauge pressure is (1.97 atm – 1.00 atm) = 0.97 atm = 9.8×10^4 Pa.

65. **THINK** The design principles of the Venturi meter, a device that measures the flow speed of a fluid in a pipe, involve both the continuity equation and Bernoulli's equation.

EXPRESS The continuity equation yields $AV = av$, and Bernoulli's equation yields $\frac{1}{2}\rho V^2 = \Delta p + \frac{1}{2}\rho v^2$, where $\Delta p = p_2 - p_1$ with p_2 equal to the pressure in the throat and p_1 the pressure in the pipe. The first equation gives $v = (A/a)V$. We use this to substitute for v in the second equation and obtain

$$\tfrac{1}{2}\rho V^2 = \Delta p + \tfrac{1}{2}\rho (A/a)^2 V^2.$$

The equation can be used to solve for V.

ANALYZE (a) The above equation gives the following expression for V:

$$V = \sqrt{\frac{2\Delta p}{\rho\left(1-(A/a)^2\right)}} = \sqrt{\frac{2a^2\Delta p}{\rho\left(a^2 - A^2\right)}}.$$

(b) We substitute the values given to obtain

$$V = \sqrt{\frac{2a^2\Delta p}{\rho\left(a^2 - A^2\right)}} = \sqrt{\frac{2(32\times10^{-4}\,\text{m}^2)^2(41\times10^3\,\text{Pa} - 55\times10^3\,\text{Pa})}{(1000\,\text{kg/m}^3)\left((32\times10^{-4}\,\text{m}^2)^2 - (64\times10^{-4}\,\text{m}^2)^2\right)}} = 3.06\,\text{m/s}.$$

Consequently, the flow rate is

$$R = AV = (64\times10^{-4}\,\text{m}^2)(3.06\text{ m/s}) = 2.0\times10^{-2}\,\text{m}^3/\text{s}.$$

LEARN The pressure difference Δp between points 1 and 2 is what causes the height difference of the fluid in the two arms of the manometer. Note that $\Delta p = p_2 - p_1 < 0$ (pressure in throat less than that in the pipe), but $a < A$, so the expression inside the

67. (a) The friction force is

$$f = A\Delta p = \rho_\omega g d A = (1.0\times10^3\ \text{kg/m}^3)\ (9.8\ \text{m/s}^2)\ (6.0\text{m})\left(\frac{\pi}{4}\right)(0.040\ \text{m})^2 = 74\ \text{N}.$$

(b) The speed of water flowing out of the hole is $v = \sqrt{2gd}$. Thus, the volume of water flowing out of the pipe in $t = 3.0$ h is

$$V = Avt = \frac{\pi^2}{4}(0.040\ \text{m})^2\sqrt{2(9.8\ \text{m/s}^2)\ (6.0\ \text{m})}\ (3.0\ \text{h})\ (3600\ \text{s/h}) = 1.5\times10^2\text{m}^3.$$

71. (a) The stream of water emerges horizontally ($\theta_0 = 0°$ in the notation of Chapter 4) with $v_0 = \sqrt{2gh}$. Setting $y - y_0 = -(H - h)$ in Eq. 4-22, we obtain the "time-of-flight"

$$t = \sqrt{\frac{-2(H-h)}{-g}} = \sqrt{\frac{2}{g}(H-h)}.$$

Using this in Eq. 4-21, where $x_0 = 0$ by choice of coordinate origin, we find

$$x = v_0 t = \sqrt{2gh}\sqrt{\frac{2(H-h)}{g}} = 2\sqrt{h(H-h)} = 2\sqrt{(10\text{ cm})(40\text{ cm} - 10\text{ cm})} = 35\text{ cm}.$$

(b) The result of part (a) (which, when squared, reads $x^2 = 4h(H - h)$) is a quadratic equation for h once x and H are specified. Two solutions for h are therefore mathematically possible, but are they both physically possible? For instance, are both solutions positive and less than H? We employ the quadratic formula:

$$h^2 - Hh + \frac{x^2}{4} = 0 \;\Rightarrow\; h = \frac{H \pm \sqrt{H^2 - x^2}}{2}$$

which permits us to see that both roots are physically possible, so long as $x < H$. Labeling the larger root h_1 (where the plus sign is chosen) and the smaller root as h_2 (where the minus sign is chosen), then we note that their sum is simply

$$h_1 + h_2 = \frac{H + \sqrt{H^2 - x^2}}{2} + \frac{H - \sqrt{H^2 - x^2}}{2} = H.$$

Thus, one root is related to the other (generically labeled h' and h) by $h' = H - h$. Its numerical value is $h' = 40\text{cm} - 10\text{ cm} = 30\text{ cm}.$

(c) We wish to maximize the function $f = x^2 = 4h(H - h)$. We differentiate with respect to h and set equal to zero to obtain

$$\frac{df}{dh} = 4H - 8h = 0 \Rightarrow h = \frac{H}{2}$$

or $h = (40\text{ cm})/2 = 20$ cm, as the depth from which an emerging stream of water will travel the maximum horizontal distance.