3. (a) The angular wave number is $k = \frac{2\pi}{\lambda} = \frac{2\pi}{1.80\,\text{m}} = 3.49\,\text{m}^{-1}$.

(b) The speed of the wave is $v = \lambda f = \frac{\lambda\omega}{2\pi} = \frac{(1.80\,\text{m})(110\,\text{rad/s})}{2\pi} = 31.5\,\text{m/s}$.

5. (a) The motion from maximum displacement to zero is one-fourth of a cycle. One-fourth of a period is 0.170 s, so the period is $T = 4(0.170\text{ s}) = 0.680\text{ s}$.

(b) The frequency is the reciprocal of the period:

$$f = \frac{1}{T} = \frac{1}{0.680\,\text{s}} = 1.47\,\text{Hz}.$$

(c) A sinusoidal wave travels one wavelength in one period:

$$v = \frac{\lambda}{T} = \frac{1.40\,\text{m}}{0.680\,\text{s}} = 2.06\,\text{m/s}.$$

7. (a) From the simple harmonic motion relation $u_m = y_m\omega$, we have

$$\omega = \frac{16\ \text{m/s}}{0.040\ \text{m}} = 400\,\text{rad/s}.$$

Since $\omega = 2\pi f$, we obtain $f = 64$ Hz.

(b) Using $v = f\lambda$, we find $\lambda = (80\ \text{m/s})/(64\ \text{Hz}) = 1.26\ \text{m} \approx 1.3\ \text{m}$.

(c) The amplitude of the transverse displacement is $y_m = 4.0\ \text{cm} = 4.0\times10^{-2}\,\text{m}$.

(d) The wave number is $k = 2\pi/\lambda = 5.0$ rad/m.

(e) As shown in (a), the angular frequency is $\omega = (16\ \text{m/s})/(0.040\ \text{m}) = 4.0\times10^{2}$ rad/s.

(f) The function describing the wave can be written as

$$y = 0.040\sin(5x - 400t + \phi)$$

where distances are in meters and time is in seconds. We adjust the phase constant ϕ to satisfy the condition $y = 0.040$ at $x = t = 0$. Therefore, sin $\phi = 1$, for which the "simplest" root is $\phi = \pi/2$. Consequently, the answer is

$$y = 0.040\sin\left(5x - 400t + \frac{\pi}{2}\right).$$

(g) The sign in front of ω is minus.

9. (a) The amplitude y_m is half of the 6.00 mm vertical range shown in the figure, that is, $y_m = 3.0$ mm.

(b) The speed of the wave is $v = d/t = 15$ m/s, where $d = 0.060$ m and $t = 0.0040$ s. The angular wave number is $k = 2\pi/\lambda$ where $\lambda = 0.40$ m. Thus,

$$k = \frac{2\pi}{\lambda} = 16 \text{ rad/m}.$$

(c) The angular frequency is found from

$$\omega = kv = (16 \text{ rad/m})(15 \text{ m/s}) = 2.4\times10^2 \text{ rad/s}.$$

(d) We choose the minus sign (between kx and ωt) in the argument of the sine function because the wave is shown traveling to the right (in the $+x$ direction, see Section 16-5). Therefore, with SI units understood, we obtain

$$y = y_m \sin(kx - kvt) \approx 0.0030 \sin(16x - 2.4\times10^2\, t).$$

15. **THINK** Numerous physical properties of a traveling wave can be deduced from its wave function.

EXPRESS We first recall that from Eq. 16-10, a general expression for a sinusoidal wave traveling along the +x direction is

$$y(x,t) = y_m \sin(kx - \omega t + \phi)$$

where y_m is the amplitude, $k = 2\pi/\lambda$ is the angular wave number, $\omega = 2\pi/T$ is the angular frequency and ϕ is the phase constant. The wave speed is given by $v = \sqrt{\tau/\mu}$, where τ is the tension in the string and μ is the linear mass density of the string.

ANALYZE (a) The amplitude of the wave is y_m=0.120 mm.

(b) The wavelength is $\lambda = v/f = \sqrt{\tau/\mu}/f$ and the angular wave number is

$$k = \frac{2\pi}{\lambda} = 2\pi f\sqrt{\frac{\mu}{\tau}} = 2\pi(100\,\text{Hz})\sqrt{\frac{0.50\,\text{kg/m}}{10\,\text{N}}} = 141\,\text{m}^{-1}.$$

(c) The frequency is f = 100 Hz, so the angular frequency is

$$\omega = 2\pi f = 2\pi(100\text{ Hz}) = 628\text{ rad/s}.$$

(d) We may write the string displacement in the form $y = y_m \sin(kx + \omega t)$. The plus sign is used since the wave is traveling in the negative x direction.

LEARN In summary, the wave can be expressed as

$$y = (0.120\,\text{mm})\sin\left[\left(141\,\text{m}^{-1}\right)x + \left(628\,\text{s}^{-1}\right)t\right].$$

17. (a) The wave speed is given by $v = \lambda/T = \omega/k$, where λ is the wavelength, T is the period, ω is the angular frequency ($2\pi/T$), and k is the angular wave number ($2\pi/\lambda$). The displacement has the form $y = y_m \sin(kx + \omega t)$, so $k = 2.0\ \text{m}^{-1}$ and $\omega = 30$ rad/s. Thus

$$v = (30\ \text{rad/s})/(2.0\ \text{m}^{-1}) = 15\ \text{m/s}.$$

(b) Since the wave speed is given by $v = \sqrt{\tau/\mu}$, where τ is the tension in the string and μ is the linear mass density of the string, the tension is

$$\tau = \mu v^2 = \left(1.6\times10^{-4}\ \text{kg/m}\right)\left(15\ \text{m/s}\right)^2 = 0.036\ \text{N}.$$

20. From $v = \sqrt{\tau / \mu}$, we have

$$\frac{v_{\text{new}}}{v_{\text{old}}} = \frac{\sqrt{\tau_{\text{new}} / \mu_{\text{new}}}}{\sqrt{\tau_{\text{old}} / \mu_{\text{old}}}} = \sqrt{2}.$$

26. Using Eq. 16–33 for the average power and Eq. 16–26 for the speed of the wave, we solve for $f = \omega/2\pi$:

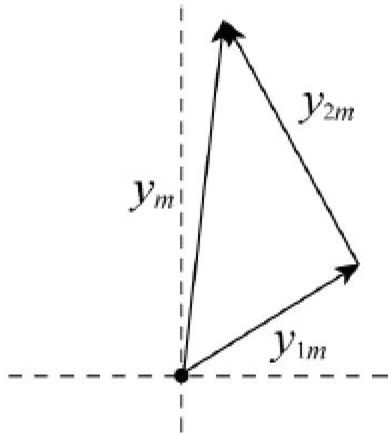
$$f = \frac{1}{2\pi y_m}\sqrt{\frac{2P_{avg}}{\mu\sqrt{\tau/\mu}}} = \frac{1}{2\pi(7.70\times10^{-3}\,\text{m})}\sqrt{\frac{2(85.0\,\text{W})}{\sqrt{(36.0\,\text{N})(0.260\,\text{kg}/2.70\,\text{m})}}} = 198\ \text{Hz}.$$

30. The wave $y(x,t) = (4.00\text{ mm})\, h[(30\text{ m}^{-1})x + (6.0\text{ s}^{-1})t]$ is of the form $h(kx - \omega t)$ with angular wave number $k = 30\text{ m}^{-1}$ and angular frequency $\omega = 6.0\text{ rad/s}$. Thus, the speed of the wave is

$$v = \omega / k = (6.0\text{ rad/s})/(30\text{ m}^{-1}) = 0.20\text{ m/s}.$$

35. **THINK** We use phasors to add the two waves and calculate the amplitude of the resultant wave.

EXPRESS The phasor diagram is shown below: y_{1m} and y_{2m} represent the original waves and y_m represents the resultant wave. The phasors corresponding to the two constituent waves make an angle of 90° with each other, so the triangle is a right triangle.



ANALYZE The Pythagorean theorem gives

$$y_m^2 = y_{1m}^2 + y_{2m}^2 = (3.0\,\text{cm})^2 + (4.0\,\text{cm})^2 = (25\,\text{cm})^2 .$$

Thus, the amplitude of the resultant wave is y_m = 5.0 cm.

LEARN When adding two waves, it is convenient to represent each wave with a phasor, which is a vector whose magnitude is equal to the amplitude of the wave. The same result, however, could also be obtained as follows: Writing the two waves as $y_1 = 3\sin(kx - \omega t)$ and $y_2 = 4\sin(kx - \omega t + \pi/2) = 4\cos(kx - \omega t)$, we have, after a little algebra,

$$y = y_1 + y_2 = 3\sin(kx - \omega t) + 4\cos(kx - \omega t) = 5\left[\frac{3}{5}\sin(kx - \omega t) + \frac{4}{5}\cos(kx - \omega t)\right]$$
$$= 5\sin(kx - \omega t + \phi)$$

where $\phi = \tan^{-1}(4/3)$. In deducing the phase ϕ, we set $\cos\phi = 3/5$ and $\sin\phi = 4/5$, and use the relation $\cos\phi\sin\theta + \sin\phi\cos\theta = \sin(\theta + \phi)$.

41. **THINK** A string clamped at both ends can be made to oscillate in standing wave patterns.

EXPRESS The wave speed is given by $v = \sqrt{\tau/\mu}$, where τ is the tension in the string and μ is the linear mass density of the string. Since the mass density is the mass per unit length, $\mu = M/L$, where M is the mass of the string and L is its length. The possible wavelengths of a standing wave are given by $\lambda_n = 2L/n$, where L is the length of the string and n is an integer.

ANALYZE (a) The wave speed is

$$v = \sqrt{\frac{\tau L}{M}} = \sqrt{\frac{(96.0\text{ N})(8.40\text{ m})}{0.120\text{ kg}}} = 82.0\text{ m/s}.$$

(b) The longest possible wavelength λ for a standing wave is related to the length of the string by $L = \lambda_1/2$ ($n = 1$), so $\lambda_1 = 2L = 2(8.40\text{ m}) = 16.8\text{ m}$.

(c) The corresponding frequency is $f_1 = v/\lambda_1 = (82.0\text{ m/s})/(16.8\text{ m}) = 4.88\text{ Hz}$.

LEARN The resonant frequencies are given by

$$f_n = \frac{v}{\lambda} = \frac{v}{2L/n} = n\frac{v}{2L} = nf_1,$$

where $f_1 = v/\lambda_1 = v/2L$. The oscillation mode with $n = 1$ is called the fundamental mode or the first harmonic.

43. **THINK** A string clamped at both ends can be made to oscillate in standing wave patterns.

EXPRESS Possible wavelengths are given by $\lambda_n = 2L/n$, where L is the length of the wire and n is an integer. The corresponding frequencies are $f_n = v/\lambda_n = nv/2L$, where v is the wave speed. The wave speed is given by $v = \sqrt{\tau/\mu} = \sqrt{\tau L/M}$, where τ is the tension in the wire, μ is the linear mass density of the wire, and M is the mass of the wire. $\mu = M/L$ was used to obtain the last form. Thus,

$$f_n = \frac{n}{2L}\sqrt{\frac{\tau L}{M}} = \frac{n}{2}\sqrt{\frac{\tau}{LM}} = \frac{n}{2}\sqrt{\frac{250\ \text{N}}{(10.0\ \text{m})\ (0.100\ \text{kg})}} = n\ (7.91\ \text{Hz}).$$

ANALYZE (a) The lowest frequency is $f_1 = 7.91$ Hz.

(b) The second lowest frequency is $f_2 = 2(7.91\ \text{Hz}) = 15.8$ Hz.

(c) The third lowest frequency is $f_3 = 3(7.91\ \text{Hz}) = 23.7$ Hz.

LEARN The frequencies are integer multiples of the fundamental frequency f_1. This means that the difference between any successive pair of the harmonic frequencies is equal to the fundamental frequency f_1.

45. **THINK** The difference between any successive pair of the harmonic frequencies is equal to the fundamental frequency.

EXPRESS The resonant wavelengths are given by $\lambda_n = 2L/n$, where L is the length of the string and n is an integer, and the resonant frequencies are

$$f_n = v/\lambda = nv/2L = nf_1,$$

where v is the wave speed. Suppose the lower frequency is associated with the integer n. Then, since there are no resonant frequencies between, the higher frequency is associated with $n + 1$. The frequency difference between successive modes is

$$\Delta f = f_{n+1} - f_n = \frac{v}{2L} = f_1.$$

ANALYZE (a) The lowest possible resonant frequency is

$$f_1 = \Delta f = f_{n+1} - f_n = 420 \text{ Hz} - 315 \text{ Hz} = 105 \text{ Hz}.$$

(b) The longest possible wavelength is $\lambda_1 = 2L$. If f_1 is the lowest possible frequency then

$$v = \lambda_1 f_1 = (2L)f_1 = 2(0.75 \text{ m})(105 \text{ Hz}) = 158 \text{ m/s}.$$

LEARN Since 315 Hz = 3(105 Hz) and 420 Hz = 4(105 Hz), the two frequencies correspond to $n = 3$ and $n = 4$, respectively.

49. (a) Equation 16-26 gives the speed of the wave:

$$v = \sqrt{\frac{\tau}{\mu}} = \sqrt{\frac{150\,\text{N}}{7.20\times10^{-3}\,\text{kg/m}}} = 144.34\ \text{m/s} \approx 1.44\times10^{2}\ \text{m/s}.$$

(b) From the figure, we find the wavelength of the standing wave to be

$$\lambda = (2/3)(90.0\ \text{cm}) = 60.0\ \text{cm}.$$

(c) The frequency is

$$f = \frac{v}{\lambda} = \frac{1.44\times10^{2}\ \text{m/s}}{0.600\,\text{m}} = 241\,\text{Hz}.$$

51. **THINK** In this problem, in order to produce the standing wave pattern, the two waves must have the same amplitude, the same angular frequency, and the same angular wave number, but they travel in opposite directions.

EXPRESS We take the two waves to be

$$y_1 = y_m \sin(kx - \omega t), \quad y_2 = y_m \sin(kx + \omega t).$$

The superposition principle gives

$$y'(x,t) = y_1(x,t) + y_2(x,t) = y_m \sin(kx - \omega t) + y_m \sin(kx + \omega t) = \left[2y_m \sin kx\right]\cos \omega t .$$

ANALYZE (a) The amplitude y_m is half the maximum displacement of the standing wave, or (0.01 m)/2 = 5.0×10^{-3} m.

(b) Since the standing wave has three loops, the string is three half-wavelengths long: $L = 3\lambda/2$, or $\lambda = 2L/3$. With $L = 3.0$m, $\lambda = 2.0$ m. The angular wave number is

$$k = 2\pi/\lambda = 2\pi/(2.0 \text{ m}) = 3.1 \text{ m}^{-1}.$$

(c) If v is the wave speed, then the frequency is

$$f = \frac{v}{\lambda} = \frac{3v}{2L} = \frac{3(100\,\text{m/s})}{2(3.0\,\text{m})} = 50 \text{ Hz}.$$

The angular frequency is the same as that of the standing wave, or

$$\omega = 2\pi f = 2\pi(50 \text{ Hz}) = 314 \text{ rad/s}.$$

(d) If one of the waves has the form $y_2(x,t) = y_m \sin(kx + \omega t)$, then the other wave must have the form $y_1(x,t) = y_m \sin(kx - \omega t)$. The sign in front of ω for $y'(x,t)$ is minus.

LEARN Using the results above, the two waves can be written as

$$y_1 = \left(5.0\times10^{-3}\,\text{m}\right)\sin\left[\left(3.14\,\text{m}^{-1}\right)x - \left(314\,\text{s}^{-1}\right)t\right]$$

and

$$y_2 = \left(5.0\times10^{-3}\,\text{m}\right)\sin\left[\left(3.14\,\text{m}^{-1}\right)x + \left(314\,\text{s}^{-1}\right)t\right].$$