5. **THINK** The frequency of oscillation of the current in the LC circuit of the generator is $f = 1/2\pi\sqrt{LC}$, where C is the capacitance and L is the inductance. This frequency is the same as the frequency of an electromagnetic wave.

EXPRESS If f is the frequency and λ is the wavelength of an electromagnetic wave, then $f\lambda = c$. Thus,

$$\frac{\lambda}{2\pi\sqrt{LC}} = c.$$

ANALYZE The solution for L is

$$L = \frac{\lambda^2}{4\pi^2 Cc^2} = \frac{(550\times10^{-9}\ \text{m})^2}{4\pi^2(17\times10^{-12}\ \text{F})(2.998\times10^{8}\ \text{m/s})^2} = 5.00\times10^{-21}\ \text{H}.$$

This is exceedingly small.

LEARN The frequency is

$$f = \frac{c}{\lambda} = \frac{3.0\times10^{8}\ \text{m/s}}{550\times10^{-9}\ \text{m}} = 5.45\times10^{14}\ \text{Hz}.$$

The EM wave is in the visible spectrum.

6. The emitted wavelength is

$$\lambda = \frac{c}{f} = 2\pi c\sqrt{LC} = 2\pi\left(2.998\times10^{8}\,\text{m/s}\right)\sqrt{\left(0.253\times10^{-6}\,\text{H}\right)\left(25.0\times10^{-12}\,\text{F}\right)} = 4.74\ \text{m}.$$

8. The intensity of the signal at Proxima Centauri is

$$I = \frac{P}{4\pi r^2} = \frac{1.0\times10^{6}\ \text{W}}{4\pi\left[\left(4.3\ \text{ly}\right)\left(9.46\times10^{15}\ \text{m/ly}\right)\right]^{2}} = 4.8\times10^{-29}\ \text{W/m}^2.$$

11. (a) The amplitude of the magnetic field is

$$B_m = \frac{E_m}{c} = \frac{2.0\text{V/m}}{2.998\times10^8\text{m/s}} = 6.67\times10^{-9}\text{T} \approx 6.7\times10^{-9}\text{T}.$$

(b) Since the $\vec{E}$-wave oscillates in the z direction and travels in the x direction, we have $B_x = B_z = 0$. So, the oscillation of the magnetic field is parallel to the y axis.

(c) The direction ($+x$) of the electromagnetic wave propagation is determined by $\vec{E}\times\vec{B}$. If the electric field points in $+z$, then the magnetic field must point in the $-y$ direction.

With SI units understood, we may write

$$B_y = B_m \cos\left[\pi\times10^{15}\left(t-\frac{x}{c}\right)\right] = \frac{2.0\cos\left[10^{15}\pi\left(t-x/c\right)\right]}{3.0\times10^8}$$

$$= \left(6.7\times10^{-9}\right)\cos\left[10^{15}\pi\left(t-\frac{x}{c}\right)\right]$$

13. (a) We use $I = E_m^2/2\mu_0 c$ to calculate E_m:

$$E_m = \sqrt{2\mu_0 I_c} = \sqrt{2\left(4\pi \times 10^{-7}\ \text{T}\cdot\text{m/A}\right)\left(1.40 \times 10^{3}\ \text{W/m}^2\right)\left(2.998 \times 10^{8}\ \text{m/s}\right)}$$

$$= 1.03 \times 10^{3}\ \text{V/m}.$$

(b) The magnetic field amplitude is therefore

$$B_m = \frac{E_m}{c} = \frac{1.03 \times 10^{4}\ \text{V/m}}{2.998 \times 10^{8}\ \text{m/s}} = 3.43 \times 10^{-6}\ \text{T}.$$

17. (a) The magnetic field amplitude of the wave is

$$B_m = \frac{E_m}{c} = \frac{2.0\ \text{V/m}}{2.998\times10^{8}\ \text{m/s}} = 6.7\times10^{-9}\ \text{T}.$$

(b) The intensity is

$$I = \frac{E_m^2}{2\mu_0 c} = \frac{(2.0\ \text{V/m})^2}{2(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A})(2.998\times10^{8}\ \text{m/s})} = 5.3\times10^{-3}\ \text{W/m}^2.$$

(c) The power of the source is

$$P = 4\pi r^2 I_{\text{avg}} = 4\pi(10\,\text{m})^2\left(5.3\times10^{-3}\ \text{W/m}^2\right) = 6.7\ \text{W}.$$

19. **THINK** The plasma completely reflects all the energy incident on it, so the radiation pressure is given by $p_r = 2I/c$, where I is the intensity.

EXPRESS The intensity is $I = P/A$, where P is the power and A is the area intercepted by the radiation.

ANALYZE Thus, the radiation pressure is

$$p_r = \frac{2I}{c} = \frac{2P}{Ac} = \frac{2\left(1.5\times10^9\,\mathrm{W}\right)}{\left(1.00\times10^{-6}\,\mathrm{m}^2\right)\left(2.998\times10^8\,\mathrm{m/s}\right)} = 1.0\times10^7\,\mathrm{Pa}.$$

LEARN In the case of total absorption, the radiation pressure would be $p_r = I/c$, a factor of 2 smaller than the case of total reflection.

20. (a) The radiation pressure produces a force equal to

$$F_r = p_r\left(\pi R_e^2\right) = \left(\frac{I}{c}\right)\left(\pi R_e^2\right) = \frac{\pi\left(1.4\times10^3\,\mathrm{W/m^2}\right)\left(6.37\times10^6\,\mathrm{m}\right)^2}{2.998\times10^8\,\mathrm{m/s}} = 6.0\times10^8\,\mathrm{N}.$$

(b) The gravitational pull of the Sun on the Earth is

$$F_{\mathrm{grav}} = \frac{GM_sM_e}{d_{es}^2} = \frac{\left(6.67\times10^{-11}\,\mathrm{N\cdot m^2/kg^2}\right)\left(2.0\times10^{30}\,\mathrm{kg}\right)\left(5.98\times10^{24}\,\mathrm{kg}\right)}{\left(1.5\times10^{11}\,\mathrm{m}\right)^2}$$

$$= 3.6\times10^{22}\,\mathrm{N},$$

22. The radiation pressure is

$$p_r = \frac{I}{c} = \frac{10\,\mathrm{W/m^2}}{2.998 \times 10^8\,\mathrm{m/s}} = 3.3 \times 10^{-8}\,\mathrm{Pa}.$$

26. The mass of the cylinder is $m=\rho(\pi D^2/4)H,$ where D is the diameter of the cylinder. Since it is in equilibrium

$$F_{\text{net}} = mg - F_r = \frac{\pi H D^2 g\rho}{4} - \left(\frac{\pi D^2}{4}\right)\left(\frac{2I}{c}\right) = 0.$$

We solve for H:

$$H = \frac{2I}{gc\rho} = \left(\frac{2P}{\pi D^2/4}\right)\frac{1}{gc\rho}$$

$$= \frac{2(4.60\,\text{W})}{[\pi(2.60\times10^{-3}\,\text{m})^2/4](9.8\,\text{m/s}^2)(3.0\times10^8\,\text{m/s})(1.20\times10^3\,\text{kg/m}^3)}$$

$$= 4.91\times10^{-7}\,\text{m}.$$

29. **THINK** The laser beam carries both energy and momentum. The total momentum of the spaceship and light is conserved.

EXPRESS If the beam carries energy U away from the spaceship, then it also carries momentum $p = U/c$ away. By momentum conservation, this is the magnitude of the momentum acquired by the spaceship. If P is the power of the laser, then the energy carried away in time t is $U = Pt$.

ANALYZE We note that there are 86400 seconds in a day. Thus, $p = Pt/c$ and, if m is mass of the spaceship, its speed is

$$v = \frac{p}{m} = \frac{Pt}{mc} = \frac{(10 \times 10^3 \text{ W})(86400 \text{ s})}{(1.5 \times 10^3 \text{ kg})(2.998 \times 10^8 \text{ m/s})} = 1.9 \times 10^{-3} \text{ m/s}.$$

LEARN As expected, the speed of the spaceship is proportional to the power of the laser beam.

32. After passing through the first polarizer the initial intensity I_0 reduces by a factor of 1/2. After passing through the second one it is further reduced by a factor of $\cos^2(\pi - \theta_1 - \theta_2) = \cos^2(\theta_1 + \theta_2)$. Finally, after passing through the third one it is again reduced by a factor of $\cos^2(\pi - \theta_2 - \theta_3) = \cos^2(\theta_2 + \theta_3)$. Therefore,

$$\frac{I_f}{I_0} = \frac{1}{2}\cos^2(\theta_1 + \theta_2)\cos^2(\theta_2 + \theta_3) = \frac{1}{2}\cos^2(50° + 50°)\cos^2(50° + 50°)$$
$$= 4.5 \times 10^{-4}.$$

Thus, 0.045% of the light's initial intensity is transmitted.

34. In this case, we replace $I_0 \cos^2 70°$ by $\frac{1}{2} I_0$ as the intensity of the light after passing through the first polarizer. Therefore,

$$I_f = \frac{1}{2} I_0 \cos^2(90° - 70°) = \frac{1}{2}(43\ \text{W/m}^2)(\cos^2 20°) = 19\ \text{W/m}^2.$$

36. (a) The fraction of light that is transmitted by the glasses is

$$\frac{I_f}{I_0} = \frac{E_f^2}{E_0^2} = \frac{E_v^2}{E_v^2 + E_h^2} = \frac{E_v^2}{E_v^2 + (2.3E_v)^2} = 0.16.$$

(b) Since now the horizontal component of $\vec{E}$ will pass through the glasses,

$$\frac{I_f}{I_0} = \frac{E_h^2}{E_v^2 + E_h^2} = \frac{(2.3E_v)^2}{E_v^2 + (2.3E_v)^2} = 0.84.$$

37. **THINK** A polarizing sheet can change the direction of polarization of the incident beam since it allows only the component that is parallel to its polarization direction to pass.

EXPRESS The 90° rotation of the polarization direction cannot be done with a single sheet. If a sheet is placed with its polarizing direction at an angle of 90° to the direction of polarization of the incident radiation, no radiation is transmitted.

ANALYZE (a) The 90° rotation of the polarization direction can be done with two sheets. We place the first sheet with its polarizing direction at some angle θ, between 0 and 90°, to the direction of polarization of the incident radiation. Place the second sheet with its polarizing direction at 90° to the polarization direction of the incident radiation. The transmitted radiation is then polarized at 90° to the incident polarization direction. The intensity is

$$I = I_0 \cos^2\theta \cos^2(90° - \theta) = I_0 \cos^2\theta \sin^2\theta,$$

where I_0 is the incident radiation. If θ is not 0 or 90°, the transmitted intensity is not zero.

(b) Consider n sheets, with the polarizing direction of the first sheet making an angle of $\theta = 90°/n$ relative to the direction of polarization of the incident radiation. The polarizing direction of each successive sheet is rotated $90°/n$ in the same sense from the polarizing direction of the previous sheet. The transmitted radiation is polarized, with its direction of polarization making an angle of 90° with the direction of polarization of the incident radiation. The intensity is

$$I = I_0 \cos^{2n}(90°/n).$$

We want the smallest integer value of n for which this is greater than $0.60 I_0$. We start with $n = 2$ and calculate $\cos^{2n}(90°/n)$. If the result is greater than 0.60, we have obtained the solution. If it is less, increase n by 1 and try again. We repeat this process, increasing n by 1 each time, until we have a value for which $\cos^{2n}(90°/n)$ is greater than 0.60. The first one will be $n = 5$.

LEARN The intensities associated with $n = 1$ to 5 are:

$$\begin{aligned}
I_{n=1} &= I_0 \cos^2(90°) = 0 \\
I_{n=2} &= I_0 \cos^4(45°) = I_0/4 = 0.25 I_0 \\
I_{n=3} &= I_0 \cos^6(30°) = 0.422 I_0 \\
I_{n=4} &= I_0 \cos^8(22.5°) = 0.531 I_0 \\
I_{n=5} &= I_0 \cos^{10}(18°) = 0.605 I_0
\end{aligned}$$

39. (a) Since the incident light is unpolarized, half the intensity is transmitted and half is absorbed. Thus the transmitted intensity is $I = 5.0$ mW/m^2. The intensity and the electric field amplitude are related by $I = E_m^2 / 2\mu_0 c$, so

$$E_m = \sqrt{2\mu_0 cI} = \sqrt{2(4\pi \times 10^{-7}\ \text{H/m})(3.00 \times 10^8\ \text{m/s})(5.0 \times 10^{-3}\ \text{W/m}^2)}$$
$$= 1.9\ \text{V/m}.$$

(b) The radiation pressure is $p_r = I_a/c$, where I_a is the absorbed intensity. Thus

$$p_r = \frac{5.0 \times 10^{-3}\ \text{W/m}^2}{3.00 \times 10^8\ \text{m/s}} = 1.7 \times 10^{-11}\ \text{Pa}.$$

41. As the polarized beam of intensity I_0 passes the first polarizer, its intensity is reduced to $I_0 \cos^2\theta$. After passing through the second polarizer, which makes a 90° angle with the first filter, the intensity is

$$I = (I_0 \cos^2\theta)\sin^2\theta = I_0/10$$

which implies $\sin^2\theta\cos^2\theta = 1/10$, or $\sin\theta\cos\theta = \sin 2\theta/2 = 1/\sqrt{10}$. This leads to $\theta = 70°$ or 20°.

45. Note that the normal to the refracting surface is vertical in the diagram. The angle of refraction is $\theta_2 = 90°$ and the angle of incidence is given by tan $\theta_1 = L/D$, where D is the height of the tank and L is its width. Thus

$$\theta_1 = \tan^{-1}\left(\frac{L}{D}\right) = \tan^{-1}\left(\frac{1.10\text{ m}}{0.850\text{ m}}\right) = 52.31°.$$

The law of refraction yields

$$n_1 = n_2\frac{\sin\theta_2}{\sin\theta_1} = (1.00)\left(\frac{\sin 90°}{\sin 52.31°}\right) = 1.26,$$

where the index of refraction of air was taken to be unity.

47. The law of refraction states

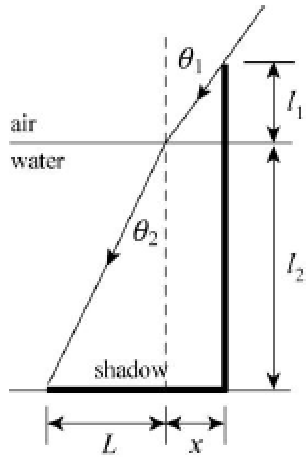
$$n_1 \sin\theta_1 = n_2 \sin\theta_2 .$$

We take medium 1 to be the vacuum, with $n_1 = 1$ and $\theta_1 = 32.0°$. Medium 2 is the glass, with $\theta_2 = 21.0°$. We solve for n_2:

$$n_2 = n_1 \frac{\sin\theta_1}{\sin\theta_2} = (1.00)\left(\frac{\sin 32.0°}{\sin 21.0°}\right) = 1.48.$$

55. **THINK** Light is refracted at the air–water interface. To calculate the length of the shadow of the pole, we first calculate the angle of refraction using the Snell's law.

EXPRESS Consider a ray that grazes the top of the pole, as shown in the diagram below.



Here $\theta_1 = 90° - \theta = 90° - 55° = 35°$, $\ell_1 = 0.50$ m, and $\ell_2 = 1.50$ m. The length of the shadow is $d = x + L$.

ANALYZE The distance x is given by

$$x = \ell_1 \tan\theta_1 = (0.50 \text{ m})\tan 35° = 0.35 \text{ m}.$$

According to the law of refraction, $n_2 \sin\theta_2 = n_1 \sin\theta_1$. We take $n_1 = 1$ and $n_2 = 1.33$ (from Table 33-1). Then,

$$\theta_2 = \sin^{-1}\left(\frac{\sin\theta_1}{n_2}\right) = \sin^{-1}\left(\frac{\sin 35.0°}{1.33}\right) = 25.55°.$$

L is given by

$$L = \ell_2 \tan\theta_2 = (1.50 \text{ m})\tan 25.55° = 0.72 \text{ m}.$$

Thus, the length of the shadow is $d = 0.35 \text{ m} + 0.72 \text{ m} = 1.07 \text{ m}$.

LEARN If the pole were empty with no water, then $\theta_1 = \theta_2$ and the length of the shadow would be

$$d' = \ell_1 \tan\theta_1 + \ell_2 \tan\theta_1 = (\ell_1 + \ell_2)\tan\theta_1$$

by simple geometric consideration.

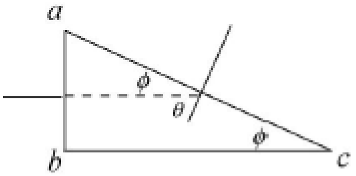
59. **THINK** Total internal reflection happens when the angle of incidence exceeds a critical angle such that Snell's law gives $\sin\theta_2 > 1$.

EXPRESS When light reaches the interfaces between two materials with indices of refraction n_1 and n_2, if $n_1 > n_2$, and the incident angle exceeds a critical value given by

$$\theta_c = \sin^{-1}\left(\frac{n_2}{n_1}\right),$$

then total internal reflection will occur.

In our case, the incident light ray is perpendicular to the face *ab*. Thus, no refraction occurs at the surface *ab*, so the angle of incidence at surface *ac* is $\theta = 90° - \phi$, as shown in the figure below.



ANALYZE (a) For total internal reflection at the second surface, $n_g \sin(90° - \phi)$ must be greater than n_a. Here n_g is the index of refraction for the glass and n_a is the index of refraction for air. Since $\sin(90° - \phi) = \cos\phi$, we want the largest value of ϕ for which $n_g \cos\phi \geq n_a$. Recall that $\cos\phi$ decreases as ϕ increases from zero. When ϕ has the largest value for which total internal reflection occurs, then $n_g \cos\phi = n_a$, or

$$\phi = \cos^{-1}\left(\frac{n_a}{n_g}\right) = \cos^{-1}\left(\frac{1}{1.52}\right) = 48.9°.$$

The index of refraction for air is taken to be unity.

(b) We now replace the air with water. If $n_w = 1.33$ is the index of refraction for water, then the largest value of ϕ for which total internal reflection occurs is

$$\phi = \cos^{-1}\left(\frac{n_w}{n_g}\right) = \cos^{-1}\left(\frac{1.33}{1.52}\right) = 29.0°.$$

LEARN Total internal reflection cannot occur if the incident light is in the medium with lower index of refraction. With $\theta_c = \sin^{-1}(n_2/n_1)$, we see that the larger the ratio n_2/n_1, the larger the value of θ_c.

62. (a) Reference to Fig. 33-24 may help in the visualization of why there appears to be a "circle of light" (consider revolving that picture about a vertical axis). The depth and the radius of that circle (which is from point *a* to point *f* in that figure) is related to the tangent of the angle of incidence. The diameter of the circle in question is given by $d = 2h \tan \theta_c$. For water $n = 1.33$, so Eq. 33-47 gives $\sin \theta_c = 1/1.33$, or $\theta_c = 48.75°$. Thus,

$$d = 2h \tan \theta_c = 2(2.00 \text{ m})(\tan 48.75°) = 4.56 \text{ m}.$$

(b) The diameter d of the circle will increase if the fish descends (increasing h).

68. (a) We use Eq. 33-49: $\theta_B = \tan^{-1} n_w = \tan^{-1}(1.33) = 53.1°$.

(b) Yes, since n_w depends on the wavelength of the light.