A statistical view of entropy

The **entropy** of a system can be defined in terms of the possible distributions of its molecules. For identical molecules, each possible distribution of molecules is called a **microstate of the system**. All equivalent microstates are grouped into a configuration of the system. The number of microstates in a configuration is the **multiplicity** W of the configuration.

For a system of N molecules that may be distributed between the two halves of a box, the multiplicity is given by

$$W = \frac{N!}{n_1!\, n_2!} \quad \text{(multiplicity of configuration).}$$

Here n_1 is the number of molecules in one half of the box and n_2 is the number of molecules in the other half. A basic assumption of statistical mechanics is that all the microstates are equally probable.

$S = k \ln W$

$S = k \ln \overline{W}$ $\qquad$ $\overline{W}$ = # of microstates

$$\overline{W} = \overline{W}(E, V)$$

E_{int}

$$S(E,V) = k \ln \overline{W}(E,V)$$

$$\boxed{\frac{1}{T} \equiv \left.\frac{\partial S(E,V)}{\partial E}\right)_V}$$

$$\cancel{dW = P\,dV}$$

$$\underline{dE_{int}} = dQ - dW = dQ = T\,d\underline{S}$$

$$dS = \frac{dQ}{T} \Rightarrow dQ = T\,dS$$

$$\boxed{\frac{dS}{dE_{int}} = \frac{1}{T}}$$

Probability and Entropy

The multiplicity W of a configuration of a system and the entropy S of the system in that configuration are related by **Boltzmann's entropy equation**:

$$S = k \ln W \quad \text{(Boltzmann's entropy equation).}$$

Here $k = 1.38 \times 10^{-23}$ J/K is the Boltzmann constant.

When N is very large (the usual case), we can approximate ln $N!$ with **Stirling's approximation**:

$$\ln N! \approx N(\ln N) - N \quad \text{(Stirling's approximation).}$$

Chapter summary

Irreversible (one-way) Process

- If an irreversible process occurs in a closed system, the entropy of the system always increases.

Entropy Change

- Entropy change for reversible process is given by

$$\Delta S = S_f - S_i = \int_i^f \frac{dQ}{T} \qquad \textbf{Eq. 20-1}$$

Second Law of Thermodynamics

- If a process occurs in a closed system, the entropy of the system increases for irreversible processes and remains constant for reversible processes.

$$\Delta S \geq 0 \qquad \textbf{Eq. 20-5}$$

Entropy Change

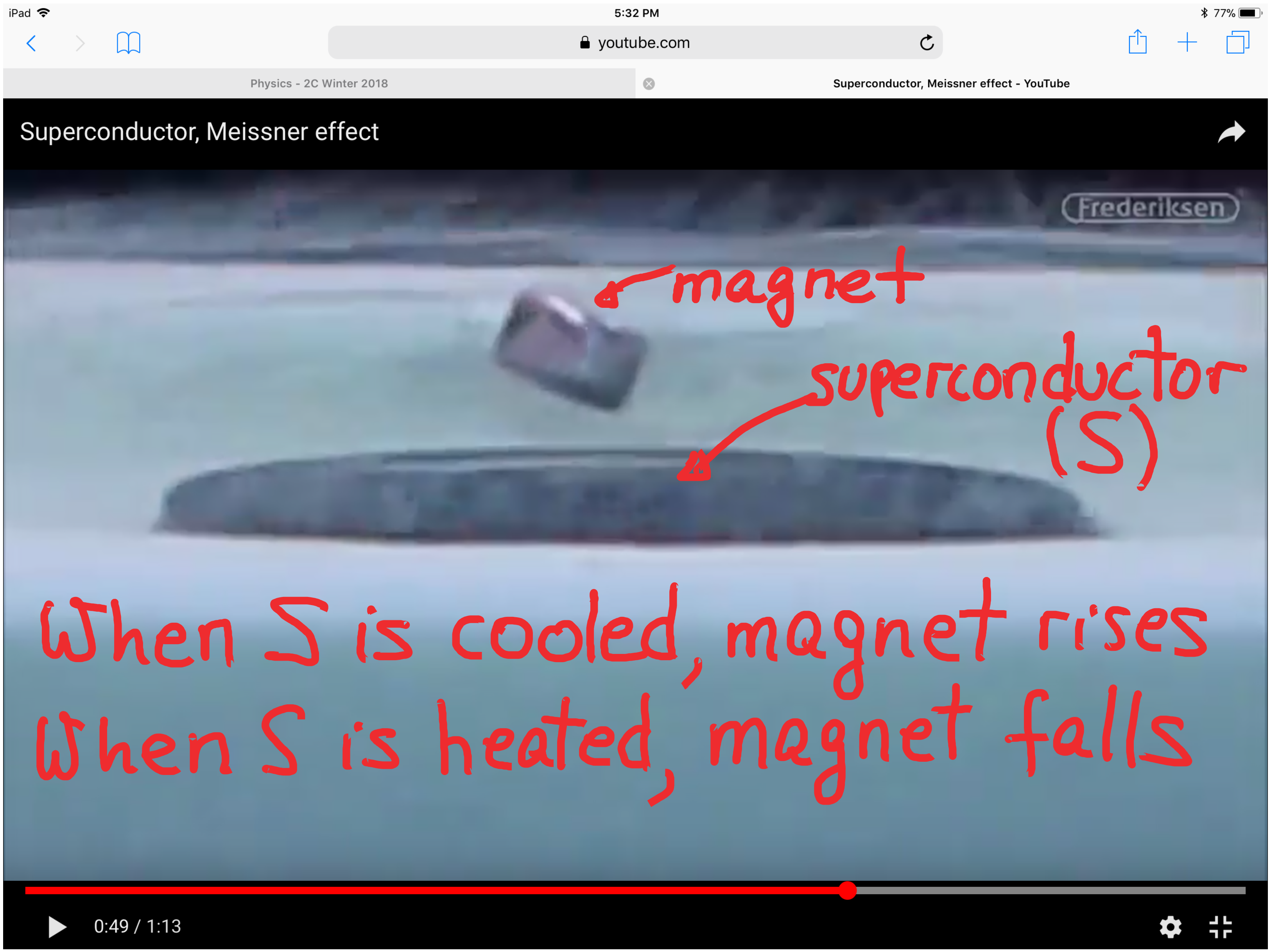
- The efficiency ε of any engine

$$\varepsilon = \frac{\text{energy we get}}{\text{energy we pay for}} = \frac{|W|}{|Q_H|} \qquad \textbf{Eq. 20-11}$$

- Efficiency of Carnot engine

$$\varepsilon_C = 1 - \frac{|Q_L|}{|Q_H|} = 1 - \frac{T_L}{T_H} \qquad \textbf{Eq. 20-12\&13}$$

Example of <u>reversible process</u>



14-5 Archimedes' Principle

- A block of wood in static equilibrium is *floating*:

When a body floats in a fluid, the magnitude F_b of the buoyant force on the body is equal to the magnitude F_g of the gravitational force on the body.

- This is expressed: $$F_b = F_g \quad \text{(floating).}$$ **Eq. (14-17)**

- Because of Eq. 14-16 we know:

When a body floats in a fluid, the magnitude F_g of the gravitational force on the body is equal to the weight $m_f g$ of the fluid that has been displaced by the body.

- Which means: $$F_g = m_f g \quad \text{(floating).}$$ **Eq. (14-18)**

14-5 Archimedes' Principle

- The apparent weight of a body in a fluid is related to the actual weight of the body by:

(*apparent weight*) = (*actual weight*) – (*buoyant force*)

- We write this as:

$$\text{weight}_{\text{app}} = \text{weight} - F_b \quad \text{(apparent weight).}$$

Eq. (14-19)

Checkpoint 2

A penguin floats first in a fluid of density ρ_0, then in a fluid of density $0.95\rho_0$, and then in a fluid of density $1.1\rho_0$. (a) Rank the densities according to the magnitude of the buoyant force on the penguin, greatest first. (b) Rank the densities according to the amount of fluid displaced by the penguin, greatest first.

Answer: (a) all the same (b) $0.95\rho_0$, $1\rho_0$, $1.1\rho_0$

14-6 The Equation of Continuity

Learning Objectives

14.16 Describe steady flow, incompressible flow, nonviscous flow, and irrotational flow.

14.17 Explain the term streamline.

14.18 Apply the equation of continuity to relate the cross-sectional area and flow speed at one point in a tube to those quantities at a different point.

14.19 Identify and calculate volume flow rate.

14.20 Identify and calculate mass flow rate.

14-6 The Equation of Continuity

- Motion of *real fluids* is complicated and poorly understood (e.g., turbulence)
- We discuss motion of an **ideal fluid**

1. Steady flow: Laminar flow, the velocity of the moving fluid at any fixed point does not change with time

2. Incompressible flow: The ideal fluid density has a constant, uniform value

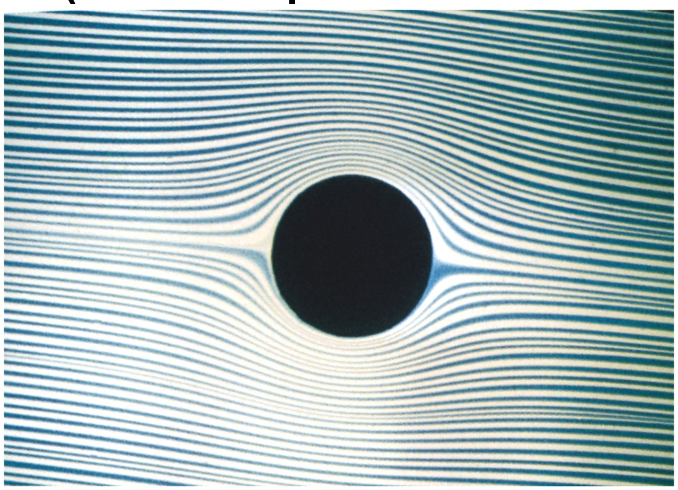
3. Nonviscous flow: Viscosity is, roughly, resistance to flow, fluid analog of friction. No resistive force here

4. Irrotational flow: May flow in a circle, but a dust grain suspended in the fluid will not rotate about com

14-6 The Equation of Continuity

- Visualize fluid flow by adding a *tracer*
- Each bit of tracer (see figure 14-13) follows a *streamline*
- A streamline is the path a tiny element of fluid follows
- Velocity is tangent to streamlines, so they can never intersect (then 1 point would experience 2 velocities)

Figure 14-13



Courtesy D. H. Peregrine, University of Bristol

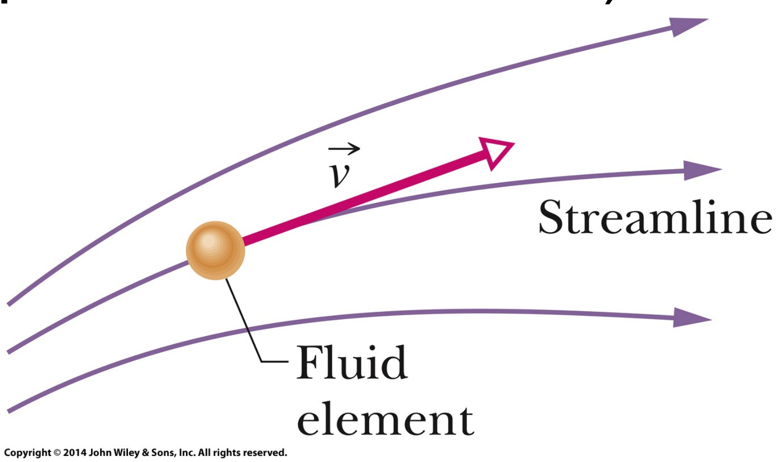


Figure 14-14

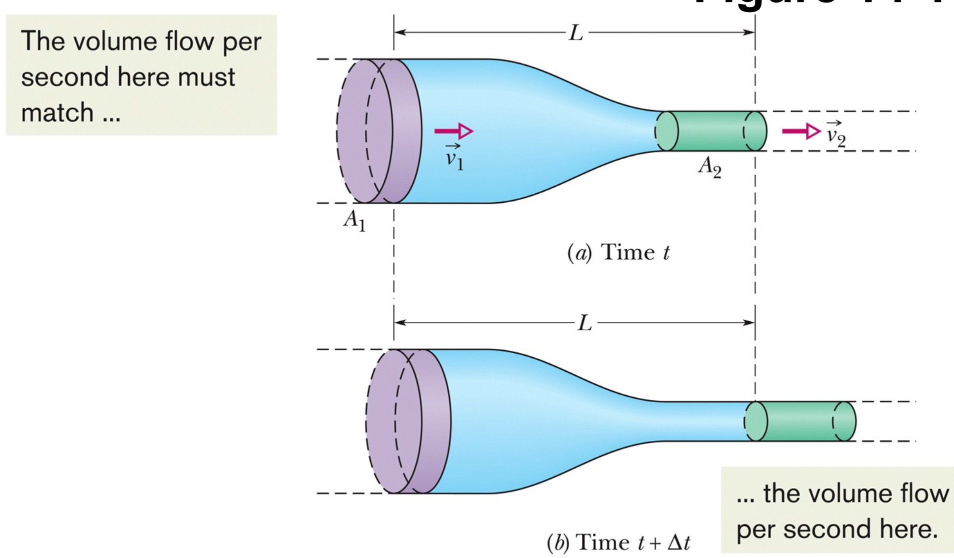
14-6 The Equation of Continuity

- Fluid speed depends on cross-sectional area
- Because of incompressibility, the volume flow rate through any cross-section must be constant
- We write the **equation of continuity**:

$$A_1 v_1 = A_2 v_2$$ **Eq. (14-23)**

- Holds for any *tube of flow* whose boundaries consist of streamlines
- Fluid elements cannot cross streamlines

Figure 14-15



14-6 The Equation of Continuity

- We can rewrite the equation as:

$$R_V = Av = \text{a constant}$$ **Eq. (14-24)**

- Where RV is the **volume flow rate** of the fluid (volume passing a point per unit time)
- If the fluid density is uniform, we can multiply by the density to get the **mass flow rate**:

$$R_m = \rho R_V = \rho A v = \text{a constant}$$ **Eq. (14-25)**

14-7 Bernoulli's Equation

Learning Objectives

14.21 Calculate the kinetic energy density in terms of a fluid's density and flow speed.

14.22 Identify the fluid pressure as being a type of energy density.

14.23 Calculate the gravitational potential energy density.

14.24 Apply Bernoulli's equation to relate the total energy density at one point on a streamline to the value at another point.

14.25 Identify that Bernoulli's equation is a statement of the conservation of energy.

14-7 Bernoulli's Equation

- Figure 14-19 represents a tube through which an ideal fluid flows
- Applying the conservation of energy to the equal volumes of input and output fluid:

$$p_1 + \frac{1}{2}\rho v_1^2 + \rho g y_1 = p_2 + \frac{1}{2}\rho v_2^2 + \rho g y_2.$$

Eq. (14-28)

- The $\frac{1}{2}\rho v^2$ term is called the fluid's **kinetic energy density**

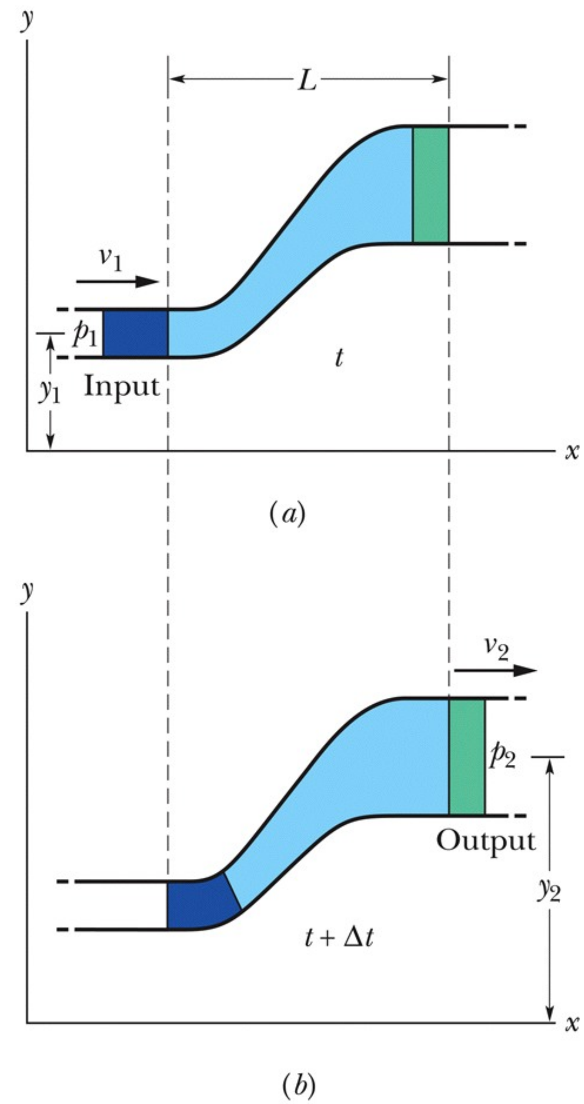


Figure 14-19

$W = \Delta K$ work-energy theorem

$\Delta K = \frac{1}{2} \rho \Delta V (v_2^2 - v_1^2)$

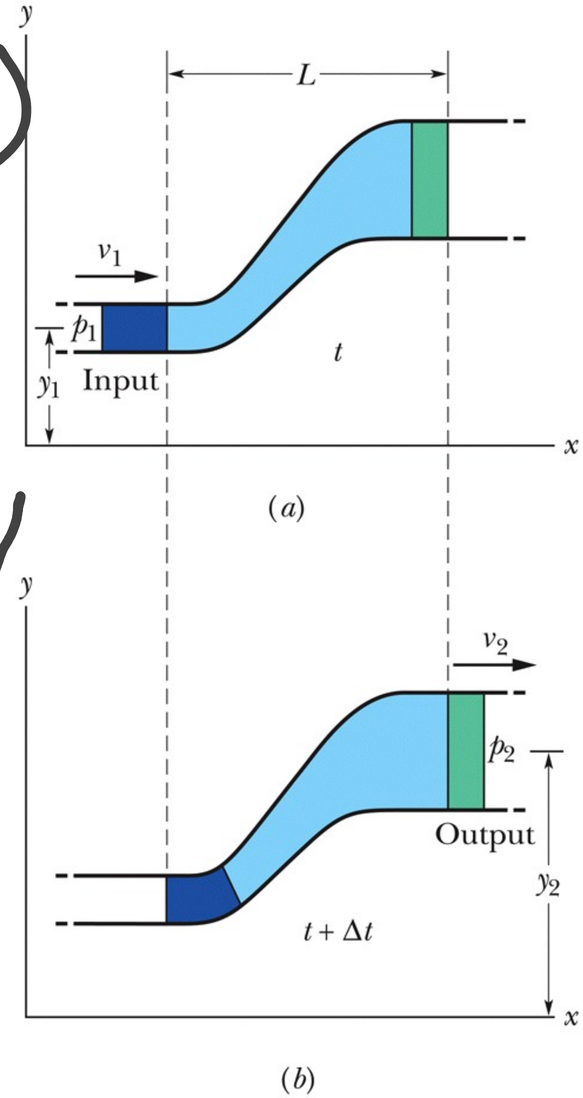
$W = W_g + W_p$ (gravity + pressure)

$W_g = \rho \Delta V g (y_1 - y_2)$

$W_p = F_1 \Delta X_1 - F_2 \Delta X_2 = (p_1 - p_2) \Delta V$

$W_g + W_p = \Delta K \Longrightarrow$

$\boxed{p_1 + \rho g y_1 + \frac{1}{2} \rho v_1^2 = \text{same w/ } 2}$



14-7 Bernoulli's Equation

- Equivalent to Eq. 14-28, we can write:

$$p + \frac{1}{2}\rho v^2 + \rho g y = \text{a constant} \qquad \textbf{Eq. (14-29)}$$

- These are both forms of **Bernoulli's Equation**
- Applying this for a fluid at rest we find Eq. 14-7
- Applying this for flow through a horizontal pipe:

$$p_1 + \frac{1}{2}\rho v_1^2 = p_2 + \frac{1}{2}\rho v_2^2, \qquad \textbf{Eq. (14-30)}$$

If the speed of a fluid element increases as the element travels along a horizontal streamline, the pressure of the fluid must decrease, and conversely.