Physics 2C — Final Exam Solutions — Winter 2018

Problem 1

10 g of steam condensing to liquid at 100°C releases

$Q_{cond} = 539 \times 10 \text{ cal} = 5390 \text{ cal}$

Cooling to 0°C releases $Q_{cooling} = 1 \times 100 \times 10 \text{ cal} = 1000 \text{ cal}$

⇒ total amount of heat released in condensing and cooling

$Q_{tot} = 6390 \text{ cal}$. This heat will melt an amount m_{ice}:

$$m_{ice} \times 79.7 \frac{\text{cal}}{\text{g}} = 6390 \text{ cal} \Rightarrow m_{ice} = 80 \text{ g}$$

We had initially 100 g of ice, 100 of water. So 20 g of ice will remain. So total water at 0°C = 180 g + 10 g = **190 g**

Problem 2

$$\frac{Q}{t} = k A \frac{T_H - T_L}{L} \; ; \quad T_H = 50°C,\ T_L = 30°C,\ L = 0.2 \text{ cm}$$

$k = 10^{-4} \text{ cal}/(\text{s cm °C})$

$A = 2\pi r h$ = lateral area of cylinder

$t = 3$ min for temp. to decrease by 1°C

The mass of the water is

$m_{H_2O} = \rho_{H_2O} \cdot \pi r^2 \cdot h$. The heat needed to change its temperature by $\Delta T = 1°C$ is $Q = C_{H_2O}\, m_{H_2O}\, \Delta T$, $C_{H_2O} = 1 \text{ cal/g °C}$. So

$$\frac{C_{H_2O} \cdot \rho_{H_2O} \cdot \pi r^2 h \cdot \Delta T}{t} = k \cdot 2\pi r h \frac{(T_H - T_L)}{L} \Rightarrow$$

$$r = \frac{k \cdot 2}{\Delta T} \cdot \frac{(T_H - T_L)}{L} \cdot \frac{t}{C_{H_2O}\, \rho_{H_2O}} = \frac{2 \times 10^{-4} \times (50-30) \cdot 60 \cdot 3 \text{ cm}}{1 \times 0.2 \times 1 \times 1} \rightarrow$$

$$\Rightarrow \boxed{r = 3.6 \text{ cm}}$$

Problem 3

Adiabatic process: $TV^{\gamma-1} = \text{const} = C \Rightarrow T = \frac{C}{V^{\gamma-1}}$

$\gamma = \frac{C_P}{C_V} = \frac{C_V + R}{C_V}$, $C_V = \frac{3}{2}R \Rightarrow \gamma = \frac{5}{3}$

Work: $W = \int_{V_1}^{V_2} P\,dV = \int_{V_1}^{V_2} \frac{nRT}{V}\,dV = \int_{V_1}^{V_2} \frac{nRC}{V^\gamma}\,dV =$

$= \frac{nRC}{1-\gamma}\left(V_2^{1-\gamma} - V_1^{1-\gamma}\right) = \frac{nRC}{\gamma-1}\left(V_1^{1-\gamma} - V_2^{1-\gamma}\right) = \frac{3}{2} nRC\left(\frac{1}{V_1^{2/3}} - \frac{1}{V_2^{2/3}}\right)$

$W(V \to 2V) = \frac{3}{2}\frac{nRC}{V^{2/3}}\left(1 - \frac{1}{2^{2/3}}\right) = \frac{3}{2}\frac{nRC}{V^{2/3}} \cdot 0.37 = 10\,J$

$W(2V \to 4V) = \frac{3}{2}\frac{nRC}{V^{2/3}}\left(\frac{1}{2^{2/3}} - \frac{1}{4^{2/3}}\right) \Rightarrow \frac{3}{2}\frac{nRC}{V^{2/3}} \cdot 0.233$

$W(2V \to 4V) = W(V \to 2V) \cdot \frac{0.233}{0.37} = \boxed{6.3\,J}$

Problem 4

$$P(v) = Cv^2 e^{-mv^2/2kT}$$

$v_1 = 300\,m/s$, $N_1 = 1000$; $v_2 = 600\,m/s$, $N_2 = 500$

$\frac{N_1}{N_2} = \frac{P(v_1)}{P(v_2)} = \frac{v_1^2}{v_2^2} e^{-\frac{m}{2kT}(v_1^2 - v_2^2)} \Rightarrow$

$\Rightarrow e^{+\frac{m}{2kT}(v_1^2 - v_2^2)} = \frac{N_2 v_1^2}{N_1 v_2^2} \Rightarrow \frac{m}{2kT}(v_1^2 - v_2^2) = \ln\frac{N_2 v_1^2}{N_1 v_2^2} \Rightarrow$

$\Rightarrow T = \frac{m}{2k}(v_1^2 - v_2^2)\frac{1}{\ln\frac{N_2 v_1^2}{N_1 v_2^2}}$; $\frac{m}{2k} = \frac{29 \times 10^{-3}\,kg\,J^{-1}K^{-1}}{6.02 \times 10^{23} \cdot 2 \cdot 1.38 \times 10^{-23}}$

$\Rightarrow \frac{m}{2k} = 1.75 \times 10^{-3}\,\frac{s^2}{m^2}K \Rightarrow \boxed{T = 227\,K}$

Problem 5

300 g of coffee at 50°C = 323 K, cool to 0°C = 273 K

$$dS = \frac{dQ}{T} = \frac{C\,dT}{T} \Rightarrow \Delta S_1 = \int_{T_i}^{T_f} \frac{dQ}{T} = -C \ln \frac{T_i}{T_f}$$

$$\Rightarrow \Delta S_1 = -\frac{1\,\text{cal}}{g\,°C} \cdot 300\,g \cdot \ln \frac{323}{273} = -50.45 \frac{\text{cal}}{K}$$

When coffee freezes, releases $Q_f = 79.5 \times 300\,\text{cal} = 23{,}850\,\text{cal}$

$$\Rightarrow \Delta S_2 = -\frac{Q_f}{273\,K} = -87.36 \frac{\text{cal}}{K} \Rightarrow$$

total change in entropy of coffee = $\Delta S_1 + \Delta S_2 = \boxed{-138 \frac{\text{cal}}{K}}$

Problem 7

In free expansion of ideal gas, no heat is absorbed

In adiabatic compression, no heat is absorbed

$\Rightarrow \boxed{\Delta S_{env} = 0}$

Problem 6

When gas is compressed isothermically, it releases heat to the environment = work done on gas. $\Delta E_{int} = 0$ if T = constant

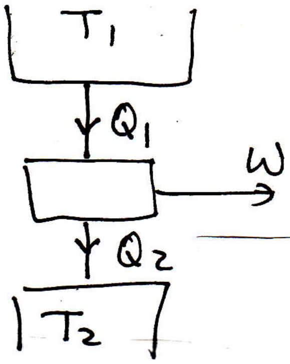
$$W = \int_{V_i}^{V_f} P\,dV = \int_{V_i}^{V_f} \frac{nRT}{V}\,dV = -nRT \ln 2$$

$\Delta E_{int} = Q - W = 0 \Rightarrow Q = W = -nRT \ln 2$ = heat absorbed by gas

$\Rightarrow$ environment absorbs heat $Q_{env} = nRT \ln 2$

$\Rightarrow$ environment increases entropy $\boxed{\Delta S_{env} = nR \ln 2}$

Problem 8



$$\mathcal{E} = \frac{W}{Q_1} = \frac{Q_1 - Q_2}{Q_1} = 1 - \frac{Q_2}{Q_1}$$

reversible $\Rightarrow \Delta S_{univ} = 0 \Rightarrow$

$$\Rightarrow \frac{Q_1}{T_1} = \frac{Q_2}{T_2} \Rightarrow \frac{Q_2}{Q_1} = \frac{T_2}{T_1}$$

$$\Rightarrow \mathcal{E} = 1 - \frac{T_2}{T_1} = 0.2, \quad T_2 = 300\,K$$

$$\Rightarrow \boxed{T_1 = 375\,K}$$

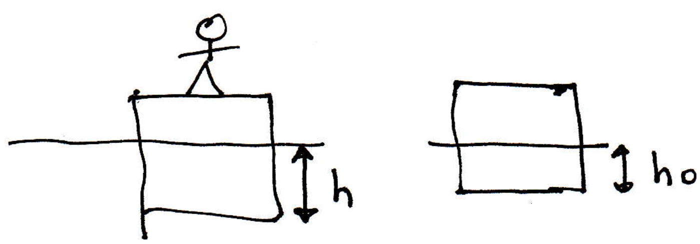
Problem 9

$$(m_{you} + m_{ice})\,g = \rho_{water} \times 0.95\, V_{ice} \times g = \frac{\rho_{water}}{\rho_{ice}} \times 0.95\, m_{ice} \times g$$

$$\Rightarrow m_{you} = \left(\frac{0.95}{0.92} - 1\right) m_{ice}, \quad m_{ice} = 2000\,kg$$

$$\Rightarrow \boxed{m_{you} = 65\,kg}$$

Problem 10



$$m_{you} + m_{ice} = \rho_{H_2O} \cdot A \cdot h, \quad m_{ice} = \rho_{H_2O} \cdot A \cdot h_o \Rightarrow$$

$$m_{you} = \rho_{H_2O} \cdot A\,(h - h_o) \Rightarrow F = \rho_{H_2O} \cdot A \cdot (h - h_o)\,g \text{ is force}$$

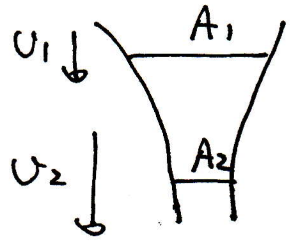
on ice when displacement is $h - h_o$. So

$$\rho_{ice} L^3 = m_{ice} \Rightarrow A = L^2 = 1.68\,m^2$$

$$\omega = \sqrt{\frac{k}{m_{ice}}} = \sqrt{\frac{\rho_{H_2O} \cdot A \cdot g}{m_{ice}}}$$

$$\Rightarrow \boxed{\omega = 2.87\,rad/s}$$

Problem 11



$$U_1 A_1 = U_2 A_2 \Rightarrow U_2 = U_1 \frac{A_1}{A_2} = U_1 \cdot \frac{2^2}{1.5^2} = 1.777\, U_1$$

$$\rho g h + \frac{1}{2}\rho U_1^2 = \frac{1}{2}\rho U_2^2 \Rightarrow$$

$$h = \frac{1}{2}\frac{U_2^2 - U_1^2}{g} = \frac{1}{2g}\left(1.777^2 - 1\right) U_1^2$$

$$U_1 = 1\,m/s,\ g = 9.81\,m/s^2 \Rightarrow \boxed{h = 11\,cm}$$

Problem 12

When not submerged, tension on string is

$T_1 = M_{al} \cdot g = \rho_{al} \cdot V_{al} \cdot g$. When half submerged,

tension is $T_2 = \rho_{al} V_{al} g - \rho_{H_2O} \frac{V_{al}}{2} g = \rho_{al} V_{al} g \left(1 - \frac{\rho_{H_2O}}{2\rho_{Al}}\right)$

$$\Rightarrow \frac{T_2}{T_1} = 1 - \frac{\rho_{H_2O}}{2\rho_{Al}} = 1 - \frac{1}{2 \times 2.7} = 0.815$$

The frequency is $f = n\frac{v}{2L}$, $v = \sqrt{\frac{T}{\mu}}$ changes,

so $f_2 = \sqrt{\frac{T_2}{T_1}}\, f_1 = 0.90 f_1 = \boxed{270.8\ Hz}$

Problem 13

$$\lambda_{sound} = \frac{v_{sound}}{f}, \quad \lambda_{string} = \frac{v_{string}}{f} \Rightarrow$$

$$\Rightarrow \lambda_{string} = \frac{v_{string}}{v_{sound}} \lambda_{sound} = 2l \quad \text{where } l = \text{length of string}$$

$$v_{string} = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{T \cdot l}{m}} \quad m = \text{mass of string} \Rightarrow$$

$$\Rightarrow 2l = \sqrt{\frac{T\,l}{m}} \frac{\lambda_{sound}}{v_{sound}} \Rightarrow l = \left(\frac{1}{2}\sqrt{\frac{T}{m}} \frac{\lambda_{sound}}{v_{sound}}\right)^2$$

$T = 216\ N$, $m = 4 \cdot 10^{-3}\ kg$, $v_{sound} = 340\ m/s$, $\lambda_{sound} = 2.27\ m$

$$\Rightarrow \boxed{l = 60\ cm}$$

Problem 14

$$\frac{1}{p} + \frac{1}{i} = \frac{1}{f} = \frac{2}{r}, \quad m = -\frac{i}{p} = 2 \Rightarrow i = -2p$$

$$\Rightarrow \frac{1}{p} - \frac{1}{2p} = \frac{2}{r} \Rightarrow \frac{1}{2p} = \frac{2}{r} \Rightarrow \boxed{p = \frac{r}{4}}$$

Problem 15

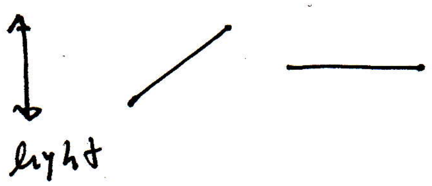
$I = \frac{E_{rms}^2}{c\mu_0}$ is power per unit area incident on black body

$\Rightarrow \sigma T^4 = \frac{E_{rms}^2}{c\mu_0} \Rightarrow T = \left(\frac{E_{rms}^2}{c\mu_0\sigma}\right)^{1/4}$

$\Rightarrow T = \left(\frac{100^2}{3\times10^8 \times 4\pi\times10^{-7} \times 5.67\times10^{-8}}\right)^{1/4} K = \boxed{147K}$

Problem 16

$I = \cos^2\theta \, I_0$ where θ is angle



2 polarizers: $\theta = 45°$

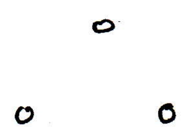
$I = I_0 \cdot \left(\cos^2\frac{\pi}{4}\right)^2 = \frac{I_0}{4}$ not enough

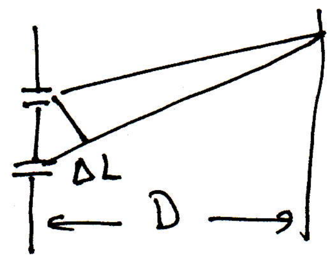
4 polarizers: $\theta = \frac{\pi}{8} = 22.5°$ $\quad I = I_0 \cdot (\cos^2 22.5°)^4 = 0.53\, I_0$ not enough

8 polarizers: $\theta = \frac{\pi}{16} = 11.25°$ $\quad I = I_0(\cos^2 11.25°)^8 = 0.73\, I_0$ " "

9 polarizers: $\theta = \frac{\pi}{18} = 10°$ $\quad I = I_0(\cos^2 10°)^9 = 0.759\, I_0$ o.k.

Problem 17





If they arrive at screen with phase difference φ

$$E = E_m(1 + 2\cos\varphi) = 0 \Rightarrow \cos\varphi = -\frac{1}{2} \Rightarrow$$

$$\Rightarrow \varphi = \frac{2\pi}{3}. \quad \varphi = 2\pi\frac{\Delta L}{\lambda} = \frac{2\pi d\sin\theta}{\lambda} \Rightarrow \lambda = \frac{2\pi d\sin\theta}{\varphi}$$

$$\sin\theta = \frac{d}{D} \Rightarrow \lambda = \frac{2\pi d^2}{D\varphi} = \frac{2\pi d^2 \cdot 3}{D\cdot 2\pi} = \frac{3d^2}{D}$$

$$\Rightarrow \lambda = \frac{3\times 1.5^2\times 10^{-6}}{2}\,m = \boxed{338\ nm}$$

Problem 18

$$I = I_m\cos^2\beta\left(\frac{\sin\alpha}{\alpha}\right)^2$$

$$\beta = \frac{\pi d}{\lambda}\sin\theta, \quad \alpha = \frac{\pi a}{\lambda}\sin\theta$$

First maximum off-center $\beta = \pi = \pi\frac{d}{\lambda}\sin\theta$

$$\Rightarrow \alpha = \frac{a}{d}\pi \Rightarrow \boxed{I = I_m \times 0.97}$$

Problem 19

$$\Delta\theta_{hw} = \frac{\lambda}{Nd\cos\theta} = \frac{500\,nm}{1000\times 0.1\times 10^{-3}m} = 5.10^{-6}m$$

$$\theta = \frac{10\,cm}{2\,m} = 0.05 \Rightarrow \cos\theta \sim 1$$

$$y = D.\Delta\theta_{hw} = 10^{-5}m = \boxed{0.01\,mm}$$

Problem 20

$$d\sin\theta = m\lambda \Rightarrow m = \frac{d\sin\theta}{\lambda} = 0.05\frac{d}{\lambda}$$

$$\Rightarrow m = \frac{0.05 . 0.1\times10^{-3}}{5\times10^{-7}} = 10$$

$\Rightarrow$ $\boxed{8 \text{ peaks}}$ in-between