Physics 2C Quiz 1 Solutions Winter 2018

Problem 1

$L = 20\,\text{cm}, \quad \Delta L = 0.0075\,\text{cm}, \quad \Delta T = 15^\circ C, \quad \Delta T' = 50^\circ C, \quad V = L^3$

$$\Delta L = L \alpha \Delta T \Rightarrow \alpha = \frac{\Delta L}{L \Delta T}$$

$$\Delta V = 3\alpha V \cdot \Delta T' = \frac{3\,\Delta L}{L \cdot \Delta T} L^3 \Delta T' = 3\,\Delta L\, L^2 \frac{\Delta T'}{\Delta T}$$

$$\Delta V = 3\,\Delta L\, L^2 \frac{\Delta T'}{\Delta T} = 3 \times 0.0075\,\text{cm} \times 20^2\,\text{cm}^2 \times \frac{50}{15} = 30\,\text{cm}^3$$

$$\boxed{\Delta V = 30\,\text{cm}^3}$$

Problem 2

Q_{ice} = heat absorbed by water-ice to get to final temperature T_f.
initial temperature = $0^\circ C$. $m_{ice} = 50\,g$, $m_{H_2O} = 50\,g$. $L_f = 79.7\,\text{cal/g}$

Q_{steam} = heat 'absorbed' by steam to get to final temperature T_f
initial temperature = $100^\circ C$. $m_{steam} = 10\,g$. $L_v = 539\,\text{cal/g}$

$$C_{H_2O} = \frac{1\,\text{cal}}{g^\circ C}$$

$$Q_{ice} = m_{ice} \cdot L_f + (m_{ice} + m_{H_2O})(T_f - 0^\circ C) \cdot C_{H_2O}$$

$$Q_{steam} = -m_{steam} \cdot L_v - m_{steam}(100^\circ C - T_f) \cdot C_{H_2O}$$

$$Q_{ice} + Q_{steam} = 0 \Rightarrow m_{ice} \cdot L_f - m_{steam} \cdot L_v + (m_{ice} + m_{H_2O} + m_{steam})\, C_{H_2O}\, T_f - 100\, m_{steam}\, C_{H_2O} = 0$$

$$\Rightarrow T_f = \frac{m_{steam} \cdot L_v - m_{ice} \cdot L_f + 100\, m_{steam} \cdot C_{H_2O}}{(m_{ice} + m_{H_2O} + m_{steam})\, C_{H_2O}} = \frac{10 \cdot 539 - 50 \cdot 79.7 + 100 \cdot 10}{50 + 50 + 10}\,^\circ C$$

$$\Rightarrow \boxed{T_f = 22^\circ C}$$

Problem 3

$\frac{1}{2} m(v^2)_{av} = \frac{1}{2} m v_{rms}^2 = \frac{3}{2} k T \Rightarrow$

$v_{rms} = const. \left(\frac{T}{m}\right)^{1/2}$. T is absolute temperature, in K.

So, $v_{rms}^N(40°C) = v_{rms}^O(20°C) \cdot \left(\frac{m_O}{m_N}\right)^{1/2} \cdot \left(\frac{273+40}{273+20}\right)^{1/2}$

$\Rightarrow v_{rms}^N(40°C) = 477 \frac{m}{s} \left(\frac{8}{7}\right)^{1/2} \left(\frac{313}{293}\right)^{1/2} = 527 \frac{m}{s}$

$$\boxed{v_{rms}^N(40°C) = 527 \frac{m}{s}}$$

Problem 4

$P_{cond} = \frac{Q}{t} = k A \frac{T_H - T_L}{L}$ we need to find t to raise T_L by 1°C.

$T_L = 10°C$, $T_H = 30°C$. $L = 0.3 cm$ = thickness of glass.

A = area of cylinder ~ lateral area = $2\pi r \cdot h$, where r is the radius of cylinder, h its height.

Q is heat needed to raise T_L by 1°C. $Q = C_{H_2O} \cdot m_{H_2O} \cdot \Delta T$, $\Delta T = 1°C$

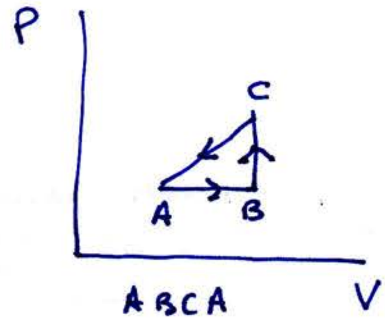
$m_{H_2O} = \rho_{H_2O} \cdot volume = \rho_{H_2O} \cdot \pi r^2 \cdot h$, $\rho_{H_2O} = 1 g/cm^3$. So

$$t = \frac{Q \cdot L}{k A (T_H - T_L)} = \frac{C_{H_2O} m_{H_2O} \Delta T \cdot L}{k A (T_H - T_L)} = \frac{C_{H_2O} \cdot \rho_{H_2O} \cdot \cancel{\pi} r^{\cancel{2}} \cancel{h} \Delta T L}{k \cdot 2 \cancel{\pi} \cancel{r} \cdot \cancel{h} (T_H - T_L)}$$

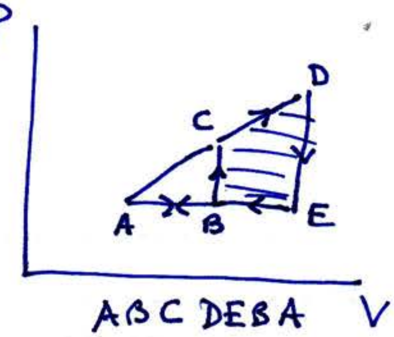
$$t = \frac{C_{H_2O} \rho_{H_2O} r \Delta T L}{k \cdot 2 (T_H - T_L)} = \frac{\frac{1 cal}{g °C} \cdot \frac{1 g}{cm^3} \cdot 2 cm \cdot 1°C \cdot 0.3 cm}{10^{-4} \frac{cal}{s\, cm\, °C} \cdot 2 (30°C - 10°C)}$$

$t = \frac{\cancel{2} \times 0.3 \times 10^4}{\cancel{2} \cdot 20} s \Rightarrow \boxed{t = 150 s = 2.5 min}$

Problem 5



ABCA



ABCDEBA

In cycle ABCA, the system does negative work $\Rightarrow$ work is done on system.

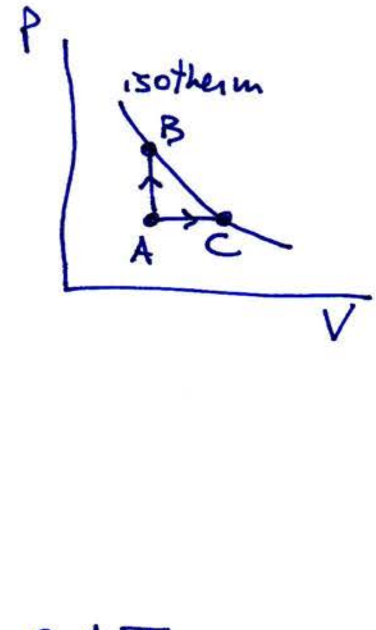
$\Delta E_{int} = Q - W = 0$ for a cycle $\Rightarrow W = Q < 0 \Rightarrow$ heat is removed from the system. 20 J of heat are removed.

In cycle ABCDEBA, system does positive work. Since $\Delta E_{int} = 0$, heat must be <u>added</u> to the system.

Work done = area in figure = <u>3 times</u> area in ABCA cycle

$\Rightarrow$ $\boxed{\text{60 J of heat must be added to the system}}$

Problem 6



$\Delta E_{int} = n C_v \Delta T$ is always valid.

Since B, C are on an isotherm, $\Delta E_{int}^{A\to B} = \Delta E_{int}^{A\to C}$

$\Delta E_{int} = Q - W$ Also, $\boxed{\Delta E_{int} = n C_v \Delta T}$

For $A \to B$, $W = 0 \Rightarrow \boxed{\Delta E_{int} = Q = 15\,J}$

For $A \to C$, $\Delta E_{int} = Q - W \Rightarrow W = Q - \Delta E_{int}$

Since it is at constant P, $Q = n C_p \Delta T \Rightarrow$

$\Rightarrow W = n C_p \Delta T - n C_v \Delta T = n \Delta T (C_p - C_v) = n R \Delta T$

For monatomic gas, $C_v = \frac{3}{2} R \Rightarrow \Delta E_{int} = \frac{3}{2} n R \Delta T$

$\Rightarrow W = n R \Delta T = \frac{2}{3} \cdot \frac{3}{2} n R \Delta T = \frac{2}{3} \cdot n C_v \Delta T = \frac{2}{3} \Delta E_{int} = \frac{2}{3} \cdot 15\,J$

$\Rightarrow \boxed{W = 10\,J}$

Problem 7

$\Delta E_{int} = Q - W = -W$ for adiabatic, $Q = 0$

So $\Delta E_{int} = n C_v \Delta T = -W \Rightarrow \boxed{W = n C_v (T_i - T_f)}$

temperature and volume are related by

$T V^{\gamma - 1} = \text{const}$. $\quad \gamma = \frac{C_p}{C_v} = \frac{\frac{5}{2}}{\frac{3}{2}} = \frac{5}{3} \Rightarrow \gamma - 1 = \frac{2}{3}$

$\Rightarrow \boxed{T V^{2/3} = \text{const}} \Rightarrow \boxed{\frac{T(V_1)}{T(V_2)} = \left(\frac{V_2}{V_1}\right)^{2/3}}$

$W(V \to 2V) = n C_v (T(V) - T(2V))$

$\Rightarrow$

$W(V \to 3V) = n C_v (T(V) - T(3V))$

$$W(V \to 3V) = \frac{T(V) - T(3V)}{T(V) - T(2V)} W(V \to 2V) = \frac{1 - \frac{T(3V)}{T(V)}}{1 - \frac{T(2V)}{T(V)}} W(V \to 2V)$$

$$\Rightarrow W(V \to 3V) = \frac{1 - \left(\frac{1}{3}\right)^{2/3}}{\left(1 - \left(\frac{1}{2}\right)^{2/3}\right)} \cdot 10\,J = 1.40 \times 10\,J$$

$\Rightarrow \boxed{W(V \to 3V) = 14\,J}$