Physics 2C   Quiz 2 Solutions   Winter 2018

Problem 1

$$f(v_x) = c\, e^{-m v_x^2/2kT}$$

$$\frac{f(v_x = 100\text{ m/s})}{f(v_x=0)} = \frac{e^{-\frac{m}{2kT}v_x^2}}{1} = \frac{5{,}000}{10{,}000} = \frac{1}{2}$$

$$\frac{f(v_x' = 200\text{m/s})}{f(v_x=0)} = \frac{f(2v_x)}{f(0)} = e^{-\frac{m}{2kT}\cdot 4v_x^2} = \left(e^{-\frac{m}{2kT}v_x^2}\right)^4 = \left(\frac{1}{2}\right)^4 = \frac{1}{16}$$

$$\Rightarrow f(200\text{ m/s}) = \frac{1}{16} f(0) \Rightarrow$$

\# of molecs with $v_x = 200\text{ m/s}$ = $10{,}000/16 = 625$

$\Rightarrow$ $\boxed{\text{approximately } 600}$

Problem 2

The tea cools to 0°C, releasing heat $Q = Cm\Delta T = \frac{1\text{ cal}}{g\,°C} \times 200\text{g} \times 60°C$

$\Rightarrow Q = 12{,}000$ cal

Change in entropy of tea: $\Delta S_{tea} = +\int_{T_i}^{T_f} \frac{dQ}{T} = -Cm \ln\frac{T_i}{T_f} = -\frac{Q}{\Delta T}\ln\frac{T_i}{T_f}$

$$\Delta S_{tea} = -\frac{12{,}000\text{ cal}}{60\text{ K}} \ln\frac{333}{273} = -39.7\text{ cal/K}$$

Heat is absorbed by ice at 0°C, change in entropy of ice is

$$\Delta S_{ice} = \frac{Q}{T_{ice}} = \frac{12{,}000\text{ cal}}{273\text{ K}} = 44.0\text{ cal/K}$$

$$\Delta S_{universe} = \Delta S_{tea} + \Delta S_{ice} = 4.26\text{ cal/K} \qquad \boxed{4\text{ cal/K}}$$

The freezing process does not change entropy of the universe

Problem 3

$dE_{int} = dQ - dW = dQ$ since it is at constant volume

$dE_{int} = nC_v\,dT = dQ$

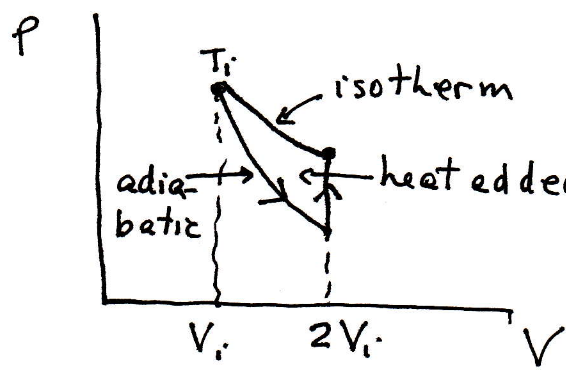
$$\Delta S = \int_i^f \frac{dQ}{T} = nC_v \ln\frac{T_f}{T_i}$$

Heat absorbed is $Q = \int_i^f dQ = nC_v(T_f - T_i) \Rightarrow nC_v = \dfrac{Q}{T_f - T_i}$

$$\Rightarrow \Delta S = \frac{Q}{T_f - T_i}\ln\frac{T_f}{T_i} = \frac{20\,J}{10\,K}\ln\frac{313}{303} = 0.065\,\frac{J}{K}$$

$$\boxed{\Delta S = 0.065\,J/K}$$

Problem 4



$\Delta S_{universe} = 0$ since reversible

$\Rightarrow \Delta S_{env} = -\Delta S_{gas}$

$\Delta S_{gas} = nR\ln\dfrac{2V_i}{V_i} = nR\ln 2$ since temperature hasn't changed

$\Rightarrow$ entropy of environment decreases by $\boxed{nR\ln 2}$

Alternative solution: in adiabatic expansion, gas cools to T_f, without absorbing heat. It absorbs heat in heating from T_f to T_i at constant volume, absorbing $dQ = nC_v\,dT$. Change in entropy of environment is $\Delta S_{env} = +\int_{T_i}^{T_f} \dfrac{nC_v\,dT}{T} = +nC_v\ln\dfrac{T_f}{T_i}$

Now $T_f V_f^{\gamma-1} = T_i V_i^{\gamma-1} \Rightarrow T_f/T_i = 1/2^{\gamma-1} \Rightarrow \Delta S_{env} = -nC_v \ln 2^{\gamma-1}$

$\Rightarrow \Delta S_{env} = -nC_v(\gamma-1)\ln 2 = -nR\ln 2$ since $\gamma = C_p/C_v \Rightarrow \gamma - 1 = R/C_v$

Problem 5

$$\varepsilon = \frac{W}{Q_H} = \frac{1}{4} = 1 - \frac{Q_L}{Q_H} \quad ; \quad Q_L = 60\,J \Rightarrow$$

$$\Rightarrow Q_H = \frac{4}{3} Q_L = 80\,J \quad ; \quad \frac{T_H}{T_L} = \frac{Q_H}{Q_L} \Rightarrow$$

$$\Rightarrow T_H = \frac{Q_H}{Q_L} T_L = \frac{80\,J}{60\,J} \times 300K = \boxed{400K}$$

Problem 6

$$W = Q_H - Q_L = \left(\frac{T_H}{T_L} - 1\right) Q_L \quad ; \quad W = 1J, \; Q_L = 20J, \; T_H = 30°C = 303K$$

$$\Rightarrow \frac{T_H}{T_L} = \frac{W}{Q_L} + 1 \Rightarrow \frac{1}{T_L} = \frac{1}{T_H}\left(\frac{W}{Q_L} + 1\right)$$

When temperature outside rises to $T_H' = 40°C$, same Q_L

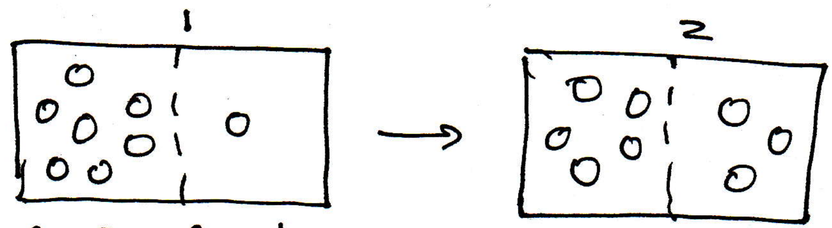
$$W' = Q_H' - Q_L = \left(\frac{T_H'}{T_L} - 1\right) Q_L = \left(\frac{T_H'}{T_H}\left(\frac{W}{Q_L} + 1\right) - 1\right) Q_L \Rightarrow$$

$$\Rightarrow W' = \frac{T_H'}{T_H}(W + Q_L) - Q_L = \frac{313}{303}(1J + 20J) - 20J$$

$$\Rightarrow \boxed{W' = 1.69\,J}$$

Problem 7

$N = 8$



$n_1 = 7, \quad n_2 = 1$ $\longrightarrow$ $n_1 = 5, \quad n_2 = 3$

$$W = \frac{N!}{n_1!\, n_2!}$$

$$W_1 = \frac{8!}{7!\,1!} = 8 \quad ; \quad W_2 = \frac{8!}{5!\,3!} = \frac{8 \cdot 7 \cdot 6}{3 \cdot 2} = 56$$

$$\Rightarrow W_2 / W_1 = 56/8 = 7$$

$$S = k \ln W \Rightarrow \Delta S = k \ln W_2/W_1 = k \ln 7$$

$$\Rightarrow \boxed{\Delta S = 1.95 k}$$