Physics 2C Quiz 4 Solutions Winter 2018

Problem 1

$\lambda = 10$ cm, $f = 100$ Hz $\Rightarrow v = \lambda f = 1000$ cm/s

$\Rightarrow \boxed{v = 10\,\text{m/s}}$ $\quad v = \sqrt{\frac{\tau}{\mu}} \Rightarrow \tau = v^2\mu$, $\mu = 0.5\,\frac{\text{kg}}{\text{m}} \Rightarrow$

$\Rightarrow \tau = 10^2 \times 0.5$ N $\Rightarrow \boxed{\tau = 50\,\text{N}}$

Problem 2

kinetic energy of mass dm is $K = \frac{1}{2}\, dm\, \dot{y}^2$

$y = y_m \sin(kx - \omega t) \Rightarrow \dot{y} = -\omega y_m \cos(kx - \omega t)$

Over a wavelength, the average of $\cos^2$ is $\frac{1}{2}$, $dm = \mu\lambda \Rightarrow$

$\boxed{K = \frac{1}{4}\mu\lambda\omega^2 y_m^2}$ $\quad y_m = 1\,\text{cm} = 0.01\,\text{m}$, $\lambda = 0.1\,\text{m}$, $\omega = 2\pi f$, $f = 100$ Hz $\Rightarrow$

$K = \frac{1}{4} \cdot 0.5 \cdot 0.1 \cdot 4\pi^2 \cdot \underbrace{100^2 \cdot 0.01^2}_{1}\ \text{J} = \boxed{0.49\,\text{J}}$

Check: $P_{av} = \frac{1}{2}\mu v \omega^2 y_m^2$ is power transmitted by wave.

That is 1/2 kinetic, 1/2 potential energy. So kinetic power is

$P_{av}^{kin} = \frac{1}{4}\mu v \omega^2 y_m^2$. The energy in one period is

the energy for 1 wavelength, $T = \frac{\lambda}{v} = \frac{1}{f}$, so

$K = T \cdot P_{av}^{kin} = \frac{1}{4}\mu\lambda\omega^2 y_m^2$ as above

Problem 3

Interference of 2 waves travelling in same direction:

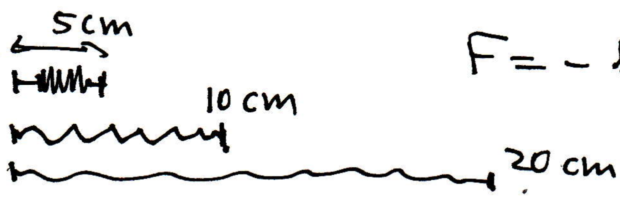
$$y' = (2y_m \cos \tfrac{1}{2}\phi)\ \sin(kx - \omega t + \tfrac{\phi}{2})$$

Power is proportional to amplitude square, so if amplitude increased from 200 W to 600 W $\Rightarrow$

$$(2\cos \tfrac{1}{2}\phi)^2 = 3 \Rightarrow \cos^2 \tfrac{1}{2}\phi = \tfrac{3}{4} \Rightarrow \cos \tfrac{1}{2}\phi = \tfrac{\sqrt{3}}{2}$$

$$\Rightarrow \tfrac{1}{2}\phi = 30^\circ \Rightarrow \boxed{\phi = 60^\circ}$$

Problem 4



$$F = -k(x - 5\text{cm})$$

Resonance frequency: $f = n \dfrac{v}{2L}$

the velocity is $v = \sqrt{\dfrac{\tau}{\mu}}$ $\quad \tau$ = tension = force

τ increases by a factor of 3. $\mu = \dfrac{m}{L}$ decreases by a factor of 2 $\Rightarrow$ v increases by factor $\sqrt{6}$

$\Rightarrow$ since L increases by factor of 2,

$$f' = \frac{\sqrt{6}}{2} f = 1.22 f$$

so $\boxed{f' = 122\ \text{Hz}}$

Problem 5

$$\beta = 10\,dB \log \frac{I}{I_0} \quad , \quad I_0 = 10^{-12} \frac{W}{m^2}$$

Fn $\beta = 20\,dB \Rightarrow \log \frac{I}{I_0} = 2 \Rightarrow I = 10^{-10} \frac{W}{m^2}$

$$I = \frac{1}{2} \rho v \omega^2 s_m^2 \quad , \quad \omega = 2\pi f$$

$$\Rightarrow s_m = \sqrt{\frac{2I}{\rho v\, 4\pi^2 f^2}} = \sqrt{\frac{2 \times 10^{-10}}{1.21 \times 343 \times 4\pi^2 \times 10^6}}\ m \Rightarrow$$

$$\Rightarrow \quad s_m = \sqrt{\frac{2}{1.21 \times 343 \times 4\pi^2}} \times 10^{-8}\ m = 1.1 \times 10^{-10}\ m$$

$$\boxed{s_m = 0.1\ nm}$$

Problem 6

Resonant frequencies in string : $f = n \frac{v_{string}}{2L}$

$$\lambda_{sound} = \frac{v_{sound}}{f} = \frac{v_{sound} \cdot 2L}{n\, v_{string}} = \frac{2L}{n} \frac{v_{sound}}{v_{string}}$$

$$\lambda_{sound} = \frac{2 \times 0.12}{3} \cdot \frac{340}{170}\ m$$

$$\Rightarrow \boxed{\lambda_{sound} = 16\ cm}$$

Problem 7

$$f_1 = f \frac{v}{v+v_s} \quad \text{car moving away}$$

$$f_2 = f \frac{v}{v-v_s} \quad \text{car approaching}$$

$$f_2 - f_1 = f v \left(\frac{1}{v-v_s} - \frac{1}{v+v_s} \right) = f v \cdot \frac{2 v_s}{v^2 - v_s^2}$$

$$\Rightarrow f_2 - f_1 \approx 2 f \frac{v_s}{v}$$

$v = 330\ m/s$, $v_s = 120\ km/h = 33\ m/s$

$$\Rightarrow \frac{v_s}{v} = 0.1 \Rightarrow \boxed{f_1 - f_2 = f_{beat} = 0.2 f}$$