

**Formulas:**

$$F = k \frac{q_1 q_2}{r^2} \quad \text{Coulomb's law} \quad ; \quad k = \frac{1}{4\pi\epsilon_0} = 8.99 \times 10^9 \text{ N} \cdot \text{m}^2 / \text{C}^2; \quad \epsilon_0 = 8.85 \times 10^{-12} \text{ C}^2 / \text{Nm}^2$$

$$\text{Electric field: } \vec{E} = \frac{q}{4\pi\epsilon_0 r^2} \hat{r} = \frac{kq}{r^2} \hat{r} \quad ; \quad \vec{E}(\vec{r}) = k \int \rho(\vec{r}') \frac{\vec{r} - \vec{r}'}{|\vec{r} - \vec{r}'|^3} d^3 r' \quad ; \quad \vec{F} = q_0 \vec{E}$$

$$\text{Linear, surface, volume charge density: } dq = \lambda d\ell \quad , \quad dq = \sigma da \quad , \quad dq = \rho dv$$

$$\text{Electric field of infinite: line of charge: } E = \frac{\lambda}{2\pi\epsilon_0 r}; \quad \text{sheet of charge: } E = \frac{\sigma}{2\epsilon_0}$$

$$\vec{\nabla} \times \vec{E} = 0 \quad (\text{electrostatics}) \quad \text{Divergence theorem: } \int d^3 r \vec{\nabla} \cdot \vec{F} = \oint \vec{F} \cdot d\vec{a}$$

$$\text{Gauss law: } \Phi = \oint \vec{E} \cdot d\vec{a} = \frac{q_{enc}}{\epsilon_0} = \frac{1}{\epsilon_0} \int d^3 r \rho(\vec{r}) \quad ; \quad \Phi = \text{electric flux}$$

$$\text{Energy: } U = k \frac{q_1 q_2}{r_{12}} \quad U = \frac{\epsilon_0}{2} \int E(\vec{r})^2 d^3 r \quad \text{Work: } W = \int \vec{F} \cdot d\vec{s}$$

$$\text{Electric potential: } \phi(P_2) - \phi(P_1) = - \int_{P_1}^{P_2} \vec{E} \cdot d\vec{s} \quad \phi(\vec{r}) = \frac{1}{4\pi\epsilon_0} \int d^3 r' \frac{\rho(\vec{r}')}{|\vec{r} - \vec{r}'|}$$

$$\text{Point charge: } \phi(\vec{r}) = \frac{1}{4\pi\epsilon_0} \frac{q}{r} \quad \text{Dipole: } \phi(r, \theta) = \frac{p \cos \theta}{4\pi\epsilon_0 r^2}$$

$$U = \frac{1}{2} \int \rho \phi d^3 r \quad \vec{E} = -\vec{\nabla} \phi \quad ; \quad \vec{\nabla} \cdot \vec{E} = \frac{\rho}{\epsilon_0} \quad ; \quad \vec{\nabla} \times \vec{E} = 0$$

$$\text{Capacitors: } Q = C\phi \quad , \quad U = \frac{Q^2}{2C}; \quad \text{Planar: } C = \frac{\epsilon_0 A}{s} \quad \text{electric field: } E = \sigma / \epsilon_0$$

$$I = \int \vec{J} \cdot d\vec{a} \quad , \quad I = \frac{dQ}{dt} \quad , \quad \vec{J} = nq\vec{u} \quad ; \quad \text{div} \vec{J} = -\frac{\partial \rho}{\partial t} \quad ; \quad \text{Power: } P = I^2 R \quad ; \quad P = \epsilon I$$

$$V = IR; \quad \vec{J} = \sigma \vec{E}; \quad \vec{E} = \rho \vec{J}; \quad R = \rho \frac{L}{A}; \quad \sigma = \frac{ne^2 \tau}{m_e}; \quad Q(t) = C\epsilon(1 - e^{-t/RC}); \quad Q(t) = Q_0 e^{-t/RC}$$

$$\text{Lorentz force: } \vec{F} = q(\vec{E} + \vec{v} \times \vec{B}); \quad \text{force on wire: } d\vec{F} = I d\vec{\ell} \times \vec{B} \quad ; \quad \text{solenoid: } B = \mu_0 n I$$

$$\text{Stokes' theorem: } \oint_C \vec{F} \cdot d\vec{s} = \int_S (\vec{\nabla} \times \vec{F}) \cdot d\vec{a} \quad ; \quad \text{vector potential: } \vec{\nabla} \times \vec{A} = \vec{B}$$

$$\text{Ampere's law: } \oint_C \vec{B} \cdot d\vec{s} = \mu_0 I_{enc} = \mu_0 \int_S \vec{J} \cdot d\vec{a} \quad ; \quad \vec{\nabla} \times \vec{B} = \mu_0 \vec{J}; \quad \vec{\nabla} \cdot \vec{B} = 0$$

$$\text{Field of a wire: } B = \frac{\mu_0 I}{2\pi r} \hat{\theta} \quad ; \quad \text{Force between wires: } F = \frac{\mu_0 I_1 I_2 \ell}{2\pi r} \quad ; \quad \text{cyclotron: } \omega = \frac{qB}{m}$$

loop (axis):  $B = \frac{\mu_0 I b^2}{2(b^2 + z^2)^{3/2}}$ ; Biot-Savart:  $d\vec{B} = \frac{\mu_0}{4\pi} \frac{Id\vec{\ell} \times \hat{r}}{r^2}$      $\vec{B} = \vec{\nabla} \times \vec{A}$

Faraday's law  $\epsilon = \frac{1}{q} \int \vec{f} \cdot d\vec{s} = -\frac{d\Phi}{dt} = \oint \vec{E} \cdot d\vec{l}$  ;  $\Phi = \int \vec{B} \cdot d\vec{a}$  ;  $\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$

Mutual inductance:  $M_{21} = \frac{N_2 \Phi_{21}}{I_1}$  ;  $\epsilon_2 = -M_{21} \frac{dI_1}{dt}$  ;  $M_{21} = M_{12} = M$

Self - inductance:  $L = \frac{N\Phi_B}{I}$  ;  $\epsilon_L = -L \frac{dI}{dt}$  ;  $\frac{L}{l} = \mu_0 n^2 A$  for solenoid ; energy  $U = \frac{1}{2} LI^2$

energy density: magnetic  $u = \frac{1}{2} \frac{B^2}{\mu_0}$ ; electric  $u = \frac{1}{2} \epsilon_0 E^2$ ,  $\mu_0 = 4\pi \times 10^{-7} \text{ kg m / C}^2$

LR circuit:  $I = \frac{V_0}{R} (1 - e^{-t/\tau_L})$  (rise) ;  $I = I_0 e^{-t/\tau_L}$  (decay) ;  $\tau_L = L/R$

LRC circuit:  $Q(t) = Q_0 e^{-\frac{R}{2L}t} \cos(\omega t + \phi)$  ;  $\omega = \sqrt{\omega_0^2 - \frac{R^2}{4L^2}}$ ;  $\omega_0 = 1/\sqrt{LC}$

AC circuit, LRC:  $\epsilon = \epsilon_0 e^{i\omega t}$  ;  $I = I_0 e^{i(\omega t + \phi)}$  ;  $I = \epsilon / Z$  ;  $\tan \phi = -\frac{Z_L + Z_C}{R}$

$Z_L = i\omega L$  ;  $Z_C = 1/i\omega C$  ;  $Z = R + Z_L + Z_C$  ;  $I_0 = \epsilon_0 / |Z|$  ;  $\bar{P} = I_{\text{rms}} V_{\text{rms}} \cos \phi$

Ampere- Maxwell law:  $\vec{\nabla} \times \vec{B} = \mu_0 \vec{J} + \mu_0 \epsilon_0 \frac{\partial \vec{E}}{\partial t} = \mu_0 \vec{J} + \mu_0 \vec{J}_d$

Electromagnetic wave in vacuum:  $c = 1/\sqrt{\mu_0 \epsilon_0} = \omega / k$

$\vec{E} = \hat{z} E_0 \sin(ky - \omega t)$  ;  $\vec{B} = \hat{x} B_0 \sin(ky - \omega t)$  ; propagates in direction  $\vec{E} \times \vec{B}$

Poynting vector:  $\vec{S} = \frac{\vec{E} \times \vec{B}}{\mu_0}$  ;  $\frac{\partial u}{\partial t} = -\vec{\nabla} \cdot \vec{S}$  ;  $u = \frac{1}{2} \epsilon_0 E^2 + \frac{1}{2\mu_0} B^2$

Electric dipole moment:  $\vec{p} = \int d^3r \rho(\vec{r}) \vec{r}$  field:  $E_r = \frac{p \cos \theta}{2\pi \epsilon_0 r^3}$  ;  $E_\theta = \frac{p \sin \theta}{4\pi \epsilon_0 r^3}$

$\vec{N} = \vec{p} \times \vec{E}$  ;  $U = -\vec{p} \cdot \vec{E}$  ;  $\vec{F} = (\vec{p} \cdot \vec{\nabla}) \vec{E}$  ;  $P = Np = \text{dipole moment / volume}$

Magnetic dipole moment:  $\vec{m} = I \vec{a}$  field  $B_r = \frac{\mu_0 m \cos \theta}{2\pi r^3}$  ;  $B_\theta = \frac{\mu_0 m \sin \theta}{2\pi r^3}$

$\vec{N} = \vec{m} \times \vec{B}$  ;  $U = -\vec{m} \cdot \vec{B}$  ;  $\vec{F} = \vec{\nabla} (\vec{m} \cdot \vec{B})$  orbital motion:  $\vec{m} = (q/2M) \vec{L}$

$M = Nm$  magnetization = magnetic moment/volume (q=charge, M=mass)