PHYSICS 4C QUIZ 2 FALL QUARTER 2019
PROF. HIRSCH OCTOBER 21, 2019

Formulas:

$F = k\dfrac{q_1 q_2}{r^2}$ Coulomb's law ; $k = \dfrac{1}{4\pi\varepsilon_0} = 8.99\times 10^9 \mathrm{N\cdot m^2/C^2}$; $\varepsilon_0 = 8.85\times 10^{-12} C^2/Nm^2$

Electric field: $\vec{E} = \dfrac{q}{4\pi\varepsilon_0 r^2}\hat{r} = \dfrac{kq}{r^2}\hat{r}$; $\vec{E}(\vec{r}) = k\int \rho(\vec{r}\,')\dfrac{\vec{r}-\vec{r}\,'}{|\vec{r}-\vec{r}\,'|^3}d^3r'$; $\vec{F} = q_0\vec{E}$

Linear, surface, volume charge density: $dq = \lambda\, d\ell$, $dq = \sigma\, da$, $dq = \rho\, dv$

Electric field of infinite: line of charge: $E = \dfrac{\lambda}{2\pi\varepsilon_0 r}$; sheet of charge: $E = \dfrac{\sigma}{2\varepsilon_0}$

Gauss law: $\Phi = \oint \vec{E}\cdot d\vec{a} = \dfrac{q_{enc}}{\varepsilon_0} = \dfrac{1}{\varepsilon_0}\int d^3r\, \rho(\vec{r})$; Φ=electric flux

Energy: $U = k\dfrac{q_1 q_2}{r_{12}}$ $U = \dfrac{\varepsilon_0}{2}\int E(\vec{r})^2 d^3r$ Work: $W = \int \vec{F}\cdot d\vec{s}$

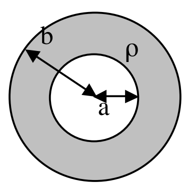
Electric potential: $\phi(P_2) - \phi(P_1) = -\int_{P_1}^{P_2} \vec{E}\cdot d\vec{s}$ $\phi(\vec{r}) = \dfrac{1}{4\pi\varepsilon_0}\int d^3r' \dfrac{\rho(\vec{r}\,')}{|\vec{r}-\vec{r}\,'|}$

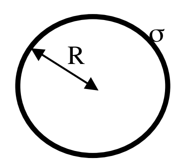
Point charge: $\phi(\vec{r}) = \dfrac{1}{4\pi\varepsilon_0}\dfrac{q}{r}$ Dipole: $\phi(r,\theta) = \dfrac{p\cos\theta}{4\pi\varepsilon_0 r^2}$

$U = \dfrac{1}{2}\int \rho\, \phi\, d^3r$ $\vec{E} = -\vec{\nabla}\phi$; $\vec{\nabla}\cdot\vec{E} = \dfrac{\rho}{\varepsilon_0}$; $\vec{\nabla}\times\vec{E} = 0$

Capacitors: $Q = C\phi$, $U = \dfrac{Q^2}{2C}$; Planar: $C = \dfrac{\varepsilon_0 A}{s}$ Spherical: $C = 4\pi\varepsilon_0 R$

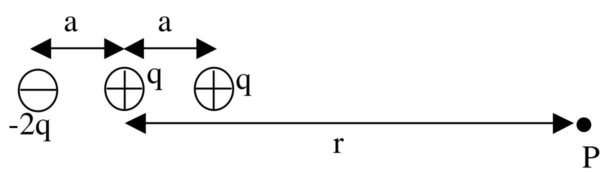
Problem 1 (10 pts)





(a) Find the electric potential at the center of the spherical shell of inner radius a and outer radius b and uniform charge density ρ shown in the figure to the left, taking the potential to be zero at infinity. (Hint: there is at least two ways to do this, one is easier than the other).
(b) Find the electric potential at the surface of the thin spherical conducting shell of radius R and surface charge density σ shown in the figure to the right.
(b') Find the electric potential at the center of this shell.
(c) Shows that the answer to (a) agrees with the answer to (b') when (b-a) is small.

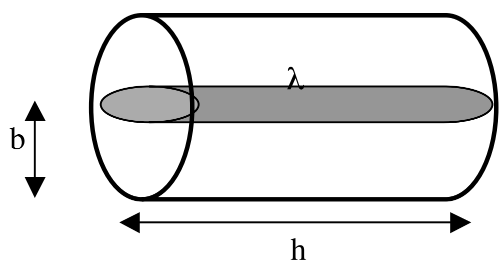
Problem 2 (10 pts)



Consider the three charges q, q, -2q shown in the figure, aligned along a line at distance a apart from each other. The point P is along the same line at distance r from the center charge, with r>>a.

(a) Find the electric potential at point P, assuming r>>a, to leading order in a/r.
(b) Find the electric field at point P, assuming r>>a, to leading order in a/r.
(c) Find the dipole moment of this charge distribution. Justify your answer.

Problem 3 (10 pts)



A cylindrical capacitor consists of a solid metallic tube of radius a and length h that has linear charge density λ, and a thin metallic shell of radius b and length h. Assume h is much larger than b.

(a) Find the surface charge density σ at the inner surface of the thin metallic shell.
(b) Find the electric field at the inner surface of the thin metallic shell.
(c) Find the difference in potential between the solid metallic tube and the thin metallic shell.
(d) What is the capacitance of this system?