

Formulas:

$$F = k \frac{q_1 q_2}{r^2} \quad \text{Coulomb's law} \quad ; \quad k = \frac{1}{4\pi\epsilon_0} = 8.99 \times 10^9 \text{ N} \cdot \text{m}^2 / \text{C}^2; \quad \epsilon_0 = 8.85 \times 10^{-12} \text{ C}^2 / \text{Nm}^2$$

$$\text{Electric field: } \vec{E} = \frac{q}{4\pi\epsilon_0 r^2} \hat{r} = \frac{kq}{r^2} \hat{r} \quad ; \quad \vec{E}(\vec{r}) = k \int \rho(\vec{r}') \frac{\vec{r} - \vec{r}'}{|\vec{r} - \vec{r}'|^3} d^3r' \quad ; \quad \vec{F} = q_0 \vec{E}$$

$$\text{Linear, surface, volume charge density: } dq = \lambda dl \quad , \quad dq = \sigma da \quad , \quad dq = \rho dv$$

$$\text{Electric field of infinite: line of charge: } E = \frac{\lambda}{2\pi\epsilon_0 r} \quad ; \quad \text{sheet of charge: } E = \frac{\sigma}{2\epsilon_0}$$

$$\vec{\nabla} \times \vec{E} = 0 \quad (\text{electrostatics})$$

$$\text{Gauss law: } \Phi = \oint \vec{E} \cdot d\vec{a} = \frac{q_{enc}}{\epsilon_0} = \frac{1}{\epsilon_0} \int d^3r \rho(\vec{r}) \quad ; \quad \Phi = \text{electric flux}$$

$$\text{Energy: } U = k \frac{q_1 q_2}{r_{12}} \quad U = \frac{\epsilon_0}{2} \int E(\vec{r})^2 d^3r \quad \text{Work: } W = \int \vec{F} \cdot d\vec{s}$$

$$\text{Electric potential: } \phi(P_2) - \phi(P_1) = - \int_{P_1}^{P_2} \vec{E} \cdot d\vec{s} \quad \phi(\vec{r}) = \frac{1}{4\pi\epsilon_0} \int d^3r' \frac{\rho(\vec{r}')}{|\vec{r} - \vec{r}'|}$$

$$\text{Point charge: } \phi(\vec{r}) = \frac{1}{4\pi\epsilon_0} \frac{q}{r} \quad \text{Dipole: } \phi(r, \theta) = \frac{p \cos \theta}{4\pi\epsilon_0 r^2}$$

$$U = \frac{1}{2} \int \rho \phi d^3r \quad \vec{E} = -\vec{\nabla} \phi \quad ; \quad \vec{\nabla} \cdot \vec{E} = \frac{\rho}{\epsilon_0} \quad ; \quad \vec{\nabla} \times \vec{E} = 0$$

$$\text{Capacitors: } Q = C\phi \quad , \quad U = \frac{Q^2}{2C} \quad ; \quad \text{Planar: } C = \frac{\epsilon_0 A}{s} \quad \text{Spherical: } C = 4\pi\epsilon_0 R$$

$$I = \int \vec{J} \cdot d\vec{a} \quad , \quad I = \frac{dQ}{dt} \quad , \quad \vec{J} = nq\vec{u} \quad ; \quad \text{div} \vec{J} = -\frac{\partial \rho}{\partial t} \quad ; \quad \text{Power: } P = I^2 R \quad ; \quad P = \epsilon I$$

$$V = IR \quad ; \quad \vec{J} = \sigma \vec{E} \quad ; \quad \vec{E} = \rho \vec{J} \quad ; \quad R = \rho \frac{L}{A} \quad ; \quad \sigma = \frac{ne^2 \tau}{m_e} \quad ; \quad Q(t) = C\epsilon(1 - e^{-t/RC}) \quad ; \quad Q(t) = Q_0 e^{-t/RC}$$

$$\text{Lorentz force: } \vec{F} = q(\vec{E} + \vec{v} \times \vec{B}) \quad ; \quad \text{force on wire: } d\vec{F} = Id\vec{\ell} \times \vec{B} \quad ; \quad \text{solenoid: } B = \mu_0 nI$$

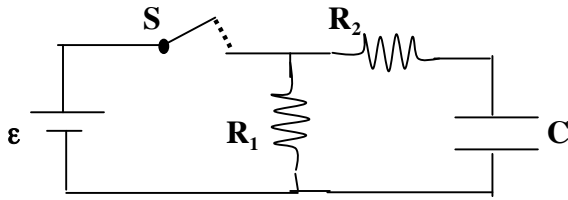
$$\text{Stokes' theorem: } \oint_C \vec{F} \cdot d\vec{s} = \int_S (\vec{\nabla} \times \vec{F}) \cdot d\vec{a} \quad ; \quad \text{vector potential: } \vec{\nabla} \times \vec{A} = \vec{B}$$

$$\text{Ampere's law: } \oint_C \vec{B} \cdot d\vec{s} = \mu_0 I_{enc} = \mu_0 \int_S \vec{J} \cdot d\vec{a} \quad ; \quad \vec{\nabla} \times \vec{B} = \mu_0 \vec{J} \quad ; \quad \vec{\nabla} \cdot \vec{B} = 0$$

$$\text{Field of a wire: } B = \frac{\mu_0 I}{2\pi r} \hat{\theta} \quad ; \quad \text{Force between wires: } F = \frac{\mu_0 I_1 I_2 \ell}{2\pi r} \quad ; \quad \text{cyclotron: } \omega = \frac{qB}{m}$$

$$\mu_0 = 4\pi \times 10^{-7} \text{ kg m} / \text{C}^2$$

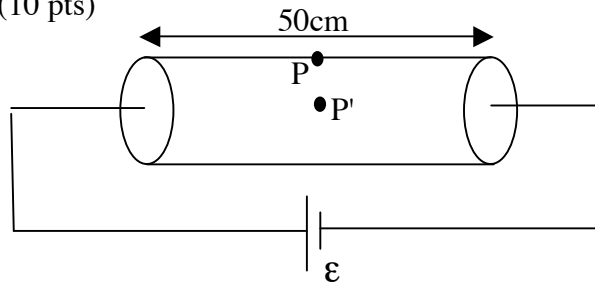
Problem 1 (10 pts)



In the circuit in the figure, the two resistors shown have the same value, $R_1=R_2=R$. Initially the charge in the capacitor C is zero and the switch S is open. Then the switch is closed, and a current $1A$ circulates through R_1 immediately after the switch is closed.

(a) A long time thereafter, how much current will circulate through R_1 ?
 (b) Assume at that time (i.e. a long time after S was closed), S is opened again. How much current will circulate through R_1 right after S is opened again?
 (c) How long after S is opened again will the power dissipated in R_1 be $\frac{1}{2}$ as large as immediately after S is opened again? Give your answer in terms of $t_0=RC$.

Problem 2 (10 pts)

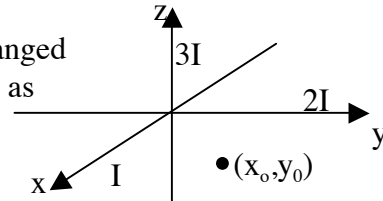


The cylindrical conductor shown in the figure has length 50cm , cross-sectional area 1mm^2 and is made of Cu , with resistivity 1.7×10^{-8} ohm-meter. The emf $\epsilon=0.2\text{V}$. Assume the other wires in the circuit have zero resistance.

(a) Find the current flowing through the cylinder, in A (amperes).
 (b) Find the magnitude of the magnetic field at point P at the surface of the cylinder, in T.
 (c) Find the magnitude of the electric field (in N/C) and of the magnetic field (in T) at point P' on the axis of the cylinder.

Problem 3 (10 pts)

Consider 3 wires with currents I , $2I$ and $3I$ respectively arranged along the x , y and z axis of a rectangular coordinate system as shown in the figure.



(a) Find the magnetic field at a point (x_0, y_0) in the xy plane. You can give your result either in Cartesian coordinates, B_x, B_y, B_z or in cylindrical coordinates B_r, B_ϕ, B_z . Give also the magnitude of B .

(b) Find the line integral $\oint_C \vec{B} \cdot d\vec{s}$ where C is a rectangle in the (x,y) plane with vertices at points $(\pm x_0, \pm y_0)$.