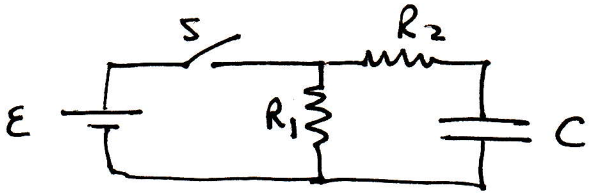
Physics 4C | Quiz 3 Solutions | Fall 2019

Problem 1



(a) When S is closed, the difference in potential across R_1 is $\mathcal{E}$.
Therefore, current through R_1 is

$$I = \frac{\mathcal{E}}{R} = 1\,A$$

A long time thereafter, difference in potential across R_1 is still $\mathcal{E}$.
Therefore, current through R_1 is $\boxed{I' = 1\,A}$

(b) A long time after S was closed, charge in capacitor is $Q = C\mathcal{E}$.
When S is opened, C starts to discharge through R_1 and R_2,
equivalence resistance is $R_{eq} = 2R$, so initial current through R_1 (and R_2) is

$$\boxed{I'' = \frac{\mathcal{E}}{2R} = 0.5\,A}$$

(c) After S is opened, current versus time is

$$I(t) = I(0)\,e^{-t/2RC} \qquad \text{since } R_{eq} = 2R$$

Power dissipated in R_1:

$$P_1(t) = I(t)^2 R = I(0)^2 e^{-t/RC} R$$

For $P_1(t) = \frac{1}{2} P_1(0) \Rightarrow e^{-t/RC} = \frac{1}{2} \Rightarrow \frac{t}{RC} = \ln 2 \Rightarrow$

$$\boxed{t = RC \ln 2 = t_0 \ln 2 = 0.69\,t_0}$$

Problem 2

$$R = \frac{\rho l}{A} = \frac{1.7\times10^{-8} \times 0.5\ \Omega m^2}{10^{-6} m^2} \Rightarrow$$

$\boxed{R = 0.0085\Omega}$ (a) $\boxed{I = \frac{\varepsilon}{R} = 23.5A}$

(b) B field is

$$B = \frac{\mu_0 I}{2\pi r}$$. r is radius. $\pi r^2 = 1 mm^2 \Rightarrow r = \sqrt{\frac{10^{-6} m^2}{\pi}}$

$$B = \frac{\mu_0 I \sqrt{\pi}}{2\pi . 10^{-3} m} = \frac{\cancel{4\pi}^{2} \times 10^{-7} \times 23.5 \times \sqrt{\pi}}{\cancel{2\pi} . 10^{-3}} T = 0.0083T$$

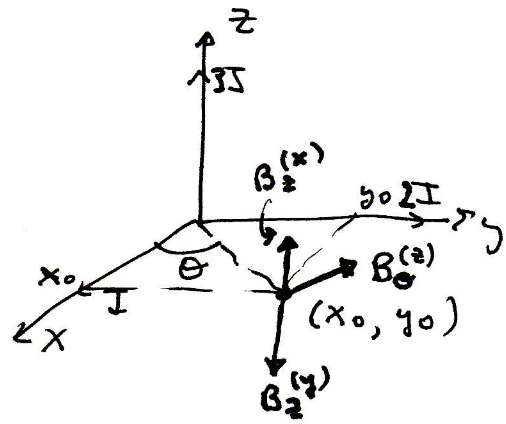
$\boxed{B = 0.0083T = 83.3G}$

(c) B = 0 at center of cylinder.

E is uniform in the ~~of~~ cylinder. $E = V/l$, with $V = \varepsilon$ the voltage drop and $l = 50$ cm the length $\Rightarrow$

$$\boxed{E = \frac{\varepsilon}{l} = \frac{0.2V}{0.5\ m} = 0.4 \frac{N}{C}}$$

Problem 3



Call $B_i^{(j)}$ the i-th component of B due to the current in wire j

$$B_z^{(x)} = \frac{\mu_0 I}{2\pi y_0}\hat{z}$$

$$B_z^{(y)} = -\frac{2\mu_0 I}{2\pi x_0}\hat{z} \quad ; \quad B_z^{(z)} = 0$$

$$\Rightarrow \boxed{B_z(x_0, y_0) = \frac{\mu_0 I}{2\pi}\left(\frac{1}{y_0} - \frac{2}{x_0}\right)}$$

The current along z gives magnetic field along $\hat{\theta}$ direction

$$B_\theta^{(z)} = \frac{3\mu_0 I}{2\pi (x_0^2 + y_0^2)^{1/2}}\hat{\theta} = \frac{3\mu_0 I}{2\pi r}\hat{\theta}$$

Cartesian components are:

$$B_x^{(z)} = -B^{(z)}\sin\theta = -\frac{3\mu_0 I}{2\pi}\frac{y_0}{x_0^2 + y_0^2}$$

$$B_y^{(z)} = B^{(z)}\cos\theta = \frac{3\mu_0 I}{2\pi}\frac{x_0}{x_0^2 + y_0^2}$$

Magnitude of B: $\boxed{B = \sqrt{B_z^2 + B_\theta^2} = \frac{\mu_0 I}{2\pi}\left[\left(\frac{1}{y_0} - \frac{2}{x_0}\right)^2 + \frac{9}{x_0^2 + y_0^2}\right]^{1/2}}$

(b) $\oint_C \vec{B}\cdot d\vec{s} = \mu_0 I_{enc}$ The wires along the x and y direction don't contribute. The answer is simply

$$\boxed{\oint_c \vec{B}\cdot d\vec{s} = 3\mu_0 I}$$

the shape of C in the xy plane also doesn't matter.