PHYSICS 4C QUIZ 4 FALL QUARTER 2019
PROF. HIRSCH NOVEMBER 18, 2019

Formulas:

$F = k\frac{q_1 q_2}{r^2}$ Coulomb's law ; $k = \frac{1}{4\pi\varepsilon_0} = 8.99\times 10^9 \mathrm{N\cdot m^2/C^2}$; $\varepsilon_0 = 8.85\times 10^{-12} C^2/Nm^2$

Electric field: $\vec{E} = \frac{q}{4\pi\varepsilon_0 r^2}\hat{r} = \frac{kq}{r^2}\hat{r}$; $\vec{E}(\vec{r}) = k\int \rho(\vec{r}\,')\frac{\vec{r}-\vec{r}\,'}{|\vec{r}-\vec{r}\,'|^3}d^3r'$; $\vec{F} = q_0\vec{E}$

Linear, surface, volume charge density: $dq = \lambda\, d\ell$, $dq = \sigma\, da$, $dq = \rho\, dv$

Electric field of infinite: line of charge: $E = \frac{\lambda}{2\pi\varepsilon_0 r}$; sheet of charge: $E = \frac{\sigma}{2\varepsilon_0}$

$\vec{\nabla}\times\vec{E} = 0$ (electrostatics)

Gauss law: $\Phi = \oint \vec{E}\cdot d\vec{a} = \frac{q_{enc}}{\varepsilon_0} = \frac{1}{\varepsilon_0}\int d^3r\, \rho(\vec{r})$; Φ=electric flux

Energy: $U = k\frac{q_1 q_2}{r_{12}}$ $U = \frac{\varepsilon_0}{2}\int E(\vec{r})^2 d^3r$ Work: $W = \int \vec{F}\cdot d\vec{s}$

Electric potential: $\phi(P_2) - \phi(P_1) = -\int_{P_1}^{P_2}\vec{E}\cdot d\vec{s}$ $\phi(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\int d^3r'\frac{\rho(\vec{r}\,')}{|\vec{r}-\vec{r}\,'|}$

Point charge: $\phi(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\frac{q}{r}$ Dipole: $\phi(r,\theta) = \frac{p\cos\theta}{4\pi\varepsilon_0 r^2}$

$U = \frac{1}{2}\int \rho\,\phi\, d^3r$ $\vec{E} = -\vec{\nabla}\phi$; $\vec{\nabla}\cdot\vec{E} = \frac{\rho}{\varepsilon_0}$; $\vec{\nabla}\times\vec{E} = 0$

Capacitors: $Q = C\phi$, $U = \frac{Q^2}{2C}$; Planar: $C = \frac{\varepsilon_0 A}{s}$ Spherical: $C = 4\pi\varepsilon_0 R$

$I = \int \vec{J}\cdot d\vec{a}$, $I = \frac{dQ}{dt}$, $\vec{J} = nq\vec{u}$; $div\vec{J} = -\frac{\partial\rho}{\partial t}$; Power: $P = I^2R$; $P = \varepsilon I$

V=IR; $\vec{J} = \sigma\vec{E}$; $\vec{E} = \rho\vec{J}$; $R = \rho\frac{L}{A}$; $\sigma = \frac{ne^2\tau}{m_e}$; $Q(t) = C\varepsilon(1-e^{-t/RC})$; $Q(t) = Q_0 e^{-t/RC}$

Lorentz force: $\vec{F} = q(\vec{E} + \vec{v}\times\vec{B})$; force on wire: $d\vec{F} = Id\vec{\ell}\times\vec{B}$; solenoid: $B = \mu_0 nI$

Stokes' theorem: $\oint_C \vec{F}\cdot d\vec{s} = \int_S (\vec{\nabla}\times\vec{F})\cdot d\vec{a}$; vector potential: $\vec{\nabla}\times\vec{A} = \vec{B}$

Ampere's law: $\oint_C \vec{B}\cdot d\vec{s} = \mu_0 I_{enc} = \mu_0\int_S \vec{J}\cdot d\vec{a}$; $\vec{\nabla}\times\vec{B} = \mu_0\vec{J}$; $\vec{\nabla}\cdot\vec{B} = 0$

Field of a wire: $B = \frac{\mu_0 I}{2\pi r}\hat{\theta}$; Force between wires: $F = \frac{\mu_0 I_1 I_2 \ell}{2\pi r}$; cyclotron: $\omega = \frac{qB}{m}$

loop (axis): $B = \frac{\mu_0 I b^2}{2(b^2+z^2)^{3/2}}$; Biot-Savart: $d\vec{B} = \frac{\mu_0}{4\pi}\frac{Id\vec{\ell}\times\hat{r}}{r^2}$

Faraday law: $\varepsilon = \frac{1}{q}\int \vec{f}\cdot d\vec{s} = -\frac{d\Phi}{dt} = \oint \vec{E}\cdot d\vec{l}$; $\Phi = \int \vec{B}\cdot d\vec{a}$; $\vec{\nabla}\times\vec{E} = -\frac{\partial \vec{B}}{\partial t}$

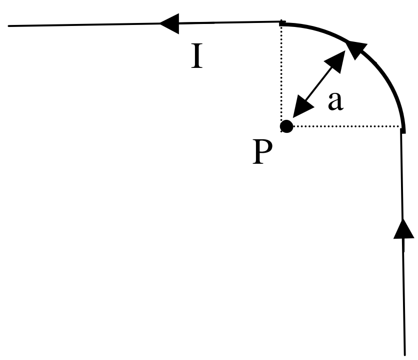
Mutual inductance: $M_{21} = \frac{N_2\Phi_{21}}{I_1}$; $\varepsilon_2 = -M_{21}\frac{dI_1}{dt}$; $M_{21} = M_{12} = M$

Self - inductance: $L = \frac{N\Phi_B}{I}$; $\varepsilon_L = -L\frac{dI}{dt}$; $\frac{L}{l} = \mu_0 n^2 A$ for solenoid ; energy $U = \frac{1}{2}LI^2$

energy density: magnetic $u = \frac{1}{2}\frac{B^2}{\mu_0}$; electric $u = \frac{1}{2}\varepsilon_0 E^2$, $\mu_0 = 4\pi\times 10^{-7}\, kg\ m/C^2$

<u>Note:</u> for all problems, please explain clearly what you do. It can help us to know that you understand what you are doing if you state which physical law(s), mathematical theorem(s), formula(s) in the formula list, etc. you are using.

Problem 1 (10 pts)



The two straight wires in the figure go to infinity, and are in directions perpendicular to each other (i.e. at a 90 degree angle). They are joined by a circular arc of radius a (in bold). The point P is at the center of the circular arc. The wires carry current I as shown.
(a) Find the magnitude and direction of the magnetic field at P.
For (b), (c), (d): Assume our coordinate system is centered at P and the z axis points perpendicular to the paper out of the paper. Consider point P'=(0,0,a), i.e. right above point P in direction out of the paper at distance a from P.
(b) Find the magnitude of the magnetic field at P' originating from one of the straight wires only.
(c) Find the z-component of the magnetic field at P' originating from the circular arc only.
(d) Find the z-component of the magnetic field at P' originated from all the wires in the figure, and the ratio of it to the magnetic field at P calculated in (a). The ratio is a number, give that number.

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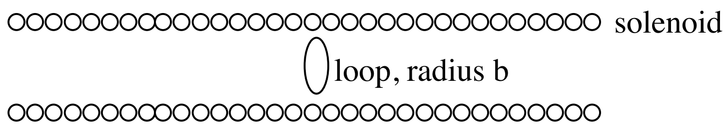
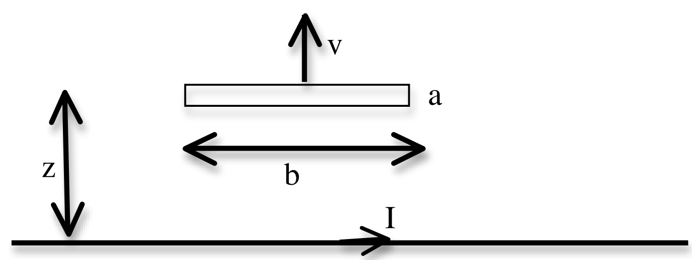
Problem 2 (10 pts)



A loop of wire of radius b and resistance R is located near the center of a long solenoid and is oriented with its normal parallel to the solenoid axis. The solenoid has n turns per meter. A constant current I_0 circulates through the loop of wire.
(a) Find an expression for the time-dependent current I(t) circulating through the solenoid in terms of I_0, b, R and n.
(b) If the current circulating through the loop is in clockwise direction when seen from the left end of the solenoid, in which direction is the current through the solenoid? Is there only one possible answer, or more than one? Give and justify all possible answers.
(c) What is the magnitude and direction of the electric field at a point on the loop of wire?

Problem 3 (10 pts)



A straight long wire carries a constant current I. A rectangular wire loop of dimensions a, b, is at distance z(t) from the wire, with $a<<z$, and is moving away from the straight wire at constant speed v as shown in the figure. The loop has resistance R. Note the the loop side of length b is parallel to the long wire, that of length a is perpendicular.
(a) Find an expression for the current i induced in the loop. Use the fact that $a<<z$ to simplify the calculation.
(b) Find an expression for the energy per unit time (power) P dissipated in the loop.
(c) Using the formula for the magnetic force on a current carrying wire, find an expression for the force F that has to be applied in order to pull the loop away from the long wire at constant speed v. Use that $a<<z$ to simplify the calculation.
(d) Compare the expressions you found for P and for F times v and explain why they are the same or different. If the latter, which of the two is larger in magnitude?