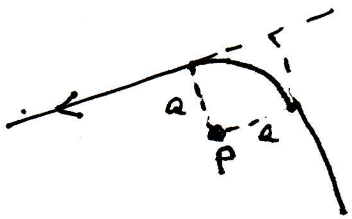
Physics 4C — Quiz 4 Solutions — Fall 2019

Problem 1



(a) Field from ∞ straight wire is $B_1 = \frac{\mu_0 I}{2\pi r}$

Here, each of the straight wires gives 1/2 of that, with $r = a$, in direction out of the paper at point P.

Field from circular loop is $B = \frac{\mu_0 I}{2b}$ for radius B. Here $b = a$ and it is 1/4 loop $\Rightarrow$ $B_2 = \frac{\mu_0 I}{8a}$. So total field at P is

$$B = \frac{\mu_0 I}{a}\left(2 \times \frac{1}{2} \times \frac{1}{2\pi} + \frac{1}{8}\right) \Rightarrow \boxed{B = \frac{\mu_0 I}{8a}\left(1 + \frac{4}{\pi}\right)}$$ points out of paper.

(b) $P' = (0, 0, a)$. From one of the straight wires: $r = \sqrt{2}\,a \Rightarrow$

$$\boxed{B_1 = \frac{1}{2}\,\frac{\mu_0 I}{2\pi \cdot \sqrt{2}\,a}}$$

(c) From the circular arc: $B_{2z} = \frac{1}{4} \cdot \frac{\mu_0 I a^2}{2\,(2a^2)^{3/2}} = \frac{1}{8 \cdot 2^{3/2}}\,\frac{\mu_0 I}{a}$

So $\boxed{B_{2z} = \frac{1}{16\sqrt{2}}\,\frac{\mu_0 I}{a}}$

(d) B_1 points at 45° from the z-axis. For z component, multiply by $\frac{\sqrt{2}}{2}$.

So total z component of field at P' is

$$B_z^{(P')} = 2 \times \frac{\sqrt{2}}{2} \times B_1 + B_{2z} = \frac{\mu_0 I}{a}\left[\frac{1}{4\pi} + \frac{1}{16\sqrt{2}}\right] = \frac{\mu_0 I}{8a}\left[\frac{2}{\pi} + \frac{1}{2\sqrt{2}}\right]$$

So ratio $$\boxed{\frac{B_z(P')}{B(P)} = \frac{\frac{2}{\pi} + \frac{1}{2\sqrt{2}}}{1 + \frac{4}{\pi}} = 0.436}$$

Problem 2

The magnetic field at the center of the solenoid is $B = \mu_0 n I$

The flux through the loop of radius b is

$$\Phi = B\pi b^2 = \mu_0 n I \pi b^2$$

$I = I(t)$, the induced emf is

$$\mathcal{E} = -\frac{d\Phi}{dt} = -\mu_0 n \pi b^2 \frac{dI}{dt}$$

and the current circulating in the loop is $I_0 = \frac{|\mathcal{E}|}{R} = \frac{\mu_0 n \pi b^2}{R} \frac{dI}{dt}$

Since I_0 is time independent,

$$\boxed{I(t) = \frac{R}{\mu_0 n \pi b^2} I_0 t + \text{const.}}$$

(b) If I_0 is clockwise, there are 2 possibilities for $I(t)$

(1) $I(t)$ is counterclockwise and increasing (in magnitude) with time

(2) $I(t)$ is clockwise and decreasing with time (in magnitude)

according to Lenz's law.

(c) The electric field on the loop has magnitude

$$\boxed{E = \frac{|\mathcal{E}|}{2\pi b}}$$ it is tangential to the loop, points in same direction as loop current, i.e. clockwise.

Problem 3

Magnetic field from long wire at the loop is

$$\boxed{B = \frac{\mu_0 I}{2\pi z}}, \quad \text{flux through loop is } \Phi = \frac{\mu_0 I}{2\pi z} \cdot a \cdot b$$

The loop moves at speed $v = dz/dt$.

emf is
$$\mathcal{E} = -\frac{\partial \Phi}{\partial t} = -\frac{d\Phi}{\partial z}\frac{\partial z}{\partial t} = \frac{\mu_0 I}{2\pi z^2} ab\, v$$

Current through loop is
$$\boxed{i = \frac{\mathcal{E}}{R} = \frac{\mu_0 I}{2\pi z^2}\frac{ab\,v}{R}}$$

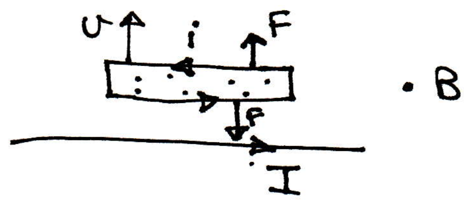
(b) Power dissipated in the loop is

$$\boxed{P = i^2 R = \frac{\mu_0^2 I^2}{4\pi^2 z^4}\frac{a^2 b^2 v^2}{R}}$$

(c) Force on side of length B is $F = i\,b\,B$

On side farther away from wire, it pushes away from long wire.

On closer side it pushes towards it.



Net force

$$F_{net} = -ib\left(B(z) - B(z-a)\right) = -ib\frac{\partial B}{\partial z} \cdot a$$

$$i - \frac{\partial B}{\partial z} = \frac{\mu_0 I}{2\pi z^2} \Rightarrow$$

$$F_{net} = iba\frac{\mu_0 I}{2\pi z^2} = \left(\frac{\mu_0 I}{2\pi z^2}\frac{ab\,v}{R}\right) ba\frac{\mu_0 I}{2\pi z^2} \Rightarrow$$

$$\Rightarrow \boxed{F_{net} = \frac{\mu_0^2 I^2}{4\pi^2 z^4}\frac{a^2 b^2 v}{R}}$$

(d) From expressions in (b) and (c), $P = F_{net} \cdot v$. They have to be the same due to energy conservation. The work done in pulling the loop away is dissipated as heat.