PHYSICS 4C | QUIZ 5 | FALL QUARTER 2019
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**Formulas:**

$F = k\frac{q_1 q_2}{r^2}$ Coulomb's law ; $k = \frac{1}{4\pi\varepsilon_0} = 8.99\times 10^9 \mathrm{N\cdot m^2/C^2}$; $\varepsilon_0 = 8.85\times 10^{-12} C^2/Nm^2$

Electric field: $\vec{E} = \frac{q}{4\pi\varepsilon_0 r^2}\hat{r} = \frac{kq}{r^2}\hat{r}$ ; $\vec{E}(\vec{r}) = k\int \rho(\vec{r}\,')\frac{\vec{r}-\vec{r}\,'}{|\vec{r}-\vec{r}\,'|^3}d^3r'$ ; $\vec{F} = q_0\vec{E}$

Linear, surface, volume charge density: $dq = \lambda\, d\ell$ , $dq = \sigma\, da$ , $dq = \rho\, dv$

Electric field of infinite: line of charge: $E = \frac{\lambda}{2\pi\varepsilon_0 r}$; sheet of charge: $E = \frac{\sigma}{2\varepsilon_0}$

$\vec{\nabla}\times\vec{E} = 0$ (electrostatics)

Gauss law: $\Phi = \oint \vec{E}\cdot d\vec{a} = \frac{q_{enc}}{\varepsilon_0} = \frac{1}{\varepsilon_0}\int d^3r\, \rho(\mathrm{r})$ ; $\Phi$=electric flux

Energy: $U = k\frac{q_1 q_2}{r_{12}}$ $U = \frac{\varepsilon_0}{2}\int E(\vec{r})^2 d^3r$ Work: $W = \int \vec{F}\cdot d\vec{s}$

Electric potential: $\phi(P_2) - \phi(P_1) = -\int_{P_1}^{P_2}\vec{E}\cdot d\vec{s}$ $\phi(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\int d^3r'\frac{\rho(\vec{r}\,')}{|\vec{r}-\vec{r}\,'|}$

Point charge: $\phi(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\frac{q}{r}$ Dipole: $\phi(r,\theta) = \frac{p\cos\theta}{4\pi\varepsilon_0 r^2}$

$U = \frac{1}{2}\int \rho\,\phi\, d^3r$ $\vec{E} = -\vec{\nabla}\phi$ ; $\vec{\nabla}\cdot\vec{E} = \frac{\rho}{\varepsilon_0}$ ; $\vec{\nabla}\times\vec{E} = 0$

Capacitors: $Q = C\phi$ , $U = \frac{Q^2}{2C}$; Planar: $C = \frac{\varepsilon_0 A}{s}$ electric field: $E = \sigma/\varepsilon_0$

$I = \int \vec{J}\cdot d\vec{a}$, $I = \frac{dQ}{dt}$ , $\vec{J} = nq\vec{u}$ ; $div\vec{J} = -\frac{\partial\rho}{\partial t}$ ; Power: $P = I^2R$ ; $P = \varepsilon I$

V=IR; $\vec{J} = \sigma\vec{E}$; $\vec{E} = \rho\vec{J}$ ; $R = \rho\frac{L}{A}$ ; $\sigma = \frac{ne^2\tau}{m_e}$; $Q(t) = C\varepsilon(1-e^{-t/RC})$; $Q(t) = Q_0 e^{-t/RC}$

Lorentz force: $\vec{F} = q(\vec{E} + \vec{v}\times\vec{B})$; force on wire: $d\vec{F} = Id\vec{\ell}\times\vec{B}$ ; solenoid: $B = \mu_0 nI$

Stokes' theorem: $\oint_C \vec{F}\cdot d\vec{s} = \int_S (\vec{\nabla}\times\vec{F})\cdot d\vec{a}$ ; vector potential: $\vec{\nabla}\times\vec{A} = \vec{B}$

Ampere's law: $\oint_C \vec{B}\cdot d\vec{s} = \mu_0 I_{enc} = \mu_0\int_S \vec{J}\cdot d\vec{a}$ ; $\vec{\nabla}\times\vec{B} = \mu_0\vec{J}$ ; $\vec{\nabla}\cdot\vec{B} = 0$

Field of a wire: $B = \frac{\mu_0 I}{2\pi r}\hat{\theta}$ ; Force between wires: $F = \frac{\mu_0 I_1 I_2 \ell}{2\pi r}$; cyclotron: $\omega = \frac{qB}{m}$

loop (axis): $B = \frac{\mu_0 I b^2}{2(b^2+z^2)^{3/2}}$; Biot-Savart: $d\vec{B} = \frac{\mu_0}{4\pi}\frac{Id\vec{\ell}\times\hat{r}}{r^2}$ $\vec{B} = \vec{\nabla}\times\vec{A}$

Faraday's law $\varepsilon = \frac{1}{q}\int \vec{f}\cdot d\vec{s} = -\frac{d\Phi}{dt} = \oint \vec{E}\cdot d\vec{l}$ ; $\Phi = \int \vec{B}\cdot d\vec{a}$ ; $\vec{\nabla}\times\vec{E} = -\frac{\partial \vec{B}}{\partial t}$

Mutual inductance: $M_{21} = \frac{N_2\Phi_{21}}{I_1}$ ; $\varepsilon_2 = -M_{21}\frac{dI_1}{dt}$ ; $M_{21} = M_{12} = M$

Self - inductance: $L = \frac{N\Phi_B}{I}$ ; $\varepsilon_L = -L\frac{dI}{dt}$ ; $\frac{L}{l} = \mu_0 n^2 A$ for solenoid ; energy $U = \frac{1}{2}LI^2$

energy density: magnetic $u = \frac{1}{2}\frac{B^2}{\mu_0}$; electric $u = \frac{1}{2}\varepsilon_0 E^2$, $\mu_0 = 4\pi\times 10^{-7} kg\ m/C^2$

LR circuit: $I = \frac{V_0}{R}(1 - e^{-t/\tau_L})$ (rise) ; $I = I_0 e^{-t/\tau_L}$ (decay) ; $\tau_L = L/R$

$LRC$ circuit: $Q(t) = Q_0 e^{-\frac{R}{2L}t}\cos(\omega t + \phi)$ ; $\omega = \sqrt{\omega_0^2 - \frac{R^2}{4L^2}}$; $\omega_0 = 1/\sqrt{LC}$

AC circuit, LRC: $\varepsilon = \varepsilon_0 e^{i\omega t}$ ; $I = I_0 e^{i(\omega t+\phi)}$ ; $I = \varepsilon / Z$ ; $\tan\phi = -\frac{Z_L + Z_C}{R}$

$Z_L = i\omega L$ ; $Z_C = 1/i\omega C$ ; $Z = R + Z_L + Z_C$ ; $I_0 = \varepsilon_0/|Z|$ ; $\bar{P} = I_{rms}V_{rms}\cos\phi$

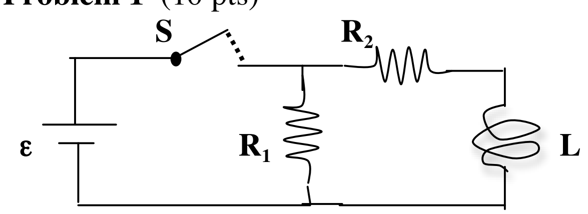
Ampere- Maxwell law: $\vec{\nabla}\times\vec{B} = \mu_0\vec{J} + \mu_0\varepsilon_0\frac{\partial\vec{E}}{\partial t} = \mu_0\vec{J} + \mu_0\vec{J}_d$

Electromagnetic wave in vacuum: $c = 1/\sqrt{\mu_0\varepsilon_0} = \omega / k$

$\vec{E} = \hat{z}E_0\sin(ky - \omega t)$ ; $\vec{B} = \hat{x}B_0\sin(ky - \omega t)$ ; propagates in direction $\vec{E}\times\vec{B}$

Poynting vector: $\vec{S} = \frac{\vec{E}\times\vec{B}}{\mu_0}$ ; $\frac{\partial u}{\partial t} = -\vec{\nabla}\cdot\vec{S}$ ; $u = \frac{1}{2}\varepsilon_0 E^2 + \frac{1}{2\mu_0}B^2$

**Problem 1** (10 pts)




In the circuit shown in the figure, $R_1$=$R_2$=R, L is the self-inductance of the inductor. At time t=0 the switch S is closed. Right thereafter (i.e. at time t=$0^+$) the current through $R_1$ is 3A. At a later time $t_0$ the current through $R_2$ is 2A. At that moment the switch S is opened again.

(a) Give an expression for $t_0$ in terms of R and L.

(b) Make an approximate graph of the current going through $R_1$, $I_{R1}$, versus time. Use the convention that $I_{R1}$ is positive at time t=$0^+$. Indicate the scale on the $I_{R1}$ axis. The time axis should start at 0, show $t_0$, and go up to t larger than $t_0$ (e.g. $3t_0$). Indicate the magnitude of $I_{R1}$ at time t=$t_0^+$, i.e. right after S is opened again.

(c) At what time t>$t_0$ does $I_{R1}$ have magnitude 1A? Give your answer in terms of $t_0$.

**Problem 2** (10 pts)
A 120 volt (rms), 60 Hz line provides power to a 75 watt light bulb.

(a) What is the resistance of the light bulb, in Ω?

(b) By what factor will the brightness of the bulb change if a 5μF capacitor is connected in series with the light bulb? Give the new power, in watts.

(c) Instead of connecting the capacitor, an inductor of inductance L is connected in series with the light bulb. If the brightness of the bulb changes by the same factor as in (b), what is the value of L, in H (henrys)?

(d) If now the capacitor of (b) and the inductor of (c) are connected in series with the lightbulb, by what factor will its brightness change from its original value (75W)? Give the new power, in watts. Justify your answer.

**Problem 3** (10 pts)
A parallel plate capacitor with round plates of radius R=0.1 m and distance between plates 0.01m is connected to wires and discharging through a resistor. When the current through the resistor is 3A:

(a) What is the magnitude of the magnetic field at a distance 1cm from the wire far from the capacitor? Assume the wire is straight. Give your answer in G (Gauss) ($1T=10^4G$).

(b) What is the magnitude of the magnetic field at a point midway between the capacitor plates at distance 1cm from the axis (line connecting the centers of the round plates)? Is it bigger, smaller or the same as that found in (a)? Give your answer in G.

(c) What is the value of dE/dt, the time derivative of the electric field between the capacitor plates? Give your answer in SI units (Volt/(meter)/seconds).