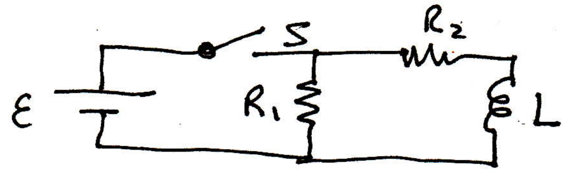
Physics 4E — **Quiz 5 Solutions** — Fall 2019

Problem 1



$R_1 = R_2 = R$

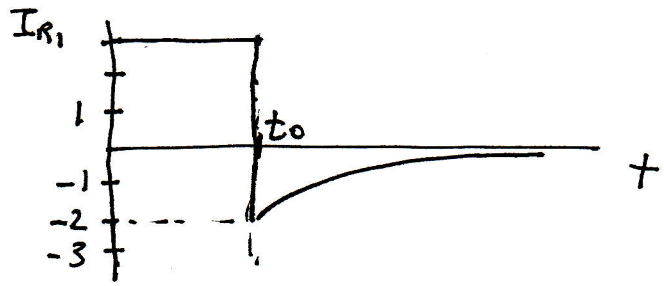
(a) The current through R_2 is given by

$$I = \frac{\mathcal{E}}{R}\left(1 - e^{-t/\tau_L}\right) \quad \text{with } \tau_L = L/R.$$

So at $t = t_0$: $\quad 1 - e^{-t_0/\tau_L} = \frac{2}{3} \Rightarrow e^{-t_0/\tau_L} = \frac{1}{3} \Rightarrow$

$$\ln \quad \frac{t_0}{\tau_L} = \ln 3 \Rightarrow \boxed{t_0 = \frac{L}{R}\ln 3 = 1.1\,\frac{L}{R}}$$

(b)



note: when S is opened again, I_{R_1} changes sign and takes the value 2A, same as through L.

(c) After t_0, current through R_1 is given by

$$I_{R_1} = I_0\, e^{-t_1/\tau_L'} \qquad \text{with } I_0 = 2\,A \text{ and } \tau_L' = \frac{L}{2R}$$

So it will be = to 1A for $\quad e^{-t_1/\tau_L'} = \frac{1}{2} \Rightarrow$

$\frac{t_1}{\tau_L'} = \ln 2 \Rightarrow t_1 = \frac{L}{2R}\ln 2$. Now this t_1 is measured from t_0 on. The total time is $t = t_0 + \frac{L}{2R}\ln 2$. Since $t_0 = \frac{L}{R}\ln 3$

$$\boxed{t = t_0 + t_0\,\frac{\ln 2}{2\ln 3} = t_0\left(1 + \frac{\ln 2}{2\ln 3}\right) = 1.315\,t_0}$$

Problem 2

(a) $P = I_{rms} V_{rms} = V_{rms}^2 / R \Rightarrow$

$R = V_{rms}^2 / P = 120^2/75\ \Omega \Rightarrow \boxed{R = 192\Omega}$

(b) With the capacitor in series, the impedance is

$$|Z| = \sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2}$$

we have $\omega = 2\pi f$, $f = 60\ Hz \Rightarrow \omega = 377 s^{-1} \Rightarrow$ with $C = 5\mu F$,

$$\Rightarrow \frac{1}{\omega C} = \frac{1}{377 \times 5 \times 10^{-6}}\ \Omega = 530\Omega$$

$$\Rightarrow |Z| = \sqrt{192^2 + 530^2}\ \Omega = 564\Omega$$

The power is now $P = V_{rms}^2 / |Z| \cos\phi = I_{rms}^2 R$

$I_{rms} = V_{rms}/|Z|$ decreased by factor $192/564 = 0.34$

$\Rightarrow$ power decreased by $0.34^2 \boxed{= 0.116}$ $\Rightarrow$ now it is $\boxed{8.7 \text{ watt}}$

Check: angle ϕ is: $\tan\phi = \frac{\frac{1}{\omega C}}{R} = \frac{530}{192} = 2.76 \Rightarrow \phi = 1.22$

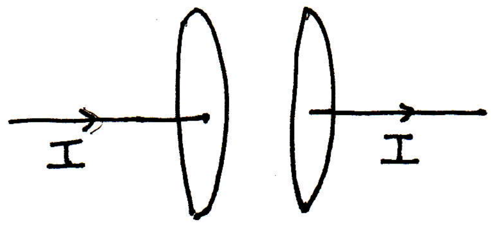
$\Rightarrow \cos\phi = 0.34$ ✓ $P = I_{rms} V_{rms} \cos\phi = \frac{V_{rms}(\cos\phi)^2}{R}$

(c) With an inductor alone, to get same impedance:

$$\omega L = \frac{1}{\omega C} \Rightarrow L = \frac{1}{\omega}\frac{1}{\omega C} = \frac{530}{377}\ H \Rightarrow \boxed{L = 1.405\ H}$$

(d) If this L and C are connected in series, their effect cancels since $Z = \sqrt{R^2 + (\omega L - \frac{1}{\omega C})^2} = R \Rightarrow$ brightness is same as initially, $\boxed{75 \text{ watts}}$

Problem 3



(a) $B = \frac{\mu_0 I}{2\pi r} = \frac{24\pi\times10^{-7}\times 3}{2\pi\times 0.01}T = 6\times10^{-5}T = \boxed{0.6\,\text{Gauss}}$

(b) Between capacitor plates:

$\oint \vec{B}\cdot d\vec{l} = \mu_0 \varepsilon_0 \int \frac{\partial \vec{E}}{\partial t}\cdot d\vec{a} \Rightarrow B\cdot 2\pi r = \mu_0 \varepsilon_0 \frac{\partial E}{\partial t}\cdot \pi r^2 \Rightarrow$

$\Rightarrow B = \mu_0 \varepsilon_0 \frac{\partial E}{\partial t}\cdot\frac{r}{2}$; $E = \frac{\sigma}{\varepsilon_0} = \frac{Q}{A\varepsilon_0} \Rightarrow \frac{\partial E}{\partial t} = \frac{I}{A\varepsilon_0} \Rightarrow$

$\Rightarrow B = \mu_0 I \frac{1}{A}\frac{r}{2} = \frac{\mu_0 I r}{2\pi R^2} = \frac{\mu_0 I}{2\pi r}\left(\frac{r}{R}\right)^2$ since $R = 0.1\,m$ and

$r = 0.01\,m \Rightarrow r/R = 0.1 \Rightarrow$

$B = 0.6\times 0.1^2\,G \Rightarrow \boxed{B = 0.006\,G}$

(c) $\frac{\partial E}{\partial t} = \frac{I}{A\varepsilon_0} = \frac{3}{\pi\cdot 0.1^2\cdot 8.85\times10^{-12}}\frac{V}{ms}$

$\Rightarrow \boxed{\frac{\partial E}{\partial t} = 1.08\times10^{13}\frac{V}{m\cdot s}}$