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# Drag reduction in two phase and polymer flows

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The basic dynamics of turbulent boundary layers of several media is described qualitatively: suspensions of several types and polymer solutions. Despite the considerable differences in these media, it is argued that a number of the flows are affected only in the buffer layer, and drag reduction can result if behavior in the sublayer and buffer layer differ. In polymer solutions, it is argued that molecular expansion is responsible for the difference, and experimental evidence of this expansion is presented, and compared with calculations.

## INTRODUCTION

Skin friction drag in turbulent flow can be reduced below that of the clear fluid (or solvent) alone by the addition of several substances.<sup>1</sup> For example, in a horizontal boundary layer above a surface, particles for which inertia may be neglected, but fallout may not, can reduce the drag in certain flow regimes. In a different situation, particles for which fallout may be neglected, but inertia may not, can also reduce the drag, but by a different mechanism. Particles for which both may be neglected can also reduce the drag if they are large enough. Polymer additives can make very substantial reductions in the drag. A detailed examination of the mechanisms involved<sup>1</sup> (in some cases highly speculative) suggests that while the heavy particle case may be explained by the effect of a stable density stratification, the other cases appear to have a basic mechanism in common: the addition of the substance affects only the dissipative scales of the turbulence, suppressing the dissipative eddies, and increasing the scale of dissipation. At the same time, the effective viscosity in steady shear is only slightly affected so that the sublayer scaling remains unchanged. In a turbulent boundary layer it is argued that such a change in dissipative scale cannot affect the momentum transport throughout most of the flow, which will, hence, have a logarithmic region with the classical slope. In the buffer zone, however, where the scales of the dissipative and energy containing eddies are roughly the same, the energy containing eddies will be suppressed, resulting in reduced momentum transport, an effectively thickened sublayer and reduced drag. The idea that a change in dissipative structure does not affect the energy containing eddies away from the wall is supported by measurements in media as disparate as slurries and polymer solutions. In polymer solutions, in particular, there is evidence that the structure of the sublayer remains essentially unchanged except in scale, leaving the mechanism of turbulence production unchanged.

The case of polymer solutions is characterized by the so-called onset phenomenon<sup>2</sup>: below a certain value of the flow parameters there is no apparent effect of the polymer. It is now clearly established that onset is well correlated with polymer terminal relaxation time<sup>3</sup> (see

Lumley<sup>2</sup> for other contributing evidence). Polymers are predicted to expand in a suitable velocity field when a criterion based on the relaxation time is exceeded<sup>4</sup> and this expansion process is felt to be responsible for drag reduction.<sup>5,6</sup> Polymer expansion in turbulent flow has been predicted, and the threshold for expansion correlates well with that for onset.<sup>7</sup> Indirect evidence<sup>8</sup> for the molecular expansion has been found in connection with Pitot tube errors, but no direct measurements have previously been reported. Here, we report direct measurements of molecular size in a two-dimensional pure strain. The molecular expansion measured, although not large due to limitations of the experimental apparatus, is that predicted<sup>7</sup> within experimental error, lending credence to other predictions regarding molecular expansion in turbulent flows and its relation to onset.

## DYNAMICS OF TURBULENT TWO-PHASE AND POLYMER FLOWS

An earlier version of some of the following material, presented from a different viewpoint, has appeared in Ref. 1. We restrict discussion to media which display little non-Newtonian behavior in steady simple shear. We adopt the position that the basic structure of turbulent boundary layers of such media (flows with suspended particles, foams, polymer solutions, etc.), with a few exceptions such as the effect of buoyancy, is similar to that of Newtonian boundary layers, although the different regions may scale differently. That is, regardless of the fine structure of the medium, and hence of the dissipative mechanism, there will be a linear region<sup>9</sup> next to the wall in which the dissipative forces dominate (the sublayer); the central part of the flow will be dominated by inertia; there will be a logarithmic region with the usual slope<sup>9</sup>; and in the buffer layer (between these two latter parts) and extending into the sublayer there will be large eddies essentially similar to those found in Newtonian flows, the form of which is determined primarily by inertial forces and kinematics,<sup>10</sup> and which produce little Reynolds stress, but do produce inflectional profiles leading to secondary instabilities and the production of Reynolds stress. Within this conceptual framework, the only question is the scaling of the various regions, which will differ in different media. Hence, our position is that a detailed, mathematical, theory for turbulent boundary layers in these media is not necessary, since their structure is so similar to that of Newtonian boundary layers, and indeed is not

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possible, since in fact Newtonian boundary layers are not sufficiently well understood to permit the construction of such a theory.

This conceptual framework is intended to give structure to the discussion which follows, and according to Townsend,<sup>11</sup> "... provide targets for criticism..." It is probably an over-simplification, and should be qualified in various ways. Support for this framework is fragmented and is somewhat negative, in the sense that the most that can be said is that the supportive bits and pieces do not seem to be a variance with the framework proposed. It has the advantage of being the simplest framework with that property, i.e., the smallest change from the Newtonian boundary layer capable of explaining the observations, which does not assume things about non-Newtonian boundary layers that are not known about the Newtonian layer.

Let us leave aside the case of heavy particles for which inertia may be neglected; this case is treated extensively in Lumley, where it is shown that the basic mechanism is a pseudo-buoyancy resulting from the drag and fallout. The pseudo-stable stratification suppresses the vertical motion, resulting in a reduction of drag relative to a single phase fluid of the same density. Whether the drag is reduced relative to the clear fluid depends on the relative contributions to the Reynolds stress of the increased density and the reduced vertical motions; for increasing stability (controlled by the ratio of terminal velocity to shear velocity) drag reduction is possible. It is evident that this mechanism is quite different from the mechanism described in the preceding paragraphs.

Let us consider particles for which fallout may be neglected, but inertia may not. This corresponds roughly to a high-speed, relatively low Reynolds number flow.<sup>1</sup> In such a flow, the particles nearly follow the motion, following well at large scales and poorly at small. We may use an essentially Lagrangian estimate for the rate at which work is done by the fluctuating relative velocities induced by the particle inertia. A crude integration<sup>1</sup> gives for the additional energy dissipation per unit total mass (particles included)

$$1.18\epsilon(2 - 1/0.74sa)\mathcal{L}/(1 + \mathcal{L}), \quad 0.74sa > 1, \quad (1)$$

where  $s$  is the (Kolmogoroff) cutoff frequency of the Lagrangian spectrum, which would ordinarily be  $(\epsilon/\nu)^{1/2}$ .  $a$  is the particle time constant, equal to  $V_T/g$  for small particles, and  $\mathcal{L}$  is the loading, or mass ratio of particles. We are considering values of  $\mathcal{L}$  of the order of unity, with particle/fluid density ratios of the order of  $10^3$  and volume concentrations of the order of  $10^{-3}$ , so that particle-particle interactions may be neglected.

It is well known in turbulence that the presence of an additional mechanism for energy dissipation at high frequencies does not imply that the over-all dissipation of energy is increased. It is an experimentally observed property of turbulence that so long as the dissipation is confined to the small scales, the large scales are dominated by inertia. Hence, if the efficiency of the dissipative mechanism is increased (still being confined to

small scales), the scale at which the dissipation occurs will be increased so as to keep the total amount dissipated the same.

In expression (1), the  $\epsilon$  is the rate at which energy is being cascaded down the spectrum, which will not be changed by the presence of the particles. The cutoff frequency  $s$  will be reduced by the presence of the particles, and the purely viscous dissipation will be given by  $\nu s^2$ , where  $\nu$  is the value of the kinematic viscosity with particles. Note that the *dynamic* viscosity is essentially unchanged by the presence of the particles, the ratio being  $1 + 5C/2$ ,<sup>12</sup> where  $C$  is the volume concentration; with  $C = 10^{-3}$ , this can safely be ignored.

Since the viscous dissipation plus the dissipation associated with the particles must dissipate all of  $\epsilon$ , we may write

$$\epsilon = \nu s^2 + 1.18\epsilon(2 - 1/0.74sa)\mathcal{L}/(1 + \mathcal{L}) \quad (2)$$

which provides an equation to determine  $s$ .

We may simplify expression (2) by defining  $\hat{s} = s(\nu_f/\epsilon)^{1/2}$  where  $\nu_f$  is the kinematic viscosity of particle-free fluid; thus,  $\hat{s}$  is the ratio of the cut-off frequency with and without particles. Also, it is convenient to define a parameter  $\gamma = 0.74a(\epsilon/\nu_f)^{1/2}$ , which is the product of the particle time scale and the cut-off frequency in the particle-free fluid (with a numerical factor included for convenience). Then, (2) becomes

$$\mathcal{L} = (1 - \hat{s}^2)/(1.36 - 1.18/\gamma\hat{s}). \quad (3)$$

The cut-off viscous wave number is more relevant to turbulence dynamics. We may form a nondimensional cutoff wavenumber by normalizing by its value when  $\mathcal{L} = 0$ , which is proportional to  $(\epsilon/\nu_f^3)^{1/4}$ , and designate it by  $\hat{k}_d$ . Then, we have

$$\hat{k}_d = [\hat{s}(1 + \mathcal{L})]^{1/2}. \quad (4)$$

We have two conflicting phenomena here: the increased density due to the presence of the particles reduces the effective  $\nu$  and tends to increase  $k_d$ , while the increased dissipation tends to reduce it. From (3) and (4) we find that there is a relatively narrow range of values of  $\mathcal{L}$  in which, for sufficiently large values of  $\gamma$  modest reductions in  $k_d$  occur (see Fig. 1).<sup>1</sup>

In the wall region of a particle-free turbulent shear flow, the occurrence of turbulence may be plotted as in Fig. 2.<sup>2</sup> The abscissa is wavenumber, scaled with the friction velocity and the kinematic viscosity of the fluid with particles, while the ordinate is distance from the wall scaled in the same way. The turbulence is con-

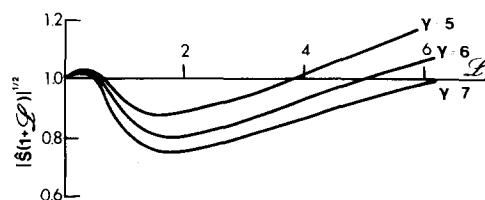


FIG. 1. Variation of dimensionless cutoff wavenumbers with loading, for various values of the particle/fluid time scale ratio.<sup>1</sup>

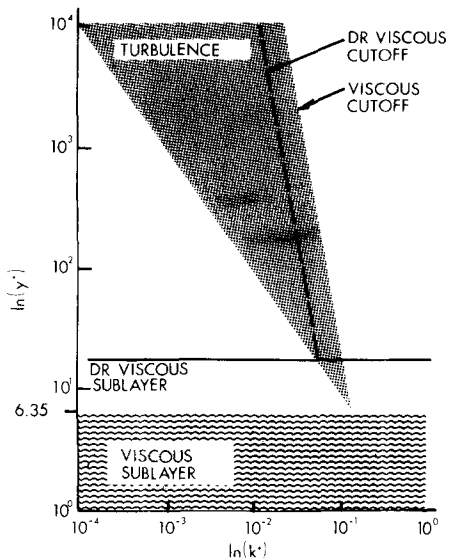


FIG. 2. Scaling relations in the viscous and inertial sublayers with and without polymers.<sup>2</sup> DR refers to drag reduction; the DR viscous cutoff and sublayer indicated correspond to an increase in viscosity in the turbulent part of the fluid, without a corresponding increase in the viscous sublayer.<sup>1</sup>

tained in the shaded region. That is, the peak of the energy containing eddies lies on the left-hand edge, while the peak of the dissipative region lies on the right-hand edge. As the wall is approached, the two scales approach each other, the turbulence Reynolds number dropping progressively, and in the viscous sublayer the two scales are of the same order.

It may be objected that a plot such as Fig. 2 ignores all the detailed information which we have about the form of the anisotropic eddies in the vicinity of the wall. Certainly, there are the large eddies to which we have already referred, which do not scale according to Fig. 2. In this figure we are referring, however, to the small scale eddies that are produced by the secondary instability of the inflectional profile produced by the big eddies, the eddies responsible for the Reynolds stress; these do scale according to Fig. 2, and they are the ones responsible for profile changes. According to Lumley,<sup>10</sup> the big eddies are essentially parasitic, in the sense that their size and intensity is determined by the mean velocity profile, while they carry little Reynolds stress.

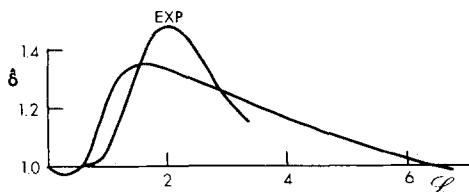


FIG. 3. Variation of dimensionless sublayer thickness with loading, for a rather large value of the particle/fluid time scale ratio.<sup>1</sup> The curve labeled EXP is from Kane *et al.*<sup>13</sup> for 36  $\mu$  glass beads, and corresponds to a value of the time scale ratio considerably beyond the range for which the present analysis is valid.

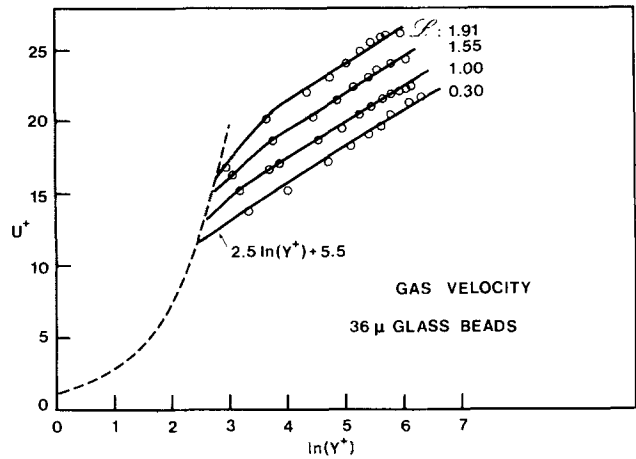


FIG. 4. Mean gas velocity profile in flows with the indicated loading of 36  $\mu$  glass beads from Kolansky *et al.*<sup>14</sup>

They are also essentially inertial, losing energy primarily at the bottom of the viscous sublayer.

According to Fig. 1, the presence of particles can, in the right parameter range, reduce the value of  $k_d$ . Away from the wall a reduction of  $k_d$  will have no effect on the energy containing eddies, and hence will leave the momentum transport and the slope of the mean velocity profile unchanged. Just outside the viscous sublayer, however, this reduction in  $k_d$  will cause the apex of the shaded region to move farther from the wall, killing the energy-containing eddies there, and effectively thickening the viscous sublayer. It is not hard to show that a thickening of the viscous sublayer at constant friction velocity corresponds to a reduction in drag, since the mean velocity at the sublayer edge increases. For small changes, the percentage change in wall shear stress is about the same as the percentage change in sublayer thickness. Of course, if the sublayer is thicker than for a clear fluid of the same density, drag has not necessarily been reduced; this will happen only if the sublayer is thicker than for the fluid without particles.

Hence, we will define  $\delta$  as the ratio of the sublayer thickness to that in the clear fluid. We will take as the definition of sublayer thickness the point of intersection of the right- and left-hand edges in Fig. 2. We obtain

$$\delta = [\hat{s}(1 + \mathcal{L})]^{-2/3} . \quad (5)$$

This is plotted in Fig. 3<sup>1</sup> for a value of  $\gamma = 7$ ,  $\gamma$  being evaluated at a distance from the wall corresponding to the sublayer thickness (as defined herein). For a qualitative comparison, we have included a measured reduction in drag due to particles.<sup>13</sup> The value of the parameter  $\gamma$  is too large (roughly 50) for our analysis to be applicable, but the general shape and range of  $\mathcal{L}$  is qualitatively similar.

There are very few detailed measurements of profiles in particle-laden flows, and none to our knowledge of turbulent structure; certainly there are none in our parameter range. In Fig. 4 we present measurements of Kolansky *et al.*,<sup>14</sup> which clearly show the unchanged logarithmic region and drag reduction relative to a clear

fluid of the same density, despite the too-large value of  $\gamma$ . In corroboration we can also offer profile measurements in pipe flows of slurries, which are observed to have a logarithmic region with the classical slope.<sup>15</sup>

We may extend these ideas to neutrally buoyant particles, which neither fall out nor dissipate extra energy through inertia; however, flows of these particles display drag reduction.<sup>16</sup> Based on our reasoning here, if the particles are in every dimension small relative to the Kolmogoroff microscale everywhere in the flow we would expect no influence, and experiment bears this out. Anomalous effects are observed only in flows in which at least one dimension of the particle exceeds the Kolmogoroff length scale near the wall. For particles smaller than the Kolmogoroff microscale, only the effect on viscosity will be important, and this will be the same both in the turbulent part of the flow and in the viscous sublayer. Hence, a diagram such as Fig. 2 will remain unchanged, and no drag reduction will occur. Some drag reduction could occur if the particles migrated out of the wall layer,<sup>17</sup> since then there would be an increase in viscosity in the turbulent fluid and not in the sublayer; however, this would require a relative Reynolds number of order unity or greater, which cannot occur if the particles are smaller than the Kolmogoroff microscale (the relative Reynolds number based on velocity differences across the particle is  $(l/\eta)^2$ , where  $l$  is the particle dimension). For particles larger than the Kolmogoroff microscale, two effects are possible: migration out of the wall layer, causing a lower viscosity there, and suppression of the small eddies that are smaller than the particles. That is, a vertical line in Fig. 2 will represent particles of a constant size; the energy containing eddies will not be affected until the vertical line crosses the left-hand boundary of the shaded part. Below this point, the energy containing eddies will be suppressed, with a reduction in Reynolds stress, causing a thickening of the sublayer and reduction of drag. The suppression of the small eddies is in addition to the increase in viscosity; otherwise, the diagram would not be affected. If the effective viscosity in the sublayer remains Newtonian, the profile slope in the logarithmic region will not change; data indicate, however, that the slope usually increases. As Tennekes<sup>9</sup> has explained, a change in slope in the log region indicates a change in the viscous stress-strain rate relation in the sublayer; most likely migration is responsible for this. It is also possible that particle interaction or particle flexibility is responsible; recent measurements in blood<sup>18</sup> indicate that flexibility is the most important contributor to non-Newtonian behavior of particulate fluids in laminar flow.

Again, we do not have measurements of detailed turbulence structure, but the observed gross behavior is at least consistent with our simple model.

In polymer drag reduction we have more information than in any of the particulate flows. The mechanism that has been proposed is the expansion of the polymers in relatively rotation-free straining flows. In the next section we shall present the evidence that exists for this ex-

pansion. Here, we will summarize briefly what is presumed to happen.

Molecules exist in solution as tangled balls, mostly filled with solvent. Thermal agitation is responsible for keeping them in this partially contracted shape, providing an effective spring constant for return to their equilibrium diameter. Under the action of a pure strain, the molecules are pulled out into an ellipsoidal shape; when the strain rate exceeds the restoring force, the molecule expands as far as it can, until it is essentially fully extended. This picture is qualitative, being complicated by the fact that the effective restoring force changes as the molecule is expanded, so that the expansion does not happen all at once. If the molecule is in a shear, so that it is rotating as well as being subjected to the straining, a greater strain rate is required to produce the same effect, since the molecule remains aligned with the principal axes of strain rate for a shorter time. If the rotation rate is too great, in fact, the expansion produced is quite limited no matter how great the strain rate. In Fig. 5<sup>7</sup> we show the behavior of an isolated molecule in various combinations of vorticity and strain rate. In particular, in a shear, vorticity is equal to strain rate, and the expansion obtainable is quite limited.

The whole picture is complicated by the fact that there is evidence<sup>19</sup> that interaction of the molecules is an important part of the drag reduction process, and by the fact that any sample of polymers contains a spectrum of molecular weights, and hence of spring constants and relaxation times. Nevertheless, it is felt that the preceding paragraph provides a crude qualitative picture of behavior of polymer molecules in drag reducing flows. In the turbulent part of the flow, at high Reynolds number, vorticity and strain rate are uncorrelated with each other, and can occur in virtually any combination; they both have distributions with long tails, so that large and small values of each are more probable than for a Gaussian distribution. Hence, there is a substantial probability of finding regions of relatively large strain rate and relatively small vorticity. Under these circumstances, the molecules can expand, if the level of strain rate is sufficiently large. This discussion is oriented primarily toward large Reynolds numbers. If we consider turbulence at low Reynolds numbers, such as that

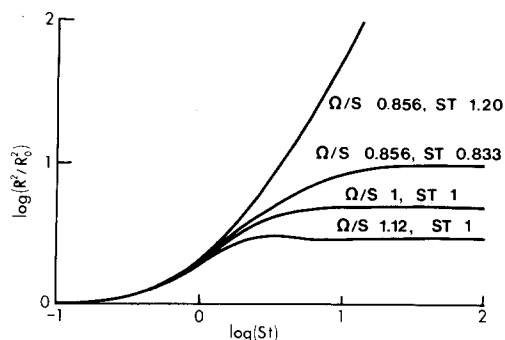


FIG. 5. Mean square molecular radius as a function of time in a two-dimensional flow, for various values of vorticity and strain rate (relative to the relaxation time  $T$ ) from Lumley.<sup>2</sup>

near a wall, there are also regions of relatively large strain rate and low vorticity; Abernathy<sup>20</sup> has shown that in the field of flow of the large counter-rotating eddy pairs in the sub- and buffer layers such regions exist.

On the other hand, in the viscous sublayer, the vorticity is equal to the strain rate, and the molecules cannot expand significantly. The diffusion coefficient for molecules of this size is so large, that by the time expanded molecules from the turbulent part of the flow have diffused down into the sublayer, they will have collapsed.

When the molecules expand, the viscosity of the solution increases dramatically. Essentially, the increased dissipation due to the presence of the molecules is dependent on the velocity differences occurring over a molecule. The more the molecule expands, the greater the velocity difference. Since the mean square diameter of the molecule can expand by a factor of the number of submolecular units, and this can be of the order of  $10^5$ , the increase in viscosity can be correspondingly great. Hence, we can have a substantial increase in viscosity in the turbulent part of the flow, and virtually no increase in viscosity in the viscous sublayer.

Again, this increase in viscosity in the turbulent part, and not in the viscous sublayer, suppresses the eddies which carry the Reynolds stress in the buffer layer, resulting in a thickening of the sublayer, and a reduction of drag. Since the majority of the turbulent part of the flow is dominated by inertia, we expect the slope of the mean velocity profile to remain unchanged, so long as the stress-strain rate relation in the viscous sublayer remains Newtonian. Since the drag reduction phenomenon occurs at concentrations so low that, if the molecules are not expanded, the total change in viscosity is of the order of 1%, the sublayer will remain Newtonian, and the slope of the mean velocity profile will be classical. This is exactly what is observed.<sup>5</sup>

In these polymer flows the form of the large eddy structure near the wall has been inferred from measurements of fluctuating wall shear stress.<sup>21</sup> It is found that the large eddies are essentially the same as in a Newtonian flow, but that the scale of the large eddies has expanded to fit the expanded sublayer. This is consistent with the predictions of Lumley<sup>10</sup> that the large eddy structure is parasitic, and determined primarily by the mean velocity profile; it is also consistent with the statement there that the large eddies lose their energy primarily at the bottom of the sublayer, where the molecules are contracted, and the viscosity is essentially unchanged from the Newtonian value. In fact, the measurements of fluctuating velocity by Rudd<sup>22</sup> are also consistent with this picture. In the buffer layer, he finds the streamwise component increasing with the mean velocity (at fixed shear velocity), while the other two components are reduced. Our model would predict that the turbulence intensities should be reduced in the buffer layer, but we have specifically restricted this to the small scale eddies that carry the Reynolds stress there. In fact, a major contributor to the streamwise fluctuating velocity are the large scale fluctuations in the mean velocity profile<sup>23</sup> induced by the big eddies. If the big

eddies extend farther from the wall, the net velocity difference over their height is greater, and the fluctuations in streamwise velocity induced will be greater. Hence, we expect the peak value of the streamwise fluctuating velocity to scale up as the sublayer thickness increases, as observed by Rudd.

The model predicts several other qualitative phenomena. For example, at low concentrations, the molecules will essentially expand fully, so that the net relative change in viscosity will be proportional to the concentration. Using our same definition for change in sublayer thickness with change in viscosity, we find drag reduction proportional to concentration for small concentrations.<sup>2</sup> Unfortunately, although early data appeared to show this, recent unpublished measurements of Berman indicate a progressive drop in drag reduction per unit concentration as concentration decreases for low concentrations, at constant flow time scale. This suggests that interaction is important and that effective intrinsic viscosity vanishes at zero concentration, implying that interaction with other molecules is necessary to expand a molecule. For larger concentrations, we have a feedback mechanism: as the molecules expand, the increased viscosity causes a reduction in the strain rate, since the dissipation must remain constant; the reduction in the strain rate stops the expansion. The equilibrium state is one of partially expanded molecules, producing an increase in viscosity just sufficient to hold them at that expansion.<sup>2</sup> The result is a saturation of drag reduction with concentration. If more polymer is added, the average expansion is reduced, to hold the effect about the same. The saturation for large concentrations is observed.<sup>24</sup>

## EVIDENCE FOR MOLECULAR EXPANSION

Our explanation for the behavior of polymer solutions in turbulent flow is based on the idea that polymer molecules expand under the right circumstances. Evidence for such expansion has been either calculations based on various models<sup>4</sup> or indirect evidence from other phenomena. For example, the calibration curve for a hot film set at an angle to the free stream,<sup>25</sup> Fig. 6, can be explained on the basis of molecular expansion in irrotational flow.<sup>2</sup> When the film is normal to the free stream, the molecules are expanded in the irrotational flow near the stagnation region, causing an increase in viscosity and boundary layer thickness and a decrease in heat transfer. As the film is placed at an increasing angle relative to the free stream, the flow along the stagnation line becomes a shear flow, in which vorticity is equal to strain rate, the molecules tumble, and contract, resulting in a reduction in viscosity and boundary layer thickness, and an increase in heat transfer. The behavior of Pitot tubes<sup>8</sup> provides similar indirect evidence. Attempts have previously been made to measure molecular size directly, in circular Couette flow<sup>26</sup>; since the flow is a shear, only relatively small molecular expansion would be expected, and the experimental measurements were in agreement with this expectation.

Recently, Dennison<sup>27</sup> has completed a series of measurements of molecular size of poly(ethylene oxide)

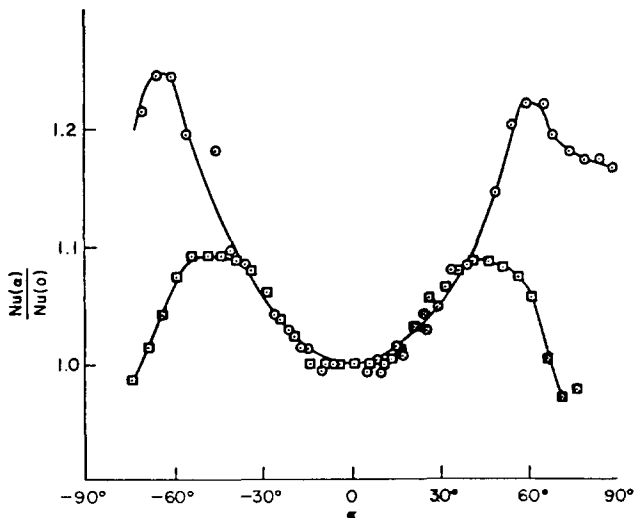


FIG. 6. Plot of angular calibration of a hot film probe at two mean speeds.  $\circ$ , 300 ppm J-100, 25 cm/sec;  $\square$ , 300 ppm J-100, 57 cm/sec.<sup>25</sup>

(Union Carbide Polyox WSR-301) by light scattering in a two-dimensional, irrotational contraction, using a modified recording goniophotometer supplied by the Shell Pipeline Division, Houston. The contraction is shown in Fig. 7,<sup>28</sup> where all dimensions are in inches. The flow region was 0.2 in. thick, the top and bottom surfaces being glass. The cell of the goniophotometer was replaced with this contraction, the beam of the light source (a He-Ne laser) passing through the contraction normal to the plane of the figure, at several points along the centerline. The upstream end of the contraction was fed from a 2.0 in. circular pipe in which the flow was laminar, the two being joined by a round-to-rectangular smooth transition.

The flow in the contraction was extensively analyzed by finite difference.<sup>28</sup> Figure 8 is a plot of strain rate along the centerline, (all values nondimensionalized by entrance velocity and entrance half-width). The analysis of the flow field for the solvent alone and for the

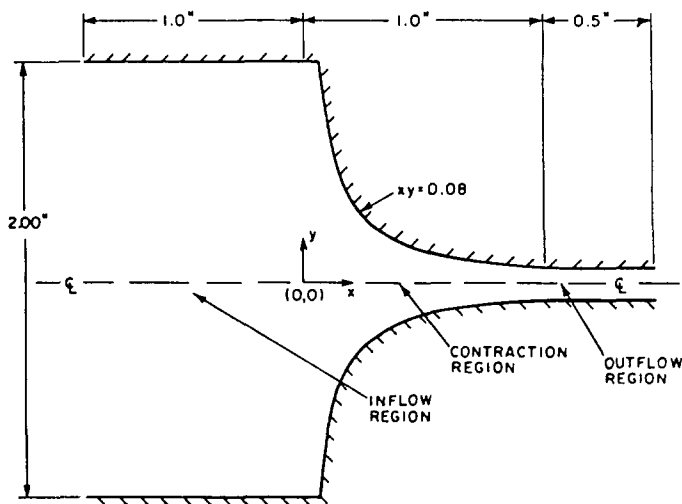


FIG. 7. Two-dimensional contraction.<sup>28</sup>

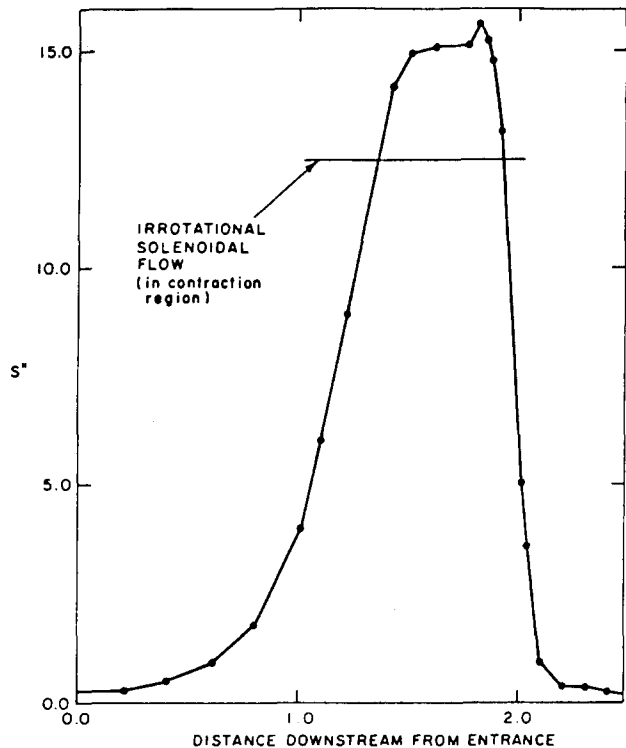


FIG. 8. Distribution of strain rate along centerline of two-dimensional contraction.<sup>28</sup>

polymer solution indicated that there was no appreciable difference. The increase of the strain rate in the contraction above the value predicted for irrotational flows is due to viscous boundary layers, which effectively reduce the dimensions of the contraction. The viscosity is computational, in the calculations of Gatski.<sup>28</sup> In the real experiment, the Reynolds number at the exit (based on width) is about 25 600, so that the flow is almost certainly turbulent. A crude estimate of turbulent boundary layer thickness, based on boundary layers in the stagnation region<sup>29</sup>; that is, proportional to  $(\nu_f/S)^{1/2}$ , where the turbulent viscosity is based on the local boundary layer thickness and free-stream velocity, and the strain rate  $S$  is that of the contraction) indicates a turbulent boundary layer of approximately the same thickness as the computational laminar boundary layer, so that Fig. 8 may be taken to be representative of the real flow. Because of its small size, of course, the real nozzle suffers from geometrical inaccuracies, which almost surely cause differences in the flow field.

The difficulty in the design of such a flow is to retain the polymer molecule in the flow field for a time great enough for it to be significantly extended. In the entrance region, the flow does not begin to follow the contraction for a distance which is probably a nearly constant fraction of the entrance width. Hence, widening the entrance region does not significantly increase the time spent by the polymer molecule in the high strain rate region. Lengthening the down stream end does not add much, first because of the difficulty of accurately machining the increasingly parallel section, and second because of the great increase in length necessary for a moderate increase in time, since the flow is traveling

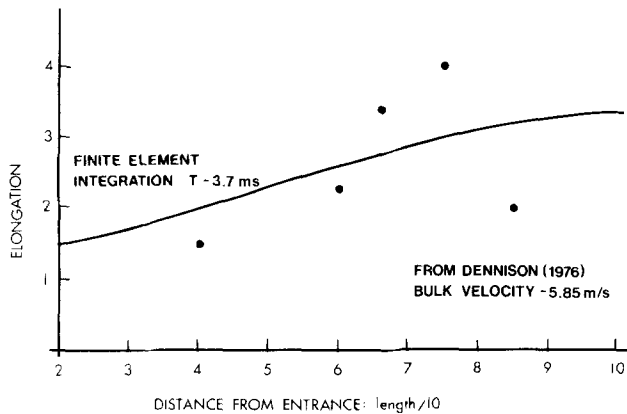


FIG. 9. Measurements of molecular expansion along centerline of two-dimensional contraction (curve is calculated from Oldroyd equation).<sup>27</sup>

faster and faster. Hence, the nozzle depicted is difficult to improve on, and the time spent in the high strain rate region is quite short.

Having the value of strain rate along the center line of the contraction, it is possible to use the Oldroyd equation, which is known to be a first approximation to the constitutive relation for a polymer solution,<sup>30,31</sup> to calculate molecular size.<sup>28</sup> Close to the critical value of the dimensionless strain rate, the finite difference technique suffers severe instabilities; we have obtained solutions in this range by finite element techniques, using eight unequal regions of stepwise constant strain rate through the contraction. In Fig. 9 we show the measured values of streamwise molecular size obtained by Dennison, together with the calculated values of molecular size from the Oldroyd equation. The terminal relaxation time has been selected to give the best fit to the data. This relaxation time is of the order of the largest times measured for this polymer by Berman *et al.*<sup>8</sup> using the observed onset of drag reduction to infer a time. It is also near the upper edge of the postulated relaxation time distribution suggested by these authors for this molecule (see Fig. 10).

The molecular expansions were measured at a concentration of 1600 ppm. Although this is substantially higher than the concentrations which are of interest in drag reduction, it was necessary to go to this concentration because of the small scattering cross section of the polymer molecule. The total light scattered by the molecules at lower concentrations was not reliably perceptible over instrumental noise. Although such a high concentration would certainly affect the behavior of this polymer in a drag reducing flow, it should have relatively little influence on its behavior in an irrotational contraction.

Although the total increase in size displayed by the molecules in this flow is necessarily limited due to the short time spent in the high strain rate region, the fact that the expansions measured agree so well with those predicted by the Oldroyd model suggests that our understanding of the behavior of such molecules in irrotational flows is substantially correct. The values observed are in good agreement with those reported by

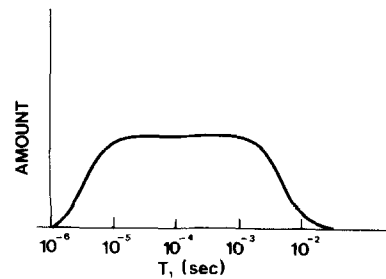


FIG. 10. Hypothetical terminal relaxation time distribution for fresh Polyox WSR-301.<sup>3</sup>

Smith *et al.*<sup>32</sup> In a turbulent flow, there would be no such limitation on the expansion of the molecules as there is here, because a finite fraction of the molecules would presumably remain in a strain rate field which was substantially rotation free for a sufficient time to expand, if the flow parameters exceed the threshold values.

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