PHYSICS 200A : CLASSICAL MECHANICS
PROBLEM SET #2

[**1**] A point mass m slides frictionlessly, under the influence of gravity, along a massive ring of radius a and mass M. The ring is affixed by horizontal springs to two fixed vertical surfaces, as depicted in Fig. 1. All motion is within the plane of the figure.

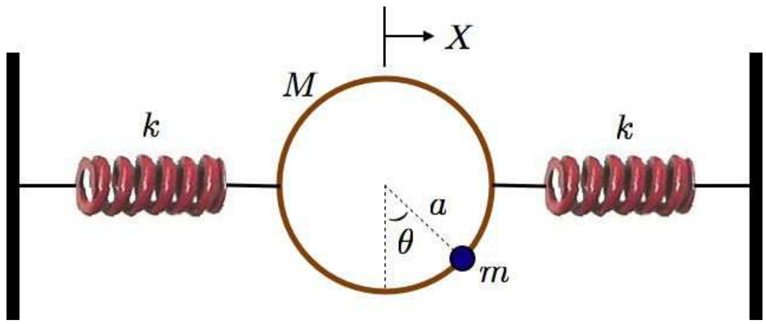

Figure 1: A point mass m slides frictionlessly along a massive ring of radius a and mass M, which is affixed by horizontal springs to two fixed vertical surfaces.

(a) Choose as generalized coordinates the horizontal displacement X of the center of the ring with respect to equilibrium, and the angle θ a radius to the mass m makes with respect to the vertical (see Fig. 1). You may assume that at $X = 0$ the springs are both unstretched. Find the Lagrangian $L(X, \theta, \dot{X}, \dot{\theta}, t)$.

(b) Find the generalized momenta p_X and p_θ, and the generalized forces F_X and F_θ.

(c) Derive the equations of motion.

(d) Find expressions for all conserved quantities.

[**2**] Two blocks connected by a spring of spring constant k are free to slide frictionlessly along a horizontal surface, as shown in Fig. 2. The unstretched length of the spring is a.

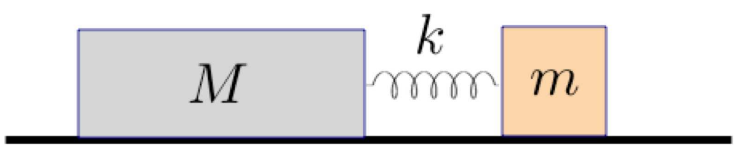

Figure 2: Two masses connected by a spring sliding horizontally along a frictionless surface.

(a) Identify a set of generalized coordinates and write the Lagrangian.

(b) Find the equations of motion.

(c) Find all conserved quantities.

(d) Find a complete solution to the equations of motion. As there are two degrees of freedom, your solution should involve 4 constants of integration. You need not match initial conditions, and you need not choose the quantities in part (c) to be among the constants.

[3] Consider a three-dimensional one-particle system whose potential energy in cylindrical polar coordinates $\{\rho, \phi, z\}$ is of the form $V(\rho, k\phi + z)$, where k is a constant.
[José and Saletan problem 3.11]

(a) Find a symmetry of the Lagrangian and use Noether's theorem to obtain the constant of the motion associated with it.

(b) Write down at least one other constant of the motion.

(c) Obtain an explicit expression for the dynamical vector field $\Delta \equiv \frac{d}{dt}$ and use it to verify that the functions found in (a) and (b) are indeed constants of the motion.

[4] A bead of mass m, subject to gravity, slides frictionlessly along a wire curve $z = x^2/2b$, where $b > 0$. The wire rotates with angular frequency ω about the $\hat{\boldsymbol{z}}$ axis.

(a) Find the Lagrangian of this system.

(b) Find the Hamiltonian.

(c) Find the effective potential $U_{\text{eff}}(x)$.

(d) Show that the motion is unbounded for $\omega^2 > \omega_{\text{c}}^2$ and find the critical value ω_{c}.

(e) Sketch the phase curves for this system for the cases $\omega^2 < \omega_{\text{c}}^2$ and $\omega^2 > \omega_{\text{c}}^2$.

(f) Find an expression for the period of the motion when $\omega^2 < \omega_{\text{c}}^2$.

(g) Find the force of constraint which keeps the bead on the wire.