UC San Diego
PHYSICAL SCIENCES
Physics

Problem Set 2

Due TBD (30 pts.)

1) Calculate the thermal conductivity χ for the 'usual' case of a hard sphere gas. Do the calculation accurately — do **not** use a Krook collision operator.

You may find it helpful to consult "Physical Kinetics", Sections 7 and 10.

Fluids: The problems below (#2-5) are intended to give you some short introduction to fluid mechanics. A basic text may be helpful. "Fluid Mechanics" by Landau and Lifshitz is a good one, but use whatever works for you.

2) Show that circulation is conserved for an ideal fluid, where $\mathrm{P} = \mathrm{P}(\rho)$. What does this imply for vorticity $\underline{\omega} = \underline{\nabla} \times \underline{v}$? Derive an equation for $\underline{\omega}$. How does $\underline{\omega}$ evolve for $\mathrm{P} = \mathrm{P}(\rho, T)$? What happens to circulation in a viscous fluid?

3) Let $\mathrm{P} = \mathrm{P}(\rho, T)$. Derive the sound wave dispersion relation and sound speed for an ideal fluid. Now allow for viscosity and thermal diffusion. Calculate the dispersion relation. Calculate the evanescence length for a wave with wave number k.

4) Give an expression for the pressure in an ideal incompressible fluid.

5) An incompressible fluid rotates with $\underline{\Omega} = \Omega(r)\hat{z}$. Calculate the wave dispersion relation. For what profile $\Omega(r)$ does instability result? Can you explain this using simple physics? Hint: Lord Rayleigh did. The argument is named for him.

6) Consider a simple system with kinetic equation

$$\frac{\partial f}{\partial t} + v\frac{\partial f}{\partial x} + \frac{\mathrm{q}}{m}E_{ext}(x,t)\frac{\partial f}{\partial v} = c(f)$$

Take f_o Maxwellian, formed by a very slow collisional process. $c(f)$ is negligible on dynamical time scales. $E_{ext}(x,t)$ varies slowly in time and space.

i) Compute the **linear** response δf to E_{ext}. From this, derive an expression for the conductivity.

ii) Use δf to derive a mean field evolution equation for f_o, on $t < \tau_{coll}$.

iii) What physics determines the evolution of f_o?

7) Read Onsager's paper on Reciprocity, posted. Write a one-page (typed) abstract of the paper, in **your own words**. Address:

— physics motivation
— key results
— exceptions, complications

Try to explain what might have led Onsager to find this famous result.