2-2   (a)   Scalar equations can be considered in this case because relativistic and classical velocities are in the same direction.

$$p = \gamma \, mv = 1.90mv = \frac{mv}{\left[1 - \left(v/c\right)^2\right]^{1/2}} \Rightarrow \frac{1}{\left[1 - \left(v/c\right)^2\right]^{1/2}} = 1.90 \Rightarrow v = \left[1 - \left(\frac{1}{1.90}\right)^2\right]^{1/2} c$$

$$= 0.85c$$

(b)   No change, because the masses cancel each other.

2-9   (a)   When $K = (\gamma - 1)mc^2 = 5mc^2$, $\gamma = 6$ and $E = \gamma \, mc^2 = 6(0.511\ 0\ \text{MeV}) = 3.07\ \text{MeV}$.

(b) $\dfrac{1}{\gamma} = \left[1 - \left(\dfrac{v}{c}\right)^2\right]^{1/2}$ and $v = c\left[1 - \left(\dfrac{1}{\gamma}\right)^2\right]^{1/2} = c\left[1 - \left(\dfrac{1}{6}\right)^2\right]^{1/2} = 0.986c$

2-12   (a)   When $K_e = K_p$, $m_e c^2\left(\gamma_e - 1\right) = m_p c^2\left(\gamma_p - 1\right)$. In this case $m_e c^2 = 0.511\ 0\ \text{MeV}$ and

$m_p c^2 = 938\ \text{MeV}$, $\gamma_e = \left[1 - (0.75)^2\right]^{1/2} = 1.511\ 9$. Substituting these values into the

first equation, we find $\gamma_p = 1 + \dfrac{m_e c^2 \left(\gamma - 1\right)}{m_p c^2} = 1 + \dfrac{(0.511\ 0)(1.511\ 9 - 1)}{939} = 1.000\ 279$.

But $\gamma_p = \dfrac{1}{\left[1 - \left(u_p/c\right)^2\right]^{1/2}}$; therefore $u_p = c\left(1 - \gamma_p^{-2}\right)^{1/2} = 0.023\ 6c$.

(b)   When $p_e = p_p$, $\gamma_p m_p u_p = \gamma_e m_e u_e$ or $u_p = \left(\dfrac{\gamma_e}{\gamma_p}\right)\left(\dfrac{m_e}{m_p}\right)u_e$,

$$u_p = \left(\frac{1.511\ 9}{1.000\ 279}\right)\left[\frac{0.511\ 0/c^2}{939/c^2}\right](0.75c) = 6.17 \times 10^{-4}c.$$

2-13   (a) $E = 400mc^2 = \gamma \, mc^2$

$\gamma = \left(1 - \dfrac{v^2}{c^2}\right)^{-1/2} = 400$

$\left(1 - \dfrac{v^2}{c^2}\right) = \left(\dfrac{1}{400}\right)^2$

$\dfrac{v}{c} = \left[1 - \dfrac{1}{400^2}\right]^{1/2}$

$v = 0.999\ 997c$

(b) $K = E - mc^2 = (400 - 1)mc^2 = 399\,mc^2 = (399)(938.3\ \text{MeV}) = 3.744 \times 10^5\ \text{MeV}$

2-15   (a) $K = \gamma \, mc^2 - mc^2 = Vq$ and so, $\gamma^2 = \left(1 + \dfrac{Vq}{mc^2}\right)^2$ and $\dfrac{v}{c} = \left\{1 - \left(1 + \dfrac{Vq}{mc^2}\right)^{-2}\right\}^{1/2}$

$$\frac{v}{c} = \left\{ 1 - \frac{1}{1 + \left(5.0 \times 10^4 \ \text{eV}/0.511 \ \text{MeV}\right)^2} \right\}^{1/2} = 0.4127$$

or $v = 0.413c$.

(b) $K = \frac{1}{2}mv^2 = Vq$

$$v = \left(\frac{2Vq}{m}\right)^{1/2} = \left\{\frac{2\left(5.0 \times 10^4 \ \text{eV}\right)}{0.511 \ \text{MeV}/c^2}\right\}^{1/2} = 0.442\,c$$

(c)   The error in using the classical expression is approximately $\frac{3}{40} \times 100\%$ or about 7.5%

2-19 $\Delta m = 6m_p + 6m_n - m_C = \left[6(1.007\ 276) + 6(1.008\ 665) - 12\right] \text{u} = 0.095\ 646 \ \text{u}$,
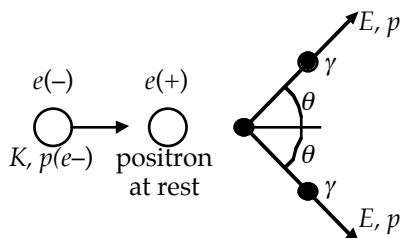
$$\Delta E = (931.49 \ \text{MeV/u})(0.095\ 646 \ \text{u}) = 89.09 \ \text{MeV}.$$

Therefore the energy per nucleon $= \frac{89.09 \ \text{MeV}}{12} = 7.42 \ \text{MeV}$.

2-20 $\Delta m = m - m_p - m_e = 1.008\ 665 \ \text{u} - 1.007\ 276 \ \text{u} - 0.000\ 548\ 5 \ \text{u} = 8.404 \times 10^{-4} \ \text{u}$
$E = c^2\left(8.404 \times 10^{-4} \ \text{u}\right) = \left(8.404 \times 10^{-4} \ \text{u}\right)(931.5 \ \text{MeV/u}) = 0.783 \ \text{MeV}$.

2-21

Conservation of mass-energy requires $K + 2mc^2 = 2E$ where K is the electron's kinetic energy, m is the electron's mass, and E is the gamma ray's energy.

$$E = \frac{K}{2} + mc^2 = (0.500 + 0.511) \ \text{MeV} = 1.011 \ \text{MeV}.$$

Conservation of momentum requires that $p_{e^-} = 2p\cos\theta$ where p_{e^-} is the initial momentum of the electron and p is the gamma ray's momentum, $\frac{E}{c} = 1.011 \ \text{MeV}/c$. Using $E_{e^-}^2 = p_{e^-}^2 c^2 + \left(mc^2\right)^2$ where E_{e^-} is the electron's total energy, $E_{e^-} = K + mc^2$, yields

$$p_{e^-} = \frac{1}{c}\sqrt{K^2 + 2Kmc^2} = \frac{\sqrt{(1.00)^2 + 2(1.00)(0.511)} \text{ MeV}}{c} = 1.422 \text{ MeV}/c.$$

Finally, $\cos\theta = \dfrac{p_{e^-}}{2p} = 0.703$; $\theta = 45.3°$.

2-23   In this problem, M is the mass of the initial particle, m_l is the mass of the lighter fragment, v_l is the speed of the lighter fragment, m_h is the mass of the heavier fragment, and v_h is the speed of the heavier fragment. Conservation of mass-energy leads to

$$Mc^2 = \frac{m_l c^2}{\sqrt{1 - v_l^2/c^2}} + \frac{m_h c^2}{\sqrt{1 - v_h^2/c^2}}$$

From the conservation of momentum one obtains

$$(m_l)(0.987\,c)(6.22) = (m_h)(0.868\,c)(2.01)$$

$$m_l = \frac{(m_h)(0.868\,c)(2.01)}{(0.987)(6.22)} = 0.284 m_h$$

Substituting in this value and numerical quantities in the mass-energy conservation equation, one obtains 3.34×10^{-27} kg $= 6.22 m_l + 2.01 m_h$ which in turn gives 3.34×10^{-27} kg $= (6.22)(0.284) m_h + 2.01 m_h$ or $m_h = \dfrac{3.34 \times 10^{-27} \text{ kg}}{3.78} = 8.84 \times 10^{-28}$ kg and $m_l = (0.284) m_h = 2.51 \times 10^{-28}$ kg.

2-26   Energy conservation: $\dfrac{1}{\sqrt{1-0^2}} 1\,400 \text{ kg} c^2 + \dfrac{900 \text{ kg} c^2}{\sqrt{1-0.85^2}} = \dfrac{Mc^2}{\sqrt{1-v^2/c^2}}$;

$3\,108 \text{ kg}\sqrt{1 - \dfrac{v^2}{c^2}} = M$. Momentum conservation: $0 + \dfrac{900 \text{ kg}(0.85c)}{\sqrt{1-0.85^2}} = \dfrac{Mv}{\sqrt{1-v^2/c^2}}$;

$1\,452 \text{ kg}\sqrt{1 - \dfrac{v^2}{c^2}} = \dfrac{Mv}{c}$.

(a)   Dividing gives $\dfrac{v}{c} = \dfrac{1\,452}{3\,108} = 0.467$ $v = 0.467\,c$.

(b)   Now by substitution $3\,108 \text{ kg}\sqrt{1 - 0.467^2} = M = 2.75 \times 10^3$ kg .

2-31   Conservation of momentum $\gamma\,mu$:

$$\frac{mu}{\sqrt{1-u^2/c^2}} + \frac{m(-u)}{3\sqrt{1-u^2/c^2}} = \frac{Mv_f}{\sqrt{1-v_f^2/c^2}} = \frac{2mu}{3\sqrt{1-u^2/c^2}}.$$

Conservation of energy $\gamma\,mc^2$:

$$\frac{mc^2}{\sqrt{1-u^2/c^2}} + \frac{mc^2}{3\sqrt{1-u^2/c^2}} = \frac{Mc^2}{\sqrt{1-v_f^2/c^2}} = \frac{4mc^2}{3\sqrt{1-u^2/c^2}}.$$

To start solving we can divide: $v_f = \frac{2u}{4} = \frac{u}{2}$. Then

$$\frac{M}{\sqrt{1-u^2/4c^2}} = \frac{4m}{3\sqrt{1-u^2/c^2}} = \frac{M}{(1/2)\sqrt{4-u^2/c^2}}$$

$$M = \frac{2m\sqrt{4-u^2/c^2}}{3\sqrt{1-u^2/c^2}}$$

Note that when $v \ll c$, this reduces to $M = \frac{4m}{3}$, in agreement with the classical result.

2-33   The energy that arrives in one year is

$$E = \mathscr{P}\,\Delta t = \left(1.79 \times 10^{17}\ \text{J/s}\right)\!\left(3.16 \times 10^7\ \text{s}\right) = 5.66 \times 10^{24}\ \text{J}.$$

Thus, $m = \dfrac{E}{c^2} = \dfrac{5.66 \times 10^{24}\ \text{J}}{\left(3.00 \times 10^8\ \text{m/s}\right)^2} = 6.28 \times 10^7\ \text{kg}$.

2-8   (a) $E_R = mc^2 = (1.67 \times 10^{-27} \text{ kg})(3 \times 10^8 \text{ m/s})^2 = 1.503 \times 10^{-10} \text{ J} = 939.4 \text{ MeV}$
          (Numerical round off gives a slightly larger value for the proton mass)

    (b) $E = \gamma\, mc^2 = \dfrac{1.503 \times 10^{-10} \text{ J}}{\left(1 - (0.95c/c)^2\right)^{1/2}} = 4.813 \times 10^{-10} \text{ J} \approx 3.01 \times 10^3 \text{ MeV}$

    (c) $K = E - mc^2 = 4.813 \times 10^{-10} \text{ J} - 1.503 \times 10^{-10} \text{ J} = 3.31 \times 10^{-10} \text{ J} = 2.07 \times 10^3 \text{ MeV}$

2-10 $E = \gamma\, mc^2;\ 1.5mc^2 = \gamma\, mc^2;\ \gamma = 1.5 \Rightarrow 1.5 = \dfrac{1}{\left[1 - (v^2/c^2)\right]^{1/2}};\ v = c\left[1 - \left(\dfrac{1}{1.5}\right)^2\right]^{1/2} = 0.75c$

2-11   (a) $K = 50 \times 10^9 \text{ eV};\ mc^2 = 938.27 \text{ MeV};$

$$E = K + mc^2 = (50 \times 10^9 \text{ eV}) + (938.27 \times 10^6 \text{ eV}) = 50\,938.3 \text{ MeV}$$

$$E^2 = p^2 c^2 + m^2 c^4 \Rightarrow p = \left[\dfrac{E^2 - m^2 c^4}{c^2}\right]^{1/2}$$

$$p = \dfrac{\left[(50\,938.3 \text{ MeV})^2 - (938.27 \text{ MeV})^2\right]^{1/2}}{c} = 5.09 \times 10^{10} \text{ eV/}c$$

$$= \dfrac{5.09 \times 10^{10} \text{ eV}}{3 \times 10^8 \text{ m/s}}(1.6 \times 10^{-19} \text{ J/eV}) = 2.71 \times 10^{-17} \text{ kg} \cdot \text{m/s}$$

    (b) $E = \gamma\, mc^2 = \dfrac{mc^2}{\left[1 - (v/c)^2\right]^{1/2}} \Rightarrow v = c\left[1 - \left(\dfrac{mc^2}{E}\right)^2\right]^{1/2}$

$$= (3 \times 10^8 \text{ m/s})\left[1 - \left(\dfrac{938.27 \text{ MeV}}{50\,938.3 \text{ MeV}}\right)^2\right]^{1/2} = 2.999\,5 \times 10^8 \text{ m/s}$$

2-17 $\Delta m = m_{\text{Ra}} - m_{\text{Rn}} - m_{\text{He}}$ (an atomic unit of mass, the u, is one-twelfth the mass of the ^{12}C atom or $1.660\,54 \times 10^{-27}$ kg)

$$\Delta m = (226.025\,4 - 22.017\,5 - 4.002\,6) \text{ u} = 0.005\,3 \text{ u}$$
$$E = (\Delta m)(931 \text{ MeV/u}) = (0.005\,3 \text{ u})(931 \text{ MeV/u}) = 4.9 \text{ MeV}$$