4-8 (a) From Equation 4.16 we have $\Delta n \propto \left(\frac{\sin\phi}{2}\right)^{-4}$ or $\Delta n_2 = \Delta n_1 \frac{\left(\frac{\sin\phi_1}{2}\right)^4}{\left(\frac{\sin\phi_2}{2}\right)^4}$. Thus the number of α's scattered at 40 degrees is given by

$$\Delta n_2 = (100\ \text{cpm})\frac{\left(\sin\frac{20}{2}\right)^4}{\left(\sin\frac{40}{2}\right)^4} = (100\ \text{cpm})\left(\frac{\sin 10}{\sin 20}\right)^4 = 6.64\ \text{cpm}.$$

Similarly

$$\Delta n \text{ at 60 degrees} = 1.45\ \text{cpm}$$
$$\Delta n \text{ at 80 degrees} = 0.533\ \text{cpm}$$
$$\Delta n \text{ at 100 degrees} = 0.264\ \text{cpm}$$

(b) From 4.16 doubling $\left(\frac{1}{2}\right)m_\alpha v_\alpha^2$ reduces Δn by a factor of 4. Thus Δn at 20 degrees $= \left(\frac{1}{4}\right)(100\ \text{cpm}) = 25\ \text{cpm}$.

(c) From 4.16 we find $\frac{\Delta n_{\text{Cu}}}{\Delta n_{\text{Au}}} = \frac{Z_{\text{Cu}}^2 N_{\text{Cu}}}{Z_{\text{Au}}^2 N_{\text{Au}}}$, $Z_{\text{Cu}} = 29$, $Z_{\text{Au}} = 79$.

$$N_{\text{Cu}} = \text{number of Cu nuclei per unit area}$$
$$= \text{number of Cu nuclei per unit volume * foil thickness}$$
$$= \left[\left(8.9\ \text{g/cm}^3\right)\left(\frac{6.02\times10^{23}\ \text{nuclei}}{63.54\ \text{g}}\right)\right]t = 8.43\times10^{22}t$$
$$N_{\text{Au}} = \left[\left(19.3\ \text{g/cm}^3\right)\left(\frac{6.02\times10^{23}\ \text{nuclei}}{197.0\ \text{g}}\right)\right]t = 5.90\times10^{22}t$$

So $\Delta n_{\text{Cu}} = \Delta n_{\text{Au}}(29)^2 \frac{8.43\times10^{22}}{(79)^2}\left(5.90\times10^2\right) = (100)\left(\frac{29}{79}\right)^2\left(\frac{8.43}{5.90}\right) = 19.3\ \text{cpm}$.

4-9 The initial energy of the system of α plus copper nucleus is 13.9 MeV and is just the kinetic energy of the α when the α is far from the nucleus. The final energy of the system may be evaluated at the point of closest approach when the kinetic energy is zero and the potential energy is $k(2e)\frac{Ze}{r}$ where r is approximately equal to the nuclear radius of copper. Invoking conservation of energy $E_i = E_f$, $K_\alpha = (k)\frac{2Ze^2}{r}$ or

$$r = (k)\frac{2Ze^2}{K_\alpha} = \frac{(2)(29)\left(1.60\times10^{-19}\right)^2\left(8.99\times10^9\right)}{\left(13.9\times10^6\ \text{eV}\right)\left(1.60\times10^{-19}\ \text{J/eV}\right)} = 6.00\times10^{-15}\ \text{m}.$$

4-11 $\frac{1}{\lambda} = R\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)$. For the Balmer series, $n_f = 2$; $n_i = 3, 4, 5, \ldots$. The first three lines in the series have wavelengths given by $\frac{1}{\lambda} = R\left(\frac{1}{2^2} - \frac{1}{n^2}\right)$ where $R = 1.097\ 37 \times 10^7\ \text{m}^{-1}$.

$$\text{1st line: } \frac{1}{\lambda} = R\left(\frac{1}{4} - \frac{1}{9}\right) = \left(\frac{5}{36}\right)R;\ \lambda = \frac{36}{5R} = 656.112\ \text{nm}$$

$$\text{2nd line: } \frac{1}{\lambda} = R\left(\frac{1}{4} - \frac{1}{16}\right) = \left(\frac{3}{16}\right)R;\ \lambda = \frac{16}{3R} = 486.009\ \text{nm}$$

$$\text{3rd line: } \frac{1}{\lambda} = R\left(\frac{1}{4} - \frac{1}{25}\right) = \left(\frac{21}{100}\right)R;\ \lambda = \frac{100}{21R} = 433.937\ \text{nm}$$

4-12 $\frac{1}{\lambda} = R\left(\frac{1-1}{n^2}\right)$ where $n = 2, 3, 4, \ldots$ and $R = 1.097\ 373\,2 \times 10^7\ \text{m}^{-1}$;

$$\text{For } n = 2:\ \lambda = R^{-1}\left(1 - \frac{1}{2^2}\right)^{-1} = 1.215\ 02 \times 10^{-7}\ \text{m} = 121.502\ \text{nm (UV)}$$

$$\text{For } n = 3:\ \lambda = R^{-1}\left(1 - \frac{1}{3^2}\right)^{-1} = 1.025\,17 \times 10^{-7}\ \text{m} = 102.517\ \text{nm (UV)}$$

$$\text{For } n = 4:\ \lambda = R^{-1}\left(1 - \frac{1}{4^2}\right)^{-1} = 1.972\ 018 \times 10^{-7}\ \text{m} = 97.201\ 8\ \text{nm (UV)}$$

4-14 (a) $r_n = \frac{n^2\hbar^2}{m_e ke^2}$; where $n = 1, 2, 3, \ldots$

$$r_n = n^2 \frac{\left(1.055 \times 10^{-34}\ \text{Js}\right)^2}{\left(9.11 \times 10^{-31}\ \text{kg}\right)\left(9.0 \times 19^9\ \text{Nm}^2/\text{C}^2\right)\left(1.6 \times 10^{-19}\ \text{C}\right)^2} = (0.052\ 9\ \text{nm})n^2$$

For $n = 1$: $r_n = 0.052\ 9$ nm

For $n = 2$: $r_n = 0.212\,1$ nm

For $n = 3$: $r_n = 0.477\ 2$ nm

(b) From Equation 4.26, $v = \left(\frac{ke^2}{m_e r}\right)^{1/2}$

$$v_1 = \left[\frac{\left(8.99 \times 10^9\ \text{Nm}^2/\text{C}^2\right)\left(1.60 \times 10^{-19}\ \text{C}\right)^2}{\left(9.11 \times 10^{-31}\ \text{kg}\right)\left(0.052\ 9 \times 10^{-9}\ \text{m}\right)}\right]^{1/2} = 2.19 \times 10^6\ \text{m/s}$$

$v_2 = 1.09 \times 10^6$ m/s

$v_3 = 7.28 \times 10^5$ m/s

(c) As $c = 3.0 \times 10^8$ m/s, $v << c$ and no relativistic correction is necessary.

4-15 (a) The energy levels of a hydrogen-like ion whose charge number is 2 is given by $E_n = (-13.6\ \text{eV})\frac{Z^2}{n^2} = \frac{-54.4\ \text{eV}}{n^2}$ for $Z = 2$. $\left(\text{He}^+\right)$

0

$E_3 = -6.04\ \text{eV}$

$E_2 = -13.6\ \text{eV}$

So $E_1 = -54.4\ \text{eV}$
$E_2 = -13.6\ \text{eV}$
$E_3 = -6.04\ \text{eV}$, etc.

$E_1 = -54.4\ \text{eV}$

(b) For He^+, $Z = 2$ so we see that the ionization energy (the energy required to take the electron from the state $n = 1$ to the state $n = \infty$ is

$$E = (-13.6\ \text{eV})\frac{2^2}{1^2} = \frac{-54.4\ \text{eV}}{n^2} \text{ for } Z = 2.\ (He^+)$$

4-17 $$r = \frac{n^2\hbar^2}{Zm_e ke^2} = \left(\frac{n^2}{Z}\right)\left(\frac{\hbar^2}{m_e ke^2}\right);\ n = 1$$

$$r = \frac{1}{Z}\left[\frac{(1.055\times10^{-34}\ \text{Js})^2}{(9.11\times10^{-31}\ \text{kg})(9\times10^9\ \text{Nm}^2/\text{C}^2)(1.6\times10^{-19}\ \text{C})^2}\right] = \frac{5.30\times10^{-11}\ \text{m}}{Z}$$

(a) For He^+, $Z = 2$, $r = \frac{5.30\times10^{-11}\ \text{m}}{2} = 2.65\times10^{-11}\ \text{m} = 0.026\ 5\ \text{nm}$

(b) For Li^{2+}, $Z = 3$, $r = \frac{5.30\times10^{-11}\ \text{m}}{3} = 1.77\times10^{-11}\ \text{m} = 0.017\ 7\ \text{nm}$

(c) For Be^{3+}, $Z = 4$, $r = \frac{5.30\times10^{-11}\ \text{m}}{4} = 1.32\times10^{-11}\ \text{m} = 0.013\ 2\ \text{nm}$

4-18 (a) $$\Delta E = (13.6\ \text{eV})\left(\frac{1}{1^2} - \frac{1}{3^2}\right) = 12.1\ \text{eV}$$

(b) Either $\Delta E = 12.1\ \text{eV}$ or $\Delta E = (13.6\ \text{eV})\left(\frac{1}{1} - \frac{1}{2^2}\right) = 10.2\ \text{eV}$ and

$$\Delta E = (13.6\ \text{eV})\left(\frac{1}{2^2} - \frac{1}{3^2}\right) = 1.89\ \text{eV}.$$

4-19 (a) $$\Delta E = (-13.6\ \text{eV})\left(\frac{1}{n_i^2} - \frac{1}{n_f^2}\right) = (-13.6\ \text{eV})\left(\frac{1}{9} - \frac{1}{4}\right) = 1.89\ \text{eV}$$

(b) $$E = \frac{hc}{\lambda} \Rightarrow \lambda = \frac{hc}{E} = (4.14\times10^{-15}\ \text{eV s})\frac{3\times10^8\ \text{m/s}}{1.89\ \text{eV}} = 658\ \text{nm}$$

(c) $$c = \lambda f \Rightarrow f = \frac{c}{\lambda} = \frac{3\times10^8\ \text{m/s}}{657\times10^{-9}\ \text{m}} = 4.56\times10^{14}\ \text{Hz}$$

4-20 $\Delta E = (-13.6\text{ eV})\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)$

(a) $\Delta E = (-13.6\text{ eV})\left(\frac{1}{25} - \frac{1}{16}\right) = 0.306\text{ eV}$

(b) $\Delta E = (-13.6\text{ eV})\left(\frac{1}{36} - \frac{1}{25}\right) = 0.166\text{ eV}$

4-22 $E = K + U = \frac{mv^2}{2} - \frac{ke^2}{r}$. But $\frac{mv^2}{2} = \left(\frac{1}{2}\right)\frac{ke^2}{r}$. Thus $E = \left(\frac{1}{2}\right)\left(\frac{-ke^2}{r}\right) = \frac{U}{2}$, so $U = 2E = 2(-13.6\text{ eV}) = -27.2\text{ eV}$ and $K = E - U = -13.6\text{ eV} - (-27.2\text{ eV}) = 13.6\text{ eV}$.

4-23 (a) $r_1 = (0.052\,9\text{ nm})n^2 = 0.052\,9\text{ nm}$ (when $n = 1$)

(b) $m_e v = m_e\left(\frac{ke^2}{m_e r}\right)^{1/2}$

$$m_e = \left[\frac{(9.1\times10^{-31}\text{ kg})(9\times10^{9}\text{ Nm}^2/\text{C}^2)}{5.29\times10^{-11}\text{ m}}\right]^{1/2}\times(1.6\times10^{-19}\text{ C})$$

$M_e v = 1.99\times10^{-24}\text{ kg m/s}$

(c) $L = m_e v r = (1.99\times10^{-24}\text{ kg m/s})(5.29\times10^{-11}\text{ m})$, $L = 1.05\times10^{-34}(\text{kg m}^2/\text{s}) = \hbar$

(d) $K = |E| = 13.6\text{ eV}$

(e) $U = -2K = -27.2\text{ eV}$

(f) $E = K + U = -13.6\text{ eV}$

4-25 (a) $\Delta E = hf = (13.6\text{ eV})\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)$ or $f = (13.6\text{ eV})\left(\frac{\frac{1}{9} - \frac{1}{16}}{4.14\times10^{-15}\text{ eV s}}\right) = 1.60\times10^{14}\text{ Hz}$

(b) $T = \frac{2\pi r_n}{v}$ so $f_{\text{rev}} = \frac{1}{T} = \frac{v}{2\pi r_n}$. Using $v = \left(\frac{ke^2}{m_e r_n}\right)^{1/2}$, $f_{\text{rev}} = \left(\frac{ke^2}{m r_n}\right)^{1/2}$. For $n = 3$, $r_3 = (3)^2 a_0$ and

$$f_{\text{rev}} = \frac{(8.99\times10^{9}\text{ Nm}^2/\text{C}^2)(1.60\times10^{-19}\text{ C})^2}{\frac{[(9.11\times10^{-31}\text{ kg})(9)(5.29\times10^{-11}\text{ m})]^{1/2}}{(2)(3.14)(9)(5.29\times10^{-11}\text{ m})}}$$

$$f_{\text{rev}} = 2.44\times10^{14}\text{ Hz } (n = 3)$$

$$f_{\text{rev}} = 1.03\times10^{14}\text{ Hz } (n = 4)$$

Thus the photon frequency is about halfway between the two frequencies of the revolution.

(b) $$R = \frac{2^2}{36\,545.6 \times 10^{-8}\ \text{cm}} = 109\,720\ \text{cm}^{-1}$$

4-26 Lyman series has $n_\text{f} = 1$, λ_max has $n_\text{i} = 2$; λ_min has $n_\text{i} = \infty$

$$\frac{1}{\lambda_\text{max}} = R\left(\frac{1}{n_\text{f}^2} - \frac{1}{n_\text{i}^2}\right) = R\left(\frac{1-1}{2^2}\right) = \frac{3R}{4}$$

$$\lambda_\text{max} = \frac{4}{3R} = \frac{4}{(3)\left(1.097 \times 10^7\ \text{m}^{-1}\right)} = 121.5\ \text{nm}$$

$$\lambda_\text{min} = \frac{1}{R} = 91.16\ \text{nm}$$

As shown on the inside front cover, the visible spectrum begins at about 350 nm, so the Lyman series is in the UV.

4-35 $\mu = \frac{m_\text{e} M}{m_\text{e} + M} = \frac{m_\text{e}}{2}$ since $m_\text{e} = M$. In general, $r_n = \frac{n^2 \hbar^2}{Z \mu k e^2}$, so for positronium $Z = 1$, $\mu = \frac{m_\text{e}}{2}$ and $r_\text{positronium} = \left(\frac{\hbar^2 n^2}{1}\right)\left(\frac{m_\text{e}}{2}\right) k e^2 = 2 a_0 n^2 = 2 r_\text{hydrogen}$. Similarly, $E_\text{positronium} = \frac{E_\text{hydrogen}}{2} = \frac{-6.80\ \text{eV}}{n^2}$.

4-31 (a) $(m_e vr) = n\hbar\,;\; v = \dfrac{n\hbar}{m_e r}$,

$$f_e = \frac{v}{2\pi r} = \frac{\frac{n\hbar}{m_e r}}{2\pi r} = \frac{n\hbar}{2\pi m_e r^2} = \frac{n\hbar}{2\pi m_e n^4 (a_0)^2} = \frac{\hbar}{2m_e (a_0)^2 n^3} = \left(\frac{m_e k^2 Z^2 e^4}{2\pi\hbar^3}\right)\left(\frac{1}{n^3}\right)$$

using $a_0 = \dfrac{\hbar^2}{m_e k e^2}$.

(b)

$$\Delta E = hv_{\text{photon}} = \left(\frac{kZ^2e^2}{2a_0 h}\right)\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)$$

$$f_{\text{photon}} = \left(\frac{kZ^2e^2}{2a_0 h}\right)\left(\frac{n_i^2 - n_f^2}{n_i^2 n_i^2}\right) = \left(\frac{kZ^2e^2}{2a_0 h}\right)\left[\frac{(n_i - n_f)(n_i - n_f)}{n_f^2 n_i^2}\right]$$

$$= \left(\frac{m_e K^2 Z^2 e^4}{2\pi\hbar^3}\right)\left(\frac{n_i + n_f}{2n_f^2 n_i^2}\right)(n_i - n_f)$$

For $n_i - n_f = 1$, $f_{\text{photon}} = \left(\dfrac{m_e k^2 Z^2 e^4}{2\pi\hbar^3}\right)\left(\dfrac{n_i + n_f}{2n_f^2 n_i^2}\right)$. For $n_i = 2$, $n_f = 1$, $n_i = 3$, $n_f = 2$, etc., $\dfrac{1}{n_i^3} < \dfrac{n_i + n_f}{2n_i^2 n_f^2} < \dfrac{1}{n_f^3}$.

(c) Frequency of emitted radiation is *in between* the initial orbital frequency and the final. As $n_i \to \infty$ the initial and final orbital frequencies squeeze closer together making the frequency of emitted radiation equal to the orbital frequency. This result agrees with Bohr's correspondence principle.

4-35 $\mu = \frac{m_e M}{m_e + M} = \frac{m_e}{2}$ since $m_e = M$. In general, $r_n = \frac{n^2\hbar^2}{Z\mu ke^2}$, so for positronium $Z = 1$,

$\mu = \frac{m_e}{2}$ and $r_{\text{positronium}} = \left(\frac{\hbar^2 n^2}{1}\right)\left(\frac{m_e}{2}\right)ke^2 = 2a_0 n^2 = 2r_{\text{hydrogen}}$. Similarly,

$E_{\text{positronium}} = \frac{E_{\text{hydrogen}}}{2} = \frac{-6.80\text{ eV}}{n^2}$.

4-37 $hf = \Delta E = \frac{4\pi^2 m_e k^2 e^4}{2h^2}\left(\frac{1}{(n-1)^2} - \frac{1}{n^2}\right)$, $f = \left(\frac{2\pi^2 m_e k^2 e^4}{h^3}\right)\left(\frac{2n-1}{(n-1)^2(n^2)}\right)$ as $n \to \infty$,

$f \to \left(\frac{2\pi^2 m_e k^2 e^4}{h^3}\right)\left(\frac{2}{n^3}\right)$. The revolution frequency is $f = \frac{v}{2\pi r} = \left(\frac{1}{2\pi}\right)\left(\frac{ke^2}{m_e}\right)^{1/2}\left(\frac{1}{r^{3/2}}\right)$

where $r = \frac{n^2 h^2}{4\pi^2 m_e ke^2}$ substituting for r, $f = \left(\frac{1}{2\pi}\right)\left(\frac{ke^2}{m_e}\right)^{1/2}\left(\frac{8\pi^3 m_e ke^3 (m_e k)^{1/2}}{n^3 h^3}\right) = \frac{4\pi^2 m_e k^2 e^4}{h^3 n^3}$

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