PHYSICS 2D QUIZ 4 FALL QUARTER 2020
PROF. HIRSCH

Justify all your answers to all 3 problems. Write clearly.
Formulas:

Time dilation; Length contraction: $\Delta t = \gamma \Delta t' \equiv \gamma\, \Delta t_p$; $L = L_p/\gamma$; $c = 3\times 10^8 m/s$

Lorentz transformation: $x' = \gamma(x - vt)$; $y' = y$; $z' = z$; $t' = \gamma(t - vx/c^2)$; inverse: $v \to -v$

Velocity transformation: $u_x' = \dfrac{u_x - v}{1 - u_x v/c^2}$; $u_y' = \dfrac{u_y}{\gamma(1 - u_x v/c^2)}$; inverse: $v \to -v$

Spacetime interval: $(\Delta s)^2 = (c\Delta t)^2 - [\Delta x^2 + \Delta y^2 + \Delta z^2]$ $\gamma = 1/\sqrt{1 - v^2/c^2}$

Relativistic Doppler shift: $f_{obs} = f_{source}\sqrt{1 + v/c}\ /\ \sqrt{1 - v/c}$

Momentum: $\vec{p} = \gamma m\vec{u}$; Energy: $E = \gamma mc^2$; Kinetic energy: $K = (\gamma - 1)mc^2$

Rest energy: $E_0 = mc^2$; $E = \sqrt{p^2c^2 + m^2c^4}$

Electron: $m_e = 0.511\, MeV/c^2$ Proton: $m_p = 938.26\, MeV/c^2$ Neutron: $m_n = 939.55\, MeV/c^2$

Atomic mass unit: $1\,u = 931.5\, MeV/c^2$; electron volt: $1\text{eV} = 1.6\times 10^{-19}$ J

Stefan's law: $e_{tot} = \sigma T^4$, e_{tot} = power/unit area ; $\sigma = 5.67\times 10^{-8} W/m^2K^4$

$e_{tot} = cU/4$, U = energy density = $\int_0^\infty u(\lambda,T)d\lambda$; Wien's law: $\lambda_m T = \dfrac{hc}{4.96k_B}$

Boltzmann distribution: $P(E) = Ce^{-E/(k_BT)}$

Planck's law: $u_\lambda(\lambda,T) = N_\lambda(\lambda)\times \overline{E}(\lambda,T) = \dfrac{8\pi}{\lambda^4}\times \dfrac{hc/\lambda}{e^{hc/\lambda k_BT} - 1}$; $N(f) = \dfrac{8\pi f^2}{c^3}$

Photons: $E = hf = pc$; $f = c/\lambda$; $hc = 12{,}400\ eV A$; $k_B = (1/11{,}600)eV/K$

Photoelectric effect: $eV_s = K_{max} = hf - \phi$, $\phi \equiv$ work function;]

Compton scattering: $\lambda' - \lambda = \dfrac{h}{m_ec}(1 - \cos\theta)$; $\dfrac{h}{m_ec} = 0.0243A$ $ke^2 = 14.4eVA$

Coulomb force: $F = \dfrac{kq_1q_2}{r^2}$; Coulomb energy: $U = \dfrac{kq_1q_2}{r}$; Coulomb potential: $V = \dfrac{kq}{r}$

Force in electric and magnetic fields (Lorentz force): $\vec{F} = q\vec{E} + q\vec{v}\times\vec{B}$

Rutherford scattering: $\Delta n(\theta) = C\dfrac{Z^2}{K_\alpha^2}\dfrac{1}{\sin^4(\theta/2)}$; $b = \dfrac{kq_\alpha Q}{2K_\alpha}\cot(\theta/2)$

Hydrogen: $\dfrac{1}{\lambda_{mn}} = R\left(\dfrac{1}{m^2} - \dfrac{1}{n^2}\right)$; $R = \dfrac{1}{911.8A}$; $\hbar c = 1973\text{eVA}$

Bohr atom: $E_n = -\dfrac{ke^2Z}{2r_n} = -E_0\dfrac{Z^2}{n^2}$; $E_0 = \dfrac{ke^2}{2a_0} = \dfrac{m_e(ke^2)}{2\hbar^2} = 13.6eV$; $K = \dfrac{m_ev^2}{2}$; $U = -\dfrac{ke^2Z}{r}$

$hf = E_i - E_f$; $r_n = r_0n^2$; $r_0 = \dfrac{a_0}{Z}$; $a_0 = \dfrac{\hbar^2}{m_eke^2} = 0.529A$; $L = m_evr = n\hbar$ angular momentum

de Broglie: $\lambda = \dfrac{h}{p}$; $f = \dfrac{E}{h}$; $\omega = 2\pi f$; $k = \dfrac{2\pi}{\lambda}$; $E = \hbar\omega$; $p = \hbar k$; $E = \dfrac{p^2}{2m}$

Wave packets: $y(x,t) = \sum_j a_j\cos(k_jx - \omega_jt)$, or $y(x,t) = \int dk\, a(k)\, e^{i(kx - \omega(k)t)}$; $\Delta k\Delta x \sim 1$; $\Delta\omega\Delta t \sim 1$

group and phase velocity : $v_g = \frac{d\omega}{dk}$; $v_p = \frac{\omega}{k}$; Heisenberg: $\Delta x \Delta p \sim \hbar$; $\Delta t \Delta E \sim \hbar$

Probability: $P(x)dx = |\Psi(x)|^2\, dx$; $P(a \le x \le b) = \int_a^b dx P(x)$ $\hbar c = 1973\ eVA$

Schrodinger equation: $-\frac{\hbar^2}{2m}\frac{\partial^2\Psi}{\partial x^2} + U(x)\Psi(x,t) = i\hbar\frac{\partial \Psi}{\partial t}$; $\Psi(x,t) = \psi(x)e^{-i\frac{E}{\hbar}t}$

Time – independent Schrodinger equation: $-\frac{\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2} + U(x)\psi(x) = E\psi(x)$; $\int_{-\infty}^{\infty} dx\ \psi^*\psi = 1$

∞ square well: $\psi_n(x) = \sqrt{\frac{2}{L}}\sin(\frac{n\pi x}{L})$; $E_n = \frac{\pi^2\hbar^2 n^2}{2mL^2}$; $\frac{\hbar^2}{2m_e} = 3.81 eVA^2$ (electron)

Expectation value of$[Q]$: $<Q> = \int \psi^*(x)[Q]\psi(x)\ dx$; Momentum operator: $p = \frac{\hbar}{i}\frac{\partial}{\partial x}$

Problem 1 (10 points)
A free electron at time t=0 is described by the wavepacket

$$\psi(x) = \int dk\ a(k)e^{ikx}$$

with a(k)=a for $0.09A^{-1} < k < 0.11A^{-1}$, a(k)=0 elsewhere.
(a) Estimate the uncertainty in the position of this electron at t=0. Give your answer in A.
(b) Estimate the uncertainty in the momentum of this electron. Give your answer in eV/c.
(c) Estimate the position of this electron at time t=1s. Give your answer in km.

Problem 2 (10 points)
An electron is in a stationary state of an infinite square well of length 8A. It is equally likely to be found at distance 1A and 3A from the left wall, and its energy is less than 8eV.
(a) Find its energy in eV and its quantum number n.
(b) How much more likely is it to find this electron at a distance 2A than at a distance 1A from the left wall?
(3) Find approximately the probability that this electron will be found in the interval from 0.96A to 1.04A as measured from the left wall. How does your answer compare to the classical value?

Problem 3 (10 points)
An electron is in a finite square well extending from 0A to 5A and height 50eV.
(a) Estimate the number of stationary states for this electron in this well.
(b) Assume the electron is in the lowest energy state. Estimate the distance that the electron will penetrate into the exterior region x>5A.
(c) Using the result found in (b), estimate (in eV) how much lower the energy of this electron will be compared to that of an electron in an infinite well of the same width in the lowest energy state.