PHYSICS 4E QUIZ 3 SPRING QUARTER 2020
PROF. HIRSCH MAY 11

<u>Open book</u>. Show all steps in your calculations. Justify all answers. Write clearly.
<u>Some information:</u>

$hc = 12,400 eVA, \ \ \hbar c = 1973 eVA \ , \ m_e c^2 = 511,000 eV \ , \ k_B = 1/11,600 eV/K$

$ke^2 = 14.4 eVA \ ; \ \ 1A = 10^{-10} m \ ; \ \ c = 3\cdot 10^8 m/s \ ; \ \ \hbar^2 / m_e = 7.62 eVA^2$

Relativity: $E = \sqrt{p^2c^2 + m^2c^4} \ , \ E = \gamma mc^2 \ , \ p = \gamma mv$

proton: $m_p / m_e = 1836$; Uncertainty principle: $\Delta x \Delta p \sim \hbar, \ \ \Delta t \Delta E \sim \hbar$

Problem 1 (10 pts)
A free electron moving in one dimension is described by the wavepacket

$$\psi(x,t) = \int_{-\infty}^{\infty} dk \ a(k) \ e^{i(kx - \omega(k)t)} \quad \text{with } a(k) = Ae^{-\alpha|k-k_0|} \quad \text{and } \alpha = 100\text{A}, \ k_0 = 200\text{A}^{-1}.$$

(a) Find the speed at which this electron is moving. Give your answer in km/s.
(b) Estimate the uncertainty in the position of this electron at time t=0. Give your anwer in A.
(c) Find the wavefunction $\psi(x, t=0)$ by doing the integral, make a schematic plot of it, and explain using the plot why the uncertainty in the position of the electron is of the order found in (b).

Problem 2 (10 pts)
A particle is confined to move in one dimension in the region $x \geq 0$ in a potential given by:
$U(x) = \lambda x$
with λ=1eV/A. Assume the particle is in the lowest possible energy state.
(a) For a classical particle, what would be its position and its energy?
Now assuming the particle is (i) an electron, and (ii) a proton, estimate using the uncertainty principle: (<u>Hint:</u> minimize the energy)
(b) What is the uncertainty in its position? Give your answer in A, for (i) and (ii).
(c) The particle's estimated kinetic energy, in eV, for (i) and (ii).
(d) Show that the ratio of its estimated kinetic energy to potential energy is independent of the particle's mass and of the value of λ. What is the value of this ratio? Smaller, equal or larger than 1? (Note: for the harmonic oscillator, the ratio is equal to 1).

Problem 3 (10 pts + 3 pts extra credit)
An electron in a square well of height $U_0 = \infty$ has ground state energy 4eV.
(a) What is the energy of the first excited state, in eV?
(b) What is the width of this well, L, in A?
(c) Assume now you have a finite well of the same width L as above, and U_0 is such that the ground state energy is $U_0/2$. What is the value of U_0 and of the ground state energy now, in eV?
(d) (Extra credit) Is there a first excited state for the electron in this well when U_0 has the value found in (c)? Justify your answer clearly.