PHYSICS 4E | QUIZ 4 | SPRING QUARTER 2020
PROF. HIRSCH | | MAY 18

<u>Open book</u>. Show all steps in your calculations. Justify all answers. Write clearly.

$hc = 12{,}400 eVA,\ \ \hbar c = 1973 eVA\ ,\ m_e c^2 = 511{,}000 eV\ ,\ k_B = 1/11{,}600 eV/K$

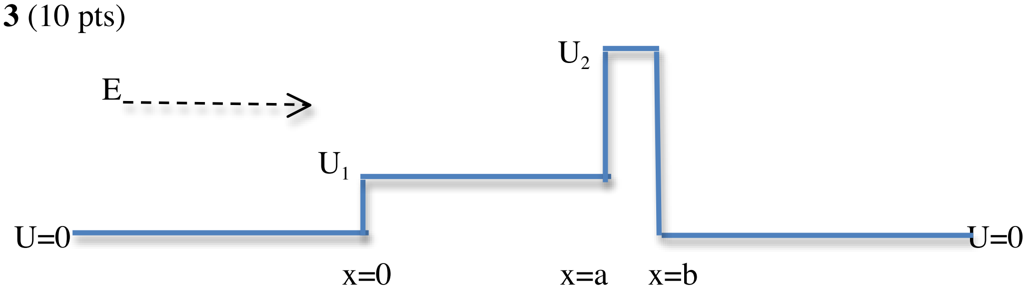
$ke^2 = 14.4 eVA\ ;\ \ 1A = 10^{-10} m\ ;\ \ c = 3\cdot 10^8 m/s\ ;\ \ \hbar^2/m_e = 7.62 eVA^2$

proton: $m_p/m_e = 1836$; Uncertainty principle: $\Delta x \Delta p \sim \hbar,\ \ \Delta t \Delta E \sim \hbar$

Problem 1 (10 pts)
An electron is described by the wavefunction
$\psi(x) = C(1-x^2)$ for $-1 \le x \le 1,\ \ \psi(x) = 0$ for $|x| > 1$, where x is measured in A.
(a) Find C.
(b) Calculate the quantum uncertainty in the position, Δx. Do not make approximations.
(c) Calculate the quantum uncertainty in the momentum, Δp, expressed as a number times $\hbar$. Do not make approximations.
(d) Verify whether your results satisfy $\Delta x \Delta p > \hbar/2$. If not, explain why not.
(e) Calculate the average kinetic energy of this electron, in eV.
Justify all your answers.

Problem 2 (10 pts)
For a quantum harmonic oscillator (also for a classical one) the average kinetic energy <K> equals the average potential energy <U>. (Hint: $K=p^2/2m$.) <u>Using that fact</u>:
(a) Prove that in the ground state of the quantum harmonic oscillator, $\Delta x \Delta p = \hbar/2$.
(b) Find the value of $\Delta x \Delta p$ for the first excited state of the quantum harmonic oscillator.
(c) An electron is in the first excited state of a harmonic oscillator potential and has classical amplitude of oscillation 3Angstrom. Find the uncertainty in its position, Δx.
(d) Find the energy of this electron in the first excited state of this potential, in eV.

Problem 3 (10 pts)



In the figure, the potential U(x) is 0 for x<0, U_1 for 0<x<a, U_2 for a<x<b, 0 for x>b, with U_1=2eV, U_2=7eV, a=5A, b=6.5A. Electrons are incident from the left with kinetic energy E=3eV, at a rate of 10,000 electrons per second.
(a) How many electrons get reflected at x=0 per second? You may ignore U_2 for this part.
(b) How many electrons per second are detected at a point x>b?
(c) Is the wavefunction $\psi(x)$ describing these electrons an eigenfunction of the momentum operator? If yes, what is its eigenvalue, in units eV/c? Justify your answer.
(d) Is $\psi(x)$ an eigenfunction of the Hamiltonian operator for this problem? If yes, what is its eigenvalue, in eV? Justify your answer.