Open book. Show all steps in your calculations. Justify all answers. Write clearly.

$hc = 12,400eVA,\ \hbar c = 1973eVA\ ,\ m_e c^2 = 511,000eV\ ,\ k_B = 1/11,600eV/K$

$ke^2 = 14.4eVA\ ;\ \ 1A = 10^{-10}m\ ;\ \ c = 3\cdot 10^8 m/s\ ;\ \ \hbar^2/m_e = 7.62eVA^2$

$m_p/m_e = 1836\ \ ;\ \ \mu_B = 5.79\times 10^{-5} eV/T\ \ ;\ \int_0^\infty dx x^n e^{-\lambda x} = n!/\lambda^{n+1}$

Problem 1 (10 pts)

An electron is in a stationary state of a two-dimensional potential given by

$$U(x,y) = 0 \quad \text{for} \quad -\frac{L}{2} \le x \le \frac{L}{2}, -\frac{L}{2} \le y \le \frac{L}{2}$$

$$U(x,y) = \infty \quad \text{for} \quad |x| > \frac{L}{2} \text{ or } |y| > \frac{L}{2} \text{ or both}$$

with L=5A. In the region $x \ge 0,\ y \ge 0$ this electron is more likely to be found around the point $(x,y) = (L/4, L/4)$ than anywhere else in that region.

(a) Find the quantum numbers n_1, n_2 and the energy for this electron, in eV.

(b) What is the longest wavelength photon (in A) that this electron can absorb in making a transition to another state? Justify your answer clearly. Assume no selection rules.

(c) What is the shortest wavelength photon (in A) that this electron can emit in making a transition to another state? Justify your answer clearly. Assume no selection rules.

(d) Assume now there are other electrons placed in this box that prevent this electron from making a transition to a lower energy state because of the Pauli exclusion principle. What is the minimum number of other electrons in this box?

Problem 2 (10 pts)

The wavefunction for an electron in a hydrogen-like ion is

$$\psi(r,\theta,\phi) = Cre^{-3r/a_0}\cos\theta e^{im_\ell\phi}$$

where C is a constant.

(a) Give the quantum numbers n, ℓ and m_ℓ and ionic charge Z. Justify all your answers.

(b) Find the most probable r for this electron, in terms of a_0, the Bohr radius.

(c) Find the average potential energy for this electron, as a multiple of ke^2/a_0

(d) Compare the results found in (b) and in (c) with the values predicted by the Bohr model for the same values of the quantum number n found in (a).

Problem 3 (10 pts)

The wavelength of photons emitted when an electron goes from the 3d to the 2p levels of hydrogen is 6564.3A, since the energy difference is E_3-E_2=1.889eV. Assume now a magnetic field of magnitude B=10 T is turned on.

(a) Ignoring electron spin, find the possible wavelengths of photons emitted in this transition in the presence of B. Keep in mind the selection rule $\Delta m_\ell = 0,\ 1$ or -1.

(b) see next page

(b) Taking into consideration electron spin and ignoring the spin-orbit interaction, find the maximum and minimum wavelength of the photons emitted in this transition. Keep in mind the selection rule $\Delta(m_\ell + m_s) = 0,\ 1$ or -1.