"Engineering Flows: Models and Mixing Length Theory"

- Law of Wall and Prandtl Mixing Length Theory
- Another Look at Wakes
- Heat Transfer: Laminar and Turbulent

→ Landau-Lifshitz → Good Coverage ✱
→ V.P. Krainov: "Qual Methods in Physical Kinetics and Hydrodynamics"
S.B. Pope: Turbulent Flow (Engineering Style)

→ Macroscopic Problems in Turbulence

⇒ especially eddy viscosity and boundary layers.

- K41, inertial range problem ⇒

Richardson
"small" scales ⇒ $l_d < l < l_{INT}$
$l_d \sim (\nu^3/\epsilon)^{1/4}$, $l_{INT} \sim l_0$

Key ideas: universality
⇒ self-similarity, scale invariance
- dissipation indep. Re, ν (C_D(Re) flat)
⇔ singularity formation
$P_d \sim \nu \langle (\nabla v)^2 \rangle \rightarrow \nu \langle \omega^2 \rangle$

- Galilean invariance.

$\epsilon \sim v(l)^2 \frac{v(l)}{l}$ $\qquad \epsilon \sim \frac{\delta v(l)^3}{l}$

indep. ν, Re

and one rigorous result: 4/5 Law

$\langle \delta v^3 \rangle \sim -\frac{4}{5} \epsilon l$

i.e. real

⇕

Here, discuss macroscopic turbulence problems...

⟺ "Engineering flows"

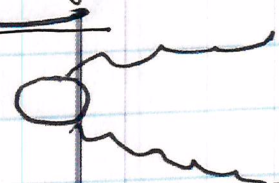
[Wakes
[Pipe flow BL
[Thermal BL
(Heat Transfer)

threads:

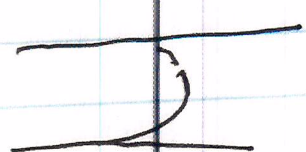
- universality ⇒ common flow structure
- self-similarity ⇒ flows same, up to/within rescaling
- mixing driven ⇒ i.e. [new aspect]

Non/linearly evolved state.

wakes



pipe flow profiles:



- central question, if diffusive model of mixing

$D \cong V_T \, l_{mix}$ { what are l_{mix}, V_T ?

(analogy with K.T. (gas)). { l_{mix} non-trivial i.e. Rich.

ℓ_{mix} usually left variable,

→ due absence of other scales

⇒ scale invariance.

- mixing length sometimes emergent i.e. Rhines scale

Mixing length {theory / models} → crude

but useful tool, heavily {utilized / maligned}.

→ [Momentum Turbulence] Flux Driven

· Turbulent Pipe Flow

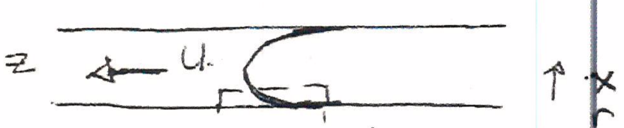
(cf. Landau, Lifshitz "Fluid Mechanics")

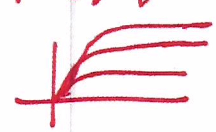
Till now → homogeneous flow in a periodic box
→ cascade in scale space (Kolmogorov) 1941

Now → inhomogeneous flow in a pipe
→ momentum transport in a turbulent boundary layer $\boxed{\text{(Prandtl)}}$ 1931, 32

Consider turbulent pipe flow: $Re \gg 1$ $\gtrsim 10^6$

eg. in toilet





core

logarithmic profile (inertial sublayer) → universal
(virtually all high Re experiments → "profile consistency")

linear profile (viscous sublayer)

Common features of pipe flow:

- linear → logarithmic $u(x)$ profile
- logarithmic profile persists over a broad range of Re

$(Re = 2ua/\nu)$

$\therefore$ logarithmic profile "universal" (Prandtl "Law of the Wall")

- resistance ~~increases~~ discontinuously with increasing Re,

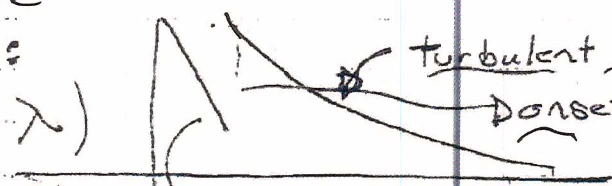
$$\lambda = \frac{2a\,\Delta P/l}{\frac{1}{2}\rho u^2} \quad (\rightarrow \tau_{KE})$$

$2a\,\Delta P/l \rightarrow$ pressure drop/length

$\frac{1}{2}\rho u^2 \rightarrow$ mean flow energy.

Resistance curve then:

$[\ln](100\,\lambda)$



turbulent branch

onset turbulence

$[\ln]\,Re$

laminar branch

log plot

incremental increase KE dropping with Re

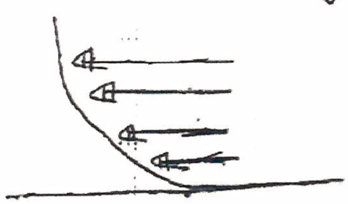
not incr. stored energy much with $\uparrow \Delta P$

some subtlety in how define Re

- turbulent resistance curve universal.

no indep scale dep.

What is going on? $\rightarrow$ physics of resistance?



no slip boundary condition

$u = u(x) \underset{x \to 0}{\longrightarrow} 0$

gradient

$\therefore u = u(x) \Rightarrow \{$ momentum flux to wall?

$$\partial_t \underline{v} + \underline{v}\cdot\nabla\underline{v} - \nu\nabla^2\underline{v} = -\frac{\nabla P}{\rho}$$

→ Momentum flux to wall ⇒ stress on the wall

→ ⊗ wall stress must balance pressure drop for steady flow

τ, ρ, some U_*

so wall stress : ρU_*^2

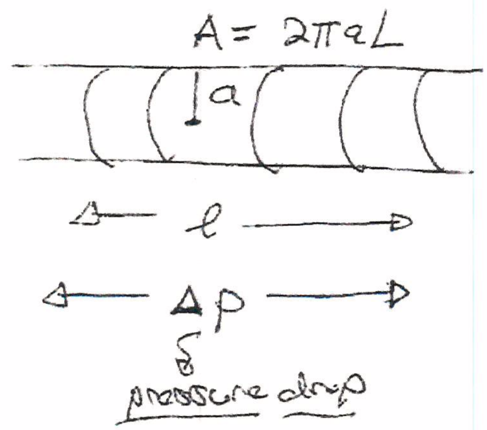
$U_* \equiv$ friction velocity

$= \frac{\Delta P}{\ell}$

$F - A$

$2\pi\, v_*^2 \ell R \sim \Delta P \pi R^2$

$\therefore \quad \boxed{\rho U_*^2 2\pi a\, \ell = \Delta p\, \pi a^2}$ defines U_*



Force on wall ≃ $\rho U_*^2 A_{wall}$

(Pressure Drop) A_{Flow} = Force on Fluid

stationarity ⇒ $\rho U_*^2 (2\pi a \ell) = (\Delta p)\, \pi a^2$

$$U_* = \left[(\Delta p / 2\rho) \left(\frac{a}{\ell} \right) \right]^{1/2}$$

Friction Velocity

$u_* \equiv$ friction velocity
$\equiv$ "typical" velocity of turbulence in turbulent pipe — think of as energy containing range.

Deriving the inertial sublayer profile:

(i) dimensional reasoning

in pipe flow inertial sublayer, have

3 dimensional parameters ρ, τ, x

ρ: density; τ: wall stress, u_*; x: distance from wall

only length in theory (neglect ν).

Key Point: ~~Assumption~~ Assumption of scale invariance

on scale $l_{vs} = \frac{\nu}{u_*} < x < a$

$\Rightarrow$ universality of logarithmic profile motivates scale invariance assumption

now, seek velocity gradient du/dx,

$\frac{du}{dx}$: u_*, x, ρ

so simplest form for dU/dx is:

$$\frac{dU}{dX} = \frac{U_*}{X}$$

$$\Rightarrow \boxed{\begin{aligned} U &= \frac{U_*}{\kappa} \ln(X/X_0) \\ &= \frac{U_*}{\kappa} \ln X \quad + \text{const.} \end{aligned}}$$

c.f. Barenblatt

→ logarithmic profile (consequence of scale invariance in pipe flow)

→ $\kappa \approx .4$ universal constant → von-Karman constant (empirical)

· $X_0 \leftrightarrow$ width of viscous sublayer $\sim \nu/u_*$

ii) Physical Reasoning

stationary flow ⇒

momentum flux to wall = pressure drop

Mixing Length Theory initiated by Boussinesq.

$$\therefore \quad \underbrace{\langle \tilde{V}_x \tilde{V}_z \rangle}_{\text{Reynolds stress}} = u_*^2$$

$$\rho \langle \tilde{V}_x \tilde{V}_z \rangle = \Gamma_\rho = \underbrace{\Pi_{x,z}}_{\text{momentum flux}}$$

$$\Pi_{x,z}/\rho = u_*^2$$

Now, to calculate $\langle \tilde{V}_x \tilde{V}_z \rangle$:

→ take velocity fluctuation as generated by mixing of $U(x)$, so

$$\rightarrow \quad \tilde{V}_z \cong -\underbrace{l}_{\text{"mixing length"}} \frac{\partial U}{\partial x}$$

$\tilde{V}_z$ results from mixing of mean profile U

analogous to Chapman-Enskog expansion, i.e.

$l \Leftrightarrow l_{mfp}$

$\tilde{V}_x \Leftrightarrow V_{th}$

here, scale invariance $\Leftrightarrow$ $\ell \sim x$

mixing length set by distance from wall

logarithmic ~~[illegible]~~ factor

no scale.

so

$$\langle \tilde{V}_x \tilde{V}_z \rangle = -\langle V_x \ell \rangle \frac{\partial U}{\partial x}$$

$$\cong -u_* x \frac{\partial U}{\partial x}$$

$u_* \rightarrow$ drive dependence

$\underline{\nu_T = u_* x}$ $\rightarrow$ "eddy viscosity" "turbulent viscosity" $\Big\}$ Key concept

Ficks

$\Rightarrow$ rate of turbulent ~~transport~~ diffusion of momentum

transport

then momentum balance $\Rightarrow$

$u_*^2 \sim \nu$

$$u_* x \frac{\partial U}{\partial x} = u_*^2$$

$$\Rightarrow \boxed{U = \frac{u_*}{\kappa} \ln(x/x_0)} \rightarrow \text{Logarithmic Profile}$$

$\rightarrow$ Law of the Wall

FAQ

Some comments:

→ as in K41, clear phenomenology critical to guiding the approximations ⟺ scale invariance

⇒ "Mixing length theory always works .... provided you know the mixing length..."
— P.D.

→ why a single value of velocity, i.e. u_*?

Consistent with mixing length hypothesis, velocity fluctuations generated by mixing of mean flow gradient, i.e.

⇒

$$\tilde{V} \sim l \frac{\partial u}{\partial x} \sim x \frac{\partial u}{\partial x}$$

$$\sim \cancel{x} \frac{u_*}{\cancel{x}} \quad l_{11}.$$

absence of preferred scale.

internal consistency

Consistent. ∴ Assumption consistent with:
- logarithmic profile
- scale invariance.

- matching, for const:

$$x_0 = \nu/u_* \quad \text{so}$$

$$u = \frac{u_*}{\kappa} \ln\left(\frac{u_* x}{\nu}\right)$$

Note: Flow in viscous sub layer is turbulent, but mixing there affected by dissipation range scales $\Rightarrow$ linear profile

Now - turbulent dissipation ?

Consider NSE:

$$\frac{\partial \hat{\underline{V}}}{\partial t} + \hat{\underline{V}} \cdot \nabla \hat{\underline{V}} + \langle V_z \rangle \frac{\partial}{\partial z} \hat{\underline{V}} + \hat{V}_x \frac{\partial}{\partial x} \langle V_z \rangle$$

$$= -\nabla \hat{p} + \nu \nabla^2 \hat{\underline{V}}$$

$\hat{\underline{V}} \cdot$ and avg $\Rightarrow$

$$\frac{\partial}{\partial t} \langle \hat{V}^2 \rangle + \langle \hat{\underline{V}} \cdot \hat{\underline{V}} \cdot \nabla \hat{\underline{V}} \rangle + \langle V_z \rangle \cancel{\langle \hat{\underline{V}} \cdot \frac{\partial}{\partial z} \hat{\underline{V}} \rangle}_{\text{odd}}$$

$$+ \langle \hat{V}_x \hat{V}_z \rangle \frac{\partial}{\partial x} \langle V_z \rangle = -\cancel{\langle \hat{\underline{V}} \cdot \nabla \hat{p} \rangle}_{\text{i.b.p}} - \nu \langle (\nabla \hat{V})^2 \rangle$$

For <u>net</u> energy budget:

$$\partial_t \mathcal{E} = -\langle \tilde{V}_x \tilde{V}_z \rangle \frac{\partial \langle V_z \rangle}{\partial x} - \nu \langle (\nabla \tilde{V})^2 \rangle$$

$$\left(+ \underbrace{T}_{\boxed{\frac{\partial}{\partial x}\langle \tilde{V}^3 \rangle}} \right)$$

sps.

[input to fluctuations by relaxation of mean shear flow (Reynolds work)

[dissipation of fluctuation energy by viscosity

∴ can define:

$$\underbrace{\epsilon}_{\text{turbulent dissipation rate}} = \langle \tilde{V}_x \tilde{V}_z \rangle \frac{\partial U}{\partial x}$$

[input to turbulence from Reynolds work mean flow.

and using mixing length theory:

$$\langle \tilde{V}_x \tilde{V}_z \rangle = U_* X \frac{\partial U}{\partial x}$$

$$\Rightarrow \epsilon = (U_* X)\left(\frac{\partial U}{\partial x}\right)^2 = \underbrace{\nu_T}_{\text{rate of "heating" by turbulent relaxation}} \left(\frac{\partial U}{\partial x}\right)^2$$

input → mean flow mixing

obviously: $\nu \langle (\nabla \tilde{v})^2 \rangle = \nu_T \left(\frac{\partial U}{\partial x} \right)^2$

small scale dissipation

and

$$\epsilon = (u_* x) \left(\frac{u_*}{x} \right)^2 \qquad \text{(ignoring } R\text{)}$$

$$= \frac{u_*^3}{x}$$

no proof, à la 4/5 Law.

→ sets dissipation rate, as fctn x

i.e. $\epsilon = \frac{v_0^3}{\ell}$ $\qquad v_0 \leftrightarrow u_*$, $\ell \leftrightarrow x$

$\ell \leftrightarrow$ wall distance.

→ ϵ finite as $\nu \to 0$ (i.e. viscous sublayer gradient diverges then)

Additional References:

- S.B. Pope, "Turbulent Flows"
- H. Tennekes and J. Lumley, "A First Course in Turbulence",

⇒ Now, interesting to tabulate comparison between Pipe Flow and K41 Problem

Pipe Flow (Prandtl)	K41 (Kolmogorov)
scales: $a, x, \nu/u_*$	ℓ_0, ℓ_n, ℓ_d
invariance: $x \to$ real space	$\ell \to$ scale space
inertial sublayer viscous sublayer	inertial range dissipation range
lance: $u_*^2 = \nu_T \frac{\partial u}{\partial x}$	$\epsilon = \frac{v(\ell)^2}{\tau(\ell)}$
namics: eddy viscosity $\nu_T = u_* x$	turn-over rate $1/\tau(\ell) = \frac{v(\ell)}{\ell}$
ult: $u = \frac{u_*}{\kappa} \ln(x)$	$v(\ell) = \epsilon^{1/3} \ell^{1/3}$
universal profile	universal spectral scaling
sipation: $\nu = \nu_T$ $x_0 = \nu/u_*$	$v(\ell)/\ell = \nu/\ell^2$ $\ell_d = \nu^{3/4}/\epsilon^{1/4}$

⇒ Practical Issues

Resistance Law ⇒ Pipe Flows

have: $\frac{\nu}{u_*} < x \leq a$
radius

can push to $x \sim a$, with logarithmic accuracy

$$u \cong \frac{u_*}{\kappa} \ln \left(\frac{u_* a}{\nu} \right)$$

but

$$v_* = u_* = \left(\frac{a}{\ell} \frac{\Delta P}{2\rho} \right)^{1/2}$$

⇒ can re-write:

$$u = \left(\frac{a \Delta P}{2\rho \ell \kappa^2} \right)^{1/2} \ln \left(a \left(\frac{a \Delta P}{2\rho \ell} \right)^{1/2} / \nu \right)$$

Convenient to define:

drive

$$\lambda = \frac{2a \Delta P / \ell}{\frac{1}{2} \rho u^2} \rightarrow \text{friction factor} \, / \, \text{resistance coefficient}$$

⇒ taking $Re = 2aU/\nu$

can re-write friction law as:

$$1/\sqrt{\lambda} = .88 \ln\left(Re\sqrt{\lambda}\right) - .85$$

phenom.

$Re \doteq 2aU/\nu$

$$\lambda = \frac{2a\,\Delta p/\ell}{1/2\,\rho U^2}$$

→ good fit to pipe flow data.

Turbulent Wakes, Thermal Boundary Layers

Here:

→ turbulent wakes, completes wake story begun earlier
- → bckgnd
- → Scaling
- → eddy mixing

→ Thermal BL / Heat Transfer

- → bckgnd, set up, types
  - Pr
- → heat transfer problem
  - — heat transfer coeff
  - — Nu
  - — laminar, turbulent
- → intro to temp fluctn turbulence — passive scalar.

References : Boundary layers, wakes, heat transfer

→ Landau & Lifshitz : excellent, 'physicist style' treatment of these 'engineering' subjects

→ V. Krasnov : Good summary, many examples

→ H. Tennekes, J. Lumley : Basic discussion, 'Good first course'.

→ S. B. Pope : classic Engineering text. Detailed zoology.

B.) Wakes - Simple physics

cf: Prandtl - Tietjens, Falkovich, Lautrup

Wake is:

- region of departure from potential flow behind object moving thru water and experiencing drag

- wake is inexorably coupled to drag

- Message of wakes:

A little ν forces a global adjustment of flow structure

- drag - thinking in frame where object at rest, drag results from loss of flow momentum to object.

What happens at wall ?

⇒ viscous sublayer / cut-off of inertial layer ?

∴ when : $\nu_T < \nu$ { molecular viscosity dominates mixing

$\Rightarrow \quad u_* x \lesssim \nu$

$$x \lesssim \nu / u_* \equiv x_0 \qquad \rightarrow \qquad x_0 = \frac{\nu}{u_*}$$

x_0: viscous sublayer scale. → dissip'n scale

In viscous sublayer, flow linear :

$$\nu \frac{\partial u}{\partial x} = u_*^2$$

$$\therefore \quad u = \frac{u_*^2 x}{\nu}$$

⇒ note effect of turbulence is to:

- flatten profile — { higher transport at fixed wall stress
- reduce central velocity
- limit Q (quality factor)

[redacted]

27.

(ii) Turbulent Wakes $Re \sim UR/\nu \gg 1$

$$\underline{U}\cdot\nabla\underline{v} + \underline{v}\cdot\nabla\underline{v} - \cancel{\nu\nabla^2\underline{v}} = -\frac{\nabla p}{\rho}$$

ignore

$$\Rightarrow \quad \frac{U}{x} v_x \sim \frac{\tilde{v}_y}{W} v_x$$

wave spreads by advection, not diffusion

$\tilde{v}_y \sim$ turbulent velocity

$$\boxed{W \sim \frac{\tilde{v}_y}{U} x}$$

Take wake turbulence isotropic,

so $\boxed{\tilde{v}_x \sim \tilde{v}_y}$.

Fair? Test?

$$W \sim x\tilde{v}_x/U$$

but from drag:

$$\tilde{v}_x \sim F_d/\rho U W^2$$

$\Rightarrow$

[redacted]

28.

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$$W \sim x \frac{F_d}{\rho u^2 W^2} \sim x\left(F_d/\rho u^2 W^2\right)$$

$$W^3 \sim F_d\, x/\rho u^2$$

$$\Rightarrow \boxed{\begin{aligned} W &\sim \left(F_d/\rho u^2\right)^{1/3} x^{1/3} \\ &\sim \left(C_D R^2\right)^{1/3} x^{1/3} \end{aligned}}$$

then, comparing widths:

laminar: $w/R \sim (x/R)^{1/2}\, Re^{-1/2}$

$Re \sim uR/\nu$

turbulent: $w/R \sim (x/R)^{1/3}\, C_D^{1/3}$

interestingly Laminar wake expands with downstream length more rapidly!

6,

29 [redacted]

Why?

→ turbulence can relax ∇V behind object (due separation) rapidly, and faster than ν. Thus surrounding flow penetrates the dead water region more rapidly.

Also observe: <u>Wake</u> Re drops with x.

⇒

$$Re \sim \frac{w\,v_y}{\nu} \sim \frac{w\,V_x}{\nu} \sim \frac{\cancel{w}}{\nu}\,\frac{F_d}{\rho u w^2}$$

y direction (spr)

wake flow Re

$$Re \sim F_d/\rho u w\,\nu$$

$$\sim \frac{u^{\cancel{2}} R^2 \cancel{\rho_0} C_D}{\nu\,\cancel{\rho_0}\,\cancel{u}\,(C_D R^2)^{1/3} x^{1/3}}$$

$C_D \sim 1$

$$\sim \left(\frac{uR}{\nu}\right)\left(R/x\right)^{1/3}$$

$$\boxed{Re(x) \sim Re_0 \, (R/x)^{1/3}}$$

and $Re(x) \to 1$ at

$$x_L \sim R \, (Re_0)^3$$

distance behind boat where turbulent wake transitions to laminar

i.e. akin l_d: transition from turbulent mixing to viscous mixing

N.B. In wake, vortical/rotational region can expand into irrotational region, but never reverse!

i.e. would really violate H-Thm ...

Wakes - Supplement Skip

→ Revisit turbulent wake, using turbulent viscosity, i.e.

$$w \sim (\nu x / u)^{1/2} \qquad (\nu \to D_T)$$

$$\to (D_T x / u)^{1/2}$$

i.e. width of turbulent wake set by turbulent diffn, following Blasius Law

but $D_T \sim w \tilde{v}$ ⇒ turbulent viscosity at mixing length level.

$$\sim w \left(F_d / \rho u w^2 \right)$$

$$\sim F_d / \rho u w \quad \sim \quad \text{const} / w$$

⇒

$$w \sim \left(F_d x / \rho u^2 w \right)^{1/2}$$

$$w^{3/2} \sim \left(F_d / \rho u^2 \right)^{1/2} x^{1/2} \sim (C_D R^2)^{1/2} x^{1/2}$$

$$\sim C_D^{1/2} R \, x^{1/2}$$

$$w \sim (C_D)^{1/3} R^{2/3} x^{1/3}$$

B2.

$$\Rightarrow \boxed{w/R \sim c_D^{1/3} (x/R)^{1/3}}$$ agrees ✓.

Now, $D_T \sim \tilde{v} w$

$$\sim \frac{(\tilde{v} w^2)}{w}$$

$$\sim \frac{\rho u \tilde{v} w^2}{\rho u w}$$

$$\sim \frac{Q}{w} \sim Q/R\,(x/R)^{1/3}$$

∴ – Point is that turbulent viscosity, mixing drops downstream, relative to constant viscous mixing.

– follows from $\tilde{v} w \sim \underbrace{Q}_{\to\,\text{const.}} / w$

– explains why turbulent wake spreads more slowly than laminar wake.

→ Some Observations re: Wake Flows

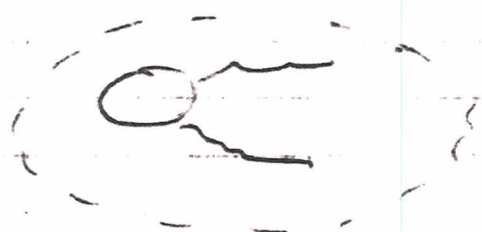
→ note,

$$F_x = -\rho U \int_{\text{wake}} v_x \, dy \, dz$$

Now $Q = \rho \int_{\text{[redacted]}} V_x \, dy \, dz$

↓
mass flow due wake
⇒ deficit.

→ but if encircle body



$\rho \int \underline{v} \cdot d\underline{a} = 0$ i.e. continuity!

Now total $\underline{V}$ → $\begin{cases} \text{velocity field} \\ \text{departure from } \underline{U} \end{cases}$

= vertical wake flow + potential flow.

so, must have $\underline{V}_{\text{pot flow}}$ s/t

$$\int \underline{V} \cdot da = Q/\rho$$

then, for area at r:

$$v \pi r^2 \sim Q/\rho$$

$$\Rightarrow \quad v \sim Q/r^2$$

$$\phi \sim Q/r$$

global adjustment in potential flow due wake/viscosity (localized)

Message:

A little v forces a global adjustment in flow structure.

Thermal Boundary Layer + Heat Transfer

Consider stationary (in mean sense) flow + heat conduction

$$\cancel{\partial_t T} + \underline{v}\cdot\underline{\nabla} T = \chi \nabla^2 T$$

χ → thermal diffusivity

$$\chi = K/\rho c_p$$

K → heat conductivity

$$\rho\, \underline{v}\cdot\underline{\nabla}\underline{v} = -\underline{\nabla} P + \rho\nu\nabla^2\underline{v} + \cancel{\underline{g}\rho}$$

So: dimensionless #

→ Re, as usual

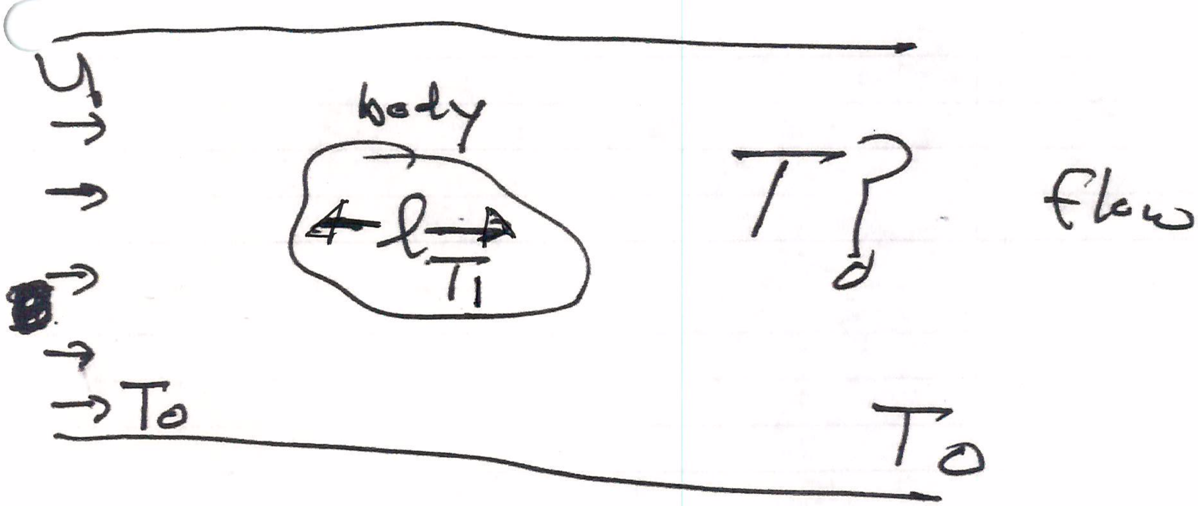
→ $Pr = \nu/\chi$

for now exclude buoyancy

n.b. → if buoyancy, $Ra = \dfrac{g\alpha(\delta T)L^3}{\nu\chi}$

Ra → Rayleigh #.

Now, generic problem:



- body scale ℓ_1 at temp T_1
- incoming flow $\underline{U}$, at T_0

⇒ what is temp field?

<u>i.e.</u> can flow cool body??

$$\frac{T-T_0}{T_1-T_0} = f^*\left(\frac{\underline{r}}{\ell}, Re, Pr\right)$$

downstr.

$$\frac{\underline{V}}{U} = f(\underline{r}/\ell, Re)$$

incomp.

is scaling of result,

Furthur ways of keeping score:

→ if concerned with cooling body

$\Gamma\!\!\to$ surface heat flux of body.

$$h = \alpha = q / (T_1 - T_0)$$

$q \sim -\chi_T \nabla T$

h ↓ heat transfer coefficient / effectiveness

T_1 ↓ body T

T_0 ↳ flow T

as $q = -k \nabla T \Big|_{\text{surface}}$ ⇒ [h is strongly tied to boundary layer dynamics.

→ dim-less ratio:

$$N = \frac{h\ell}{k} \sim \frac{D_{\text{eddy, Thermal}}}{\chi} \sim \frac{\chi_{\text{eddy}}}{\chi}.$$

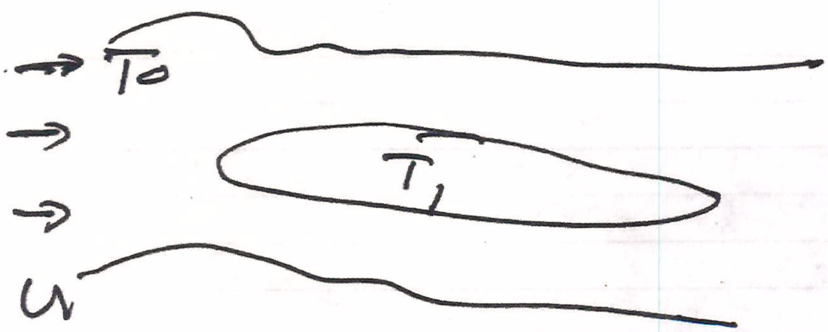
N ↓ Nusselt #

so

$$N = f(Re, P) \quad \text{for B-L heat transfer.}$$

N.B.: Note trade-offs in cooling problem
i.e. resistance of pipe, heat transfer.

so ①



How does Nu
scale in
laminar BL?

↕

How effective is
laminar flow in cooling?

$$q = -k \frac{\partial T}{\partial n}\Big|_{\text{bndry}}$$

$$\sim k \frac{(T_1 - T_0)}{\delta}$$ → surface heat flux

δ → bndry layer width

but, we know for laminar BL,

$$\delta \sim \ell / (Re)^{1/2}$$ i.e. Blasius.

so for $P \sim 1$.

$$Nu \sim \frac{hl}{k} \sim \left(\frac{q}{T_i - T_o}\right) l/k$$

$$\sim \frac{k(T_i - T_o)}{\delta} \; \frac{l}{k\,(T_i - T_o)}$$

$$\sim \sqrt{Re}$$

so

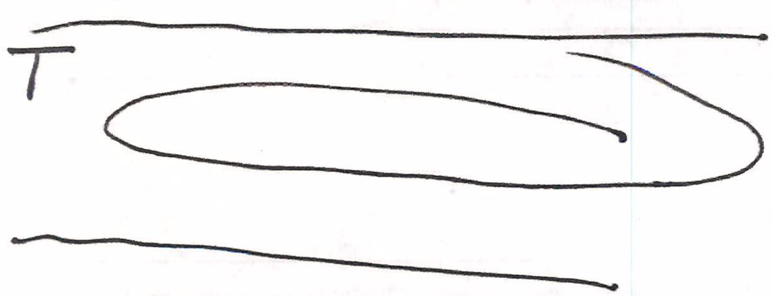
$$\boxed{N \simeq \sqrt{Re}\; f(P_r)}$$ → Nusselt number

$$\boxed{h \sim \frac{k}{l}\sqrt{Re}}$$ → heat transfer coeff.

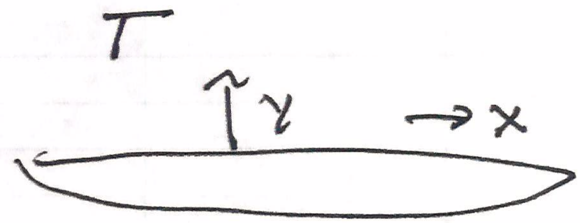
~ (note size scaling)

~ note C_p importance!

② Turbulent B.L.



sufficient to calculate temp field in flow.



$$q = -K_T \frac{dT}{dy}$$

thermal eddy vis.

th sublayer

$$K_T = \rho c_p \underbrace{V_* y}_{\nu_T}$$

$V_* \rightarrow$ friction velocity for BL

so

$$dT/dy = \frac{q}{\rho c_p V_*} / y$$

turb. boundary layer for Temp field.

$$\boxed{T = \frac{q}{\rho c_p V_*} \ln (y/y_0) + f(P)}$$

$y_0 = \nu / V_*$

additional diddle const may enter.

$(P_r \sim 1)$

$$N = \frac{K_T}{K} \sim \frac{V_* a}{K}$$

→ And, flow is turbulent, with temp fluctuations.

Production: $-\frac{Q}{T_0} \cdot \frac{\nabla T}{T_0} \rightarrow \frac{d}{dt} \frac{\tilde{T}^2}{T_0^2}$

$$\sim \left(\frac{\tilde{T}}{T_0} \right)^2 \frac{V}{L}$$

$$\equiv \alpha$$

so

$$\alpha \equiv \frac{v(\ell)}{\ell} \tilde{t}(\ell)^2$$

$$\sim \frac{\epsilon^{1/3}}{\ell^{2/3}} \tilde{t}(\ell)^2 \Rightarrow \tilde{t}(\ell) \sim \ell^{1/3} \frac{\alpha^{1/2}}{\epsilon^{1/6}}$$

$$t(\ell)^2 \sim \left(\alpha / \epsilon^{1/3} \right) \ell^{2/3} \rightarrow k^{-5/3}$$

$Pr >$
$\nu > \kappa$
$\tilde{T}$ sees smooth flow, small scales.
$\frac{\nu}{\kappa}$

i.r. scaling for $\tilde{T}/T$ fluctns.

but ? $Pr \ll 1, \gg 1$ → how reconcile dissipn ranges?

one field may see flow smooth! TBC.