Zonal Flows and Drift Wave Turbulence: A Look Back and a Look Ahead with Emphasis on L→H Transition Dynamics

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Expanded Version: Alfven Prize Lecture
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With additional input from K. Miki

Dedication

- To Marshall N. Rosenbluth
  - for fundamental contributions to this topic and to numerous others
  - for dedication indispensible to the world fusion program and the realization of ITER

Gratitude

- Family: Mei, Louise, Peter
  Harold and Patricia Diamond (deceased), Harold Jr.

- Mentors: T.H. Dupree, M.N. Rosenbluth (deceased)

- Collaborators on this topic: B.A. Carreras, S.-I. Itoh, K. Itoh, T.S. Hahm
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Outline

A) A Look Back and A Look Around: Basic Ideas of the Drift Wave-Zonal Flow System

B) A Look Ahead: Current Applications to Selected Problems of Interest

C) Focus on L→H Transition: Current Developments and Issues for Fest '11

A) A Look Back and A Look Around

Basic Ideas of the Drift Wave – Zonal Flow System

i) Physics of Zonal Flow Formation

ii) Shearing Effects on Turbulence Transport

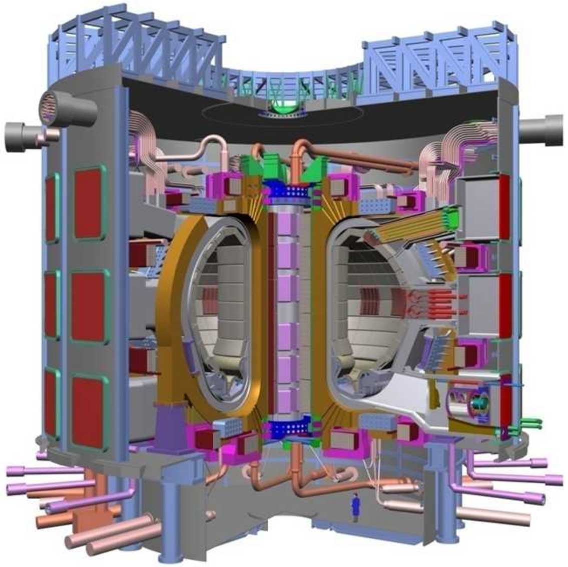
iii) Closing the Feedback Loops: Predator(s) Meet Prey

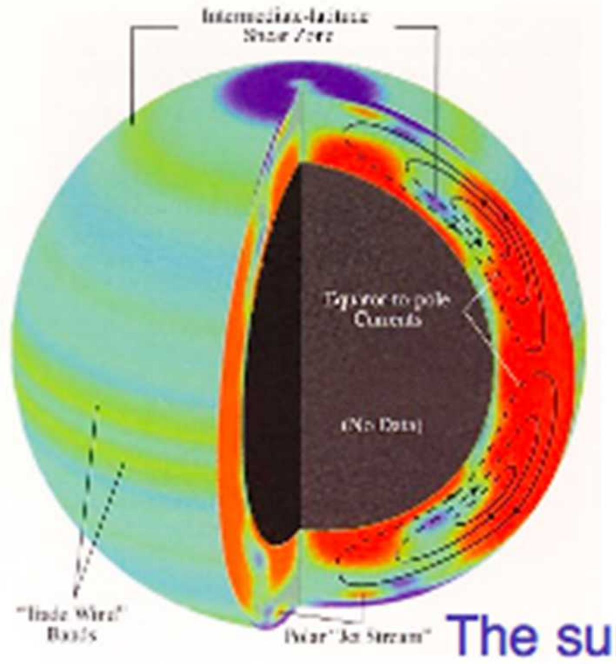
> “The difference between an idea and a theory is that the first can generate a call to action and the second cannot.”
>
> — Stanley Fish

Preamble I

- Zonal Flows Ubiquitous for:
  - ~ 2D fluids / plasmas $R_0 < 1$

    Rotation $\vec{\Omega}$, Magnetization $\vec{B}_0$, Stratification → waves
  - Ex: MFE devices, giant planets, stars…





The sun

Preamble II

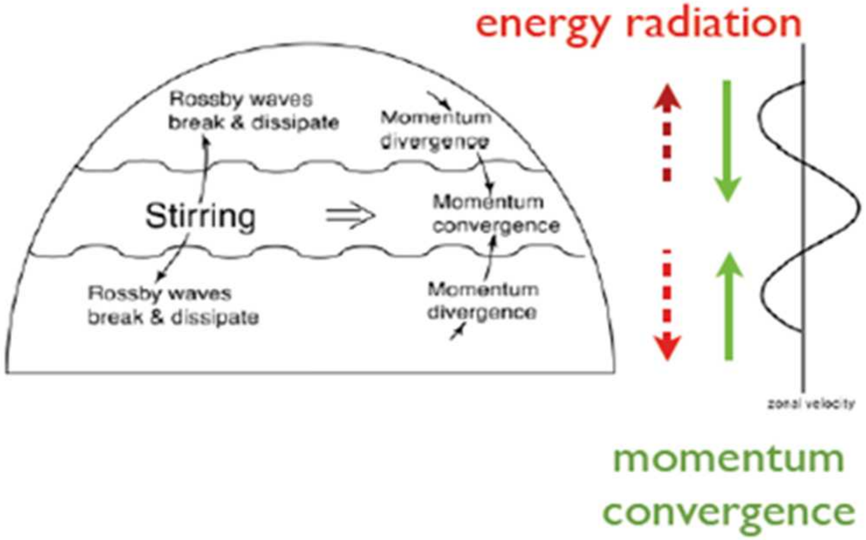
- What is a Zonal Flow?
  - *n = 0* potential mode; *m = 0* (ZFZF), with possible sideband (GAM)
  - toroidally, poloidally symmetric *ExB* shear flow
- Why are Z.F.'s important?
  - Zonal flows are secondary (nonlinearly driven):
    - modes of minimal inertia (Hasegawa et. al.; Sagdeev, et. al. '78)
    - modes of minimal damping (Rosenbluth, Hinton '98)
    - drive zero transport (*n = 0*)
  - natural predators to feed off and retain energy released by gradient-driven microturbulence

Preamble III

Heuristics of Zonal Flows a):

Simplest Possible Example: Zonally Averaged Mid-Latitude Circulation

- classic GFD example: Rossby waves + Zonal flow (c.f. Vallis '07, Held '01)
- Key Physics:



Rossby Wave:

$$\omega_k = -\frac{\beta k_x}{k_\perp^2}$$

$$v_{gy} = 2\beta \frac{k_x k_y}{k_\perp^2} \quad \left\langle \tilde{v}_y \tilde{v}_x \right\rangle = \sum_k -k_x k_y \left| \hat{\phi}_{\vec{k}} \right|^2$$

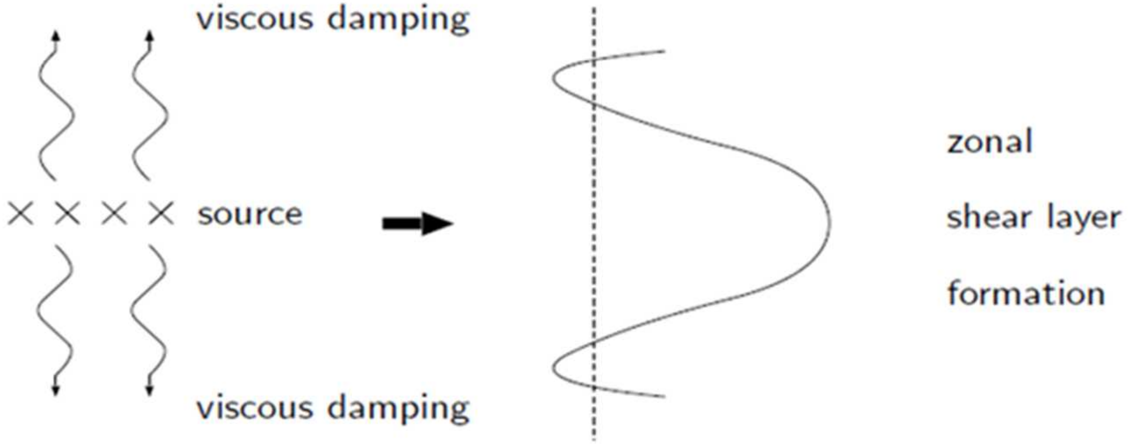
$$\therefore \; v_{gy} v_{phy} < 0$$

→ Backward wave!

⇒ Momentum convergence at stirring location

Preamble IV

- …"the central result that a rapidly rotating flow, when stirred in a localized region, will converge angular momentum into this region." (I. Held, '01)
- Outgoing waves ⇒ incoming wave momentum flux



viscous damping

source

viscous damping

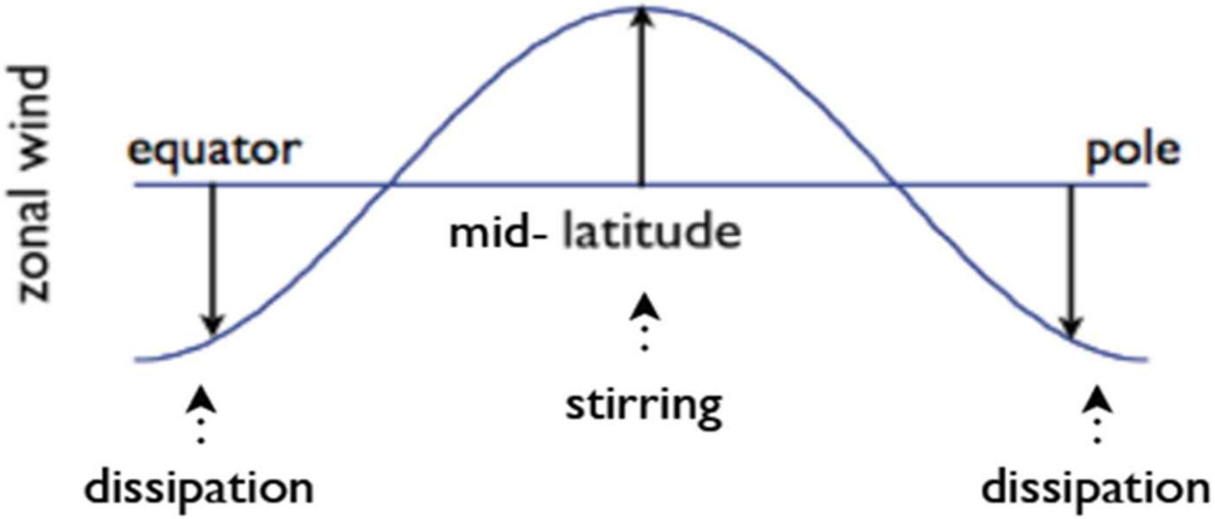
zonal

shear layer

formation

- Local Flow Direction (northern hemisphere):
  - eastward in source region
  - westward in sink region
  - set by $\beta > 0$
  - Some similarity to spinodal decomposition phenomena → both `negative diffusion' phenomena

Preamble V



Key Point: Finite Flow Structure requires *separation* of

excitation and dissipation regions.

=> Spatial structure and wave propagation within are central.

→ momentum transport by waves

Preamble VI

Key Elements:

- Waves → propagation transports momentum ↔ stresses

  → modest-weak turbulence

- vorticity transport → momentum transport → Reynolds force

  → the Taylor Identity

- Irreversibility → outgoing wave boundary conditions
- symmetry breaking → direction, boundary condition

  $\rightarrow \beta$

- Separation of forcing, damping regions

  → need damping region broads than source region

  → akin intensity profile...

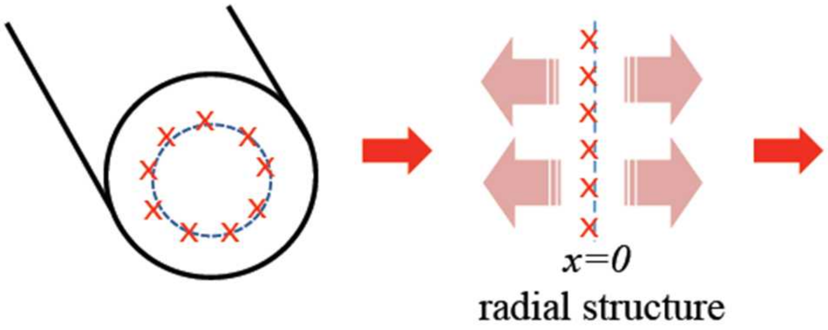
All have obvious MFE counterparts...

Preamble VII

Heuristics of Zonal Flows b.)

2) MFE perspective on Wave Transport in DW Turbulence

- localized source/instability drive intrinsic to drift wave structure



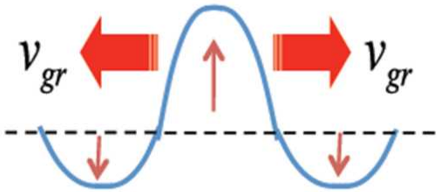
– couple to damping ↔ outgoing wave
i.e. Pearlstein-Berk eigenfunction

$x > 0 \Rightarrow v_{gr} > 0$

– $v_{gr} = -2\rho_s^2 \dfrac{k_\theta k_r v_*}{(1+k_\perp^2\rho_s^2)^2}$ $\qquad v_* < 0 \rightarrow k_r k_\theta > 0$

– $\langle v_{rE} v_{\theta E}\rangle = -\dfrac{c^2}{B^2}|\phi_{\vec{k}}|^2 k_r k_\theta < 0$

- outgoing wave energy flux → incoming wave momentum flux → counter flow spin-up!



- zonal flow layers form at excitation regions

Zonal Flows I

- Fundamental Idea:
  - Potential vorticity transport + 1 direction of translation symmetry
    → Zonal flow in magnetized plasma / QG fluid
  - Kelvin's theorem is ultimate foundation

- G.C. ambipolarity breaking → polarization charge flux → Reynolds force
  - Polarization charge → $-\rho^2\nabla^2\phi = n_{i,GC}(\phi) - n_e(\phi)$

    polarization length scale *ion GC* *electron density*

  - so $\Gamma_{i,GC} \neq \Gamma_e$ → $\rho^2\left\langle \tilde{v}_{rE}\nabla_\perp^2\tilde{\phi} \right\rangle \neq 0$ ↔ 'PV transport'

    polarization flux → What sets cross-phase?

  - If 1 direction of symmetry (or near symmetry):

    $$-\rho^2\left\langle \tilde{v}_{rE}\nabla_\perp^2\tilde{\phi} \right\rangle = -\partial_r\left\langle \tilde{v}_{rE}\tilde{v}_{\perp E} \right\rangle$$ (Taylor, 1915)

    $-\partial_r\left\langle \tilde{v}_{rE}\tilde{v}_{\perp E} \right\rangle$ → Reynolds force → Flow

Additional Comments I

- Heresy: Rigorous “inverse cascade” concept does not seem fundamental?! Well known that Z.F.’s develop on scale of flux, spectral inhomogeneity (not necessarily ‘large’)

- c.f. S. Tobias, et. al. ApJ 2011 → ZF’s appear without higher order cumulants

Additional Comments II

- Mechanisms for PV mixing: A Partial List
  - direct dissipation, as by $\gamma\nabla^2$
  - forward potential enstrophy cascade → couple to $\gamma\nabla^2$
  - local: wave absorption at critical layers, where $\omega = k_y \langle V_x(y) \rangle$

    global: overlap of neighboring 'cat's eyes' islands

    → streamline stochastization
  - nonlinear wave-fluid element interaction (akin NLLD)

Zonal Flows II

- Potential vorticity transport and momentum balance
  - Example: Simplest interesting system → Hasegawa-Wakatani
    - Vorticity: $\frac{d}{dt}\nabla^2\phi = -D_\parallel\nabla_\parallel^2(\phi - n) + D_0\nabla^2\nabla^2\phi$
    - Density: $\frac{dn}{dt} = -D_\parallel\nabla_\parallel^2(\phi - n) + D_0\nabla^2 n$

$$\left\{\begin{array}{l} D_0 \ \ \text{classical, feeble} \\ \Pr = 1 \quad \text{for simplicity} \end{array}\right.$$

  - Locally advected PV: $q = n - \nabla\phi^2$
    - PV: charge density $\left\{\begin{array}{l} n \rightarrow \text{guiding centers} \\ -\nabla\phi^2 \rightarrow \text{polarization} \end{array}\right.$
    - conserved on trajectories in inviscid theory $\boxed{dq/dt=0}$
    - PV conservation → $\left.\begin{array}{l} \text{Freezing-in law} \\ \text{Kelvin's theorem} \end{array}\right\} \rightarrow$ Dynamical constraint

Zonal Flow II, cont'd

- Potential Enstrophy (P.E.) balance

$$d\langle q^2\rangle / dt = 0$$

P.E. flux

small scale dissipation

$\langle\ \rangle$ → coarse graining

$$\text{LHS} \Rightarrow \frac{d}{dt}\langle \tilde{q}^2\rangle \equiv \partial_t\langle \tilde{q}^2\rangle + \partial_r\langle \tilde{V}_r \tilde{q}^2\rangle + D_0\langle (\nabla\tilde{q})^2\rangle$$

$\text{RHS} \Rightarrow \text{P.E. evolution} - \langle \tilde{V}_r \tilde{q}\rangle\langle q\rangle' \Rightarrow$ P.E. Production by PV mixing / flux

- PV flux: $\langle \tilde{V}_r \tilde{q}\rangle = \langle \tilde{V}_r \tilde{n}\rangle - \langle \tilde{V}_r \nabla^2\tilde{\phi}\rangle;\ \text{but}: \langle \tilde{V}_r \nabla^2\tilde{\phi}\rangle = \partial_r\langle \tilde{V}_r \tilde{V}_\theta\rangle$

∴ P.E. production directly couples driving transport and flow drive

- Fundamental Stationarity Relation for Vorticity flux

$$\langle \tilde{V}_r \nabla^2\tilde{\phi}\rangle = \langle \tilde{V}_r \tilde{n}\rangle + (\delta_t\langle \tilde{q}^2\rangle)/\langle q\rangle'$$

Reynolds force

Relaxation

Local PE decrement

∴ Reynolds force locked to driving flux and P.E. decrement; transcends quasilinear theory

Zonal Flows III

- Momentum Theorem (Charney, Drazin 1960, et. seq. P.D. et. al. ‘08)

$$\partial_t\left\{(GWMD)+\langle V_\theta\rangle\right\}=-\langle\tilde{V}_r\tilde{n}\rangle-\delta_t\langle\tilde{q}^2\rangle/\langle q\rangle'-\nu\langle V_\theta\rangle$$

drag

driving flux

Local P.E. decrement

GWMD = Generalized Wave Momentum Density; $-\langle\tilde{q}^2\rangle/\langle q\rangle'$ (pseudomomentum)

- What Does it Mean? “Non-Acceleration Theorem”:

$$\partial_t\left\{(GWMD)+\langle V_\theta\rangle\right\}=-\langle\tilde{V}_r\tilde{n}\rangle-\delta_t\langle\tilde{q}^2\rangle/\langle q\rangle'-\nu\langle V_\theta\rangle$$

  - Absent $\langle\tilde{V}_r\tilde{n}\rangle$ driving flux; $\delta_t\langle\tilde{q}^2\rangle$ — local potential enstrophy decrement

→ cannot $\left\{\begin{matrix}\text{accelerate}\\ \text{maintain}\end{matrix}\right\}$ Z.F. with stationary fluctuations!

- Fundamental constraint on models of stationary zonal flows! ↔ need explicit connection to relaxation, dissipation

Additional Comments

- What of Pr ≠ 1 ? (X.G.) (c.f. P.-C. Hsu et. al. TTF2011)

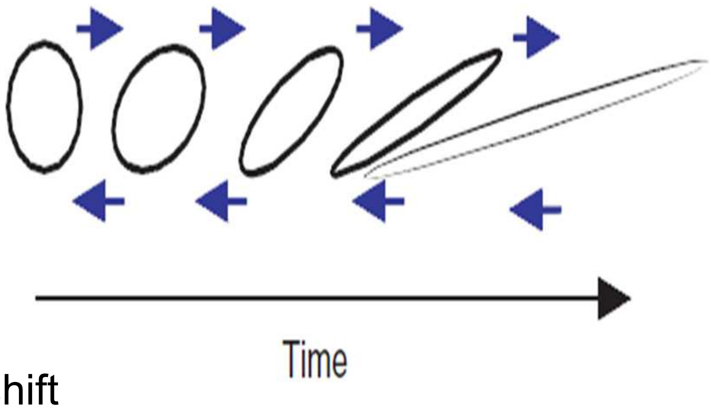
$$\partial_t \left\{ (GWMD) + \langle V_\theta \rangle \right\} = -\left\langle \tilde{V}_r \tilde{n} \right\rangle - \delta_t \left\langle \tilde{q}^2 \right\rangle / \langle q \rangle' - \nu \langle V_\theta \rangle$$
$$- D_0 (\mathrm{Pr} - 1) \left[\left(\nabla \nabla^2 \phi \right)^2 + \left(\nabla^2 \phi \right)^2 \right] / \langle q \rangle'$$

$$\left(\text{for} \quad \tilde{n} = \tilde{\phi} + \tilde{h}, \quad |\tilde{h}| < |\hat{\phi}| \right)$$

- Important: C-D theorems uncover important link between Z.F. and flux drive

Shearing I

- Coherent shearing: (Kelvin, G.I. Taylor, Dupree’66, BDT‘90)
  - radial scattering + $\langle V_E \rangle'$ → hybrid decorrelation
  - $k_r^2 D_\perp \rightarrow (k_\theta^2 \langle V_E \rangle'^2 D_\perp / 3)^{1/3} = 1/\tau_c$
  - shaping, flux compression: Hahm, Burrell ’94



- Other shearing effects (linear):

Response shift and dispersion

  - spatial resonance dispersion: $\omega - k_\parallel v_\parallel \Rightarrow \omega - k_\parallel v_\parallel - k_\theta \langle V_E \rangle'(r - r_0)$ → cross phases!
  - differential response rotation → especially for kinetic curvature effects

→ N.B. Caveat: Modes can adjust to weaken effect of external shear

(Carreras, et. al. ‘92; Scott ‘92)

Shearing II

- Zonal Shears: Wave kinetics (Zakharov et. al.; P.D. et. al. ‘98, et. seq.)

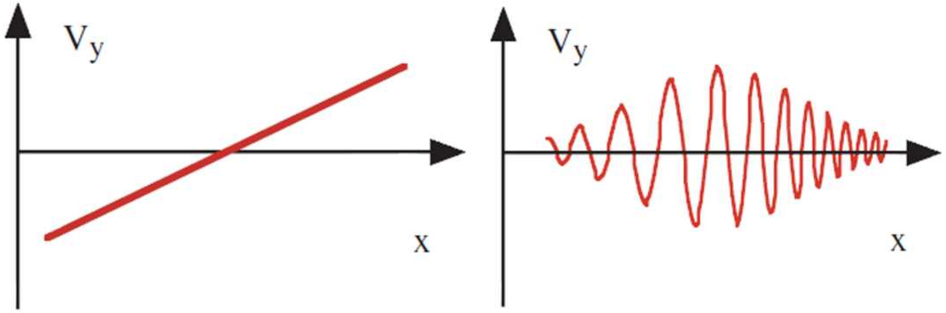
  Coherent interaction approach (L. Chen et. al.)

- $dk_r / dt = -\partial(\omega + k_\theta V_E) / \partial r$; $V_E = \langle V_E \rangle + \widetilde{V}_E$

  Mean shearing $: k_r = k_r^{(0)} - k_\theta V_E' \tau$

  Zonal Random shearing $: \langle \delta k_r^2 \rangle = D_k \tau$

  $$D_k = \sum_q k_\theta^2 \left| \widetilde{V}_{E,q}' \right|^2 \tau_{k,q}$$



- Wave ray chaos (not shear RPA) underlies D_k → induced diffusion
- Induces wave packet dispersion
- Applicable to ZFs and GAMs
- $\tau_{k,q}$ ≡ coherence time of wave packet ***k*** with shear mode ***q***

- Mean Field Wave Kinetics

$$\frac{\partial N}{\partial t} + (\vec{V}_{gr} + \vec{V}) \cdot \nabla N - \frac{\partial}{\partial r}(\omega + k_\theta V_E) \cdot \frac{\partial N}{\partial \vec{k}} = \gamma_{\vec{k}} N - C\{N\}$$

$$\Rightarrow \boxed{\frac{\partial}{\partial t}\langle N \rangle - \frac{\partial}{\partial k_r} D_k \frac{\partial}{\partial k_r} \langle N \rangle = \gamma_{\vec{k}} \langle N \rangle - \langle C\{N\} \rangle}$$

Zonal shearing

Shearing III

- Energetics: Books Balance for Reynolds Stress-Driven Flows!
- Fluctuation Energy Evolution – Z.F. shearing

$$\int d\vec{k}\,\omega\left(\frac{\partial}{\partial t}\langle N\rangle - \frac{\partial}{\partial k_r} D_k \frac{\partial}{\partial k_r}\langle N\rangle\right) \Rightarrow \frac{\partial}{\partial t}\langle \varepsilon\rangle = -\int d\vec{k}\,V_{gr}(\vec{k}) D_{\vec{k}} \frac{\partial}{\partial k_r}\langle N\rangle \qquad V_{gr} = \frac{-2k_r k_\theta V_* \rho_s^2}{\left(1+k_\perp^2\rho_s^2\right)^2}$$

Point: For $d\langle\Omega\rangle / dk_r < 0$, Z.F. shearing damps wave energy

N.B.: For zonal shears, *N ~ Ω*

- Fate of the Energy: Reynolds work on Zonal Flow

Modulational Instability

$$\partial_t \delta V_\theta + \partial\left(\delta\left\langle \tilde{V}_r \tilde{V}_\theta \right\rangle\right) / \partial r = -\gamma \delta V_\theta$$

$$\delta\left\langle \tilde{V}_r \tilde{V}_\theta \right\rangle \sim \frac{k_r k_\theta \delta\Omega}{(1+k_\perp^2\rho_s^2)^2}$$

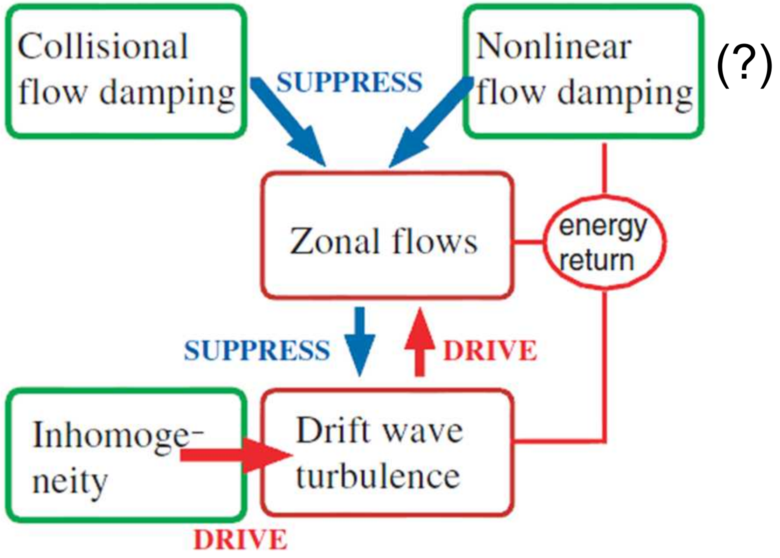
N.B.: Wave decorrelation essential:
Equivalent to PV transport/mixing
(c.f. Gurcan et. al. 2010)

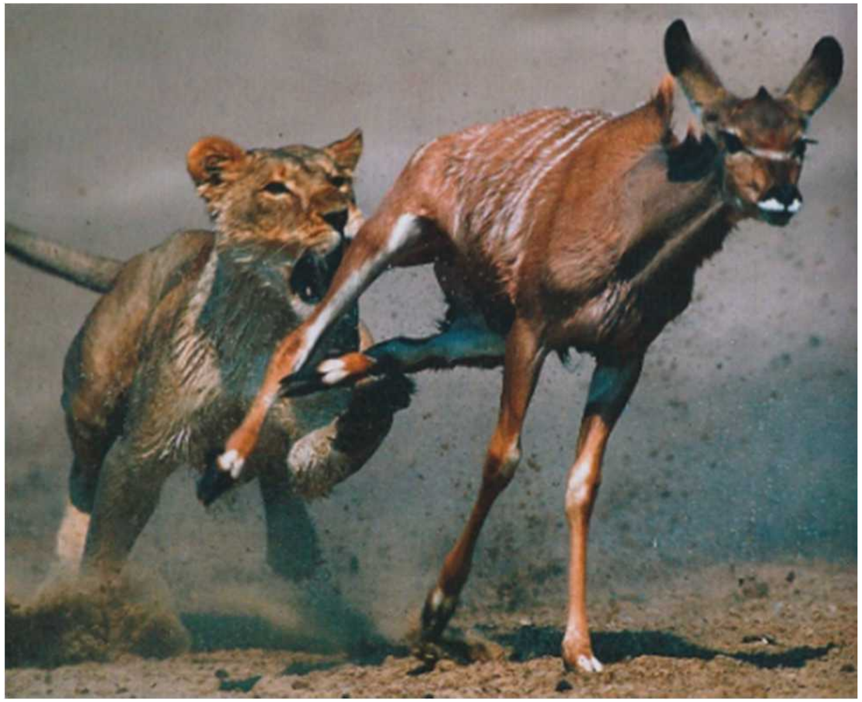
- Bottom Line:
  - Z.F. growth due to shearing of waves
  - “Reynolds work” and “flow shearing” as relabeling → books balance
  - Z.F. damping emerges as critical; MNR ‘97

Feedback Loops I

- Closing the loop of shearing and Reynolds work

- Spectral 'Predator-Prey' equations





Prey → Drift waves, <*N*>

$$\frac{\partial}{\partial t}\langle N\rangle - \frac{\partial}{\partial k_r} D_k \frac{\partial}{\partial k_r}\langle N\rangle = \gamma_k\langle N\rangle - \frac{\Delta\omega_k}{N_0}\langle N\rangle^2$$

Predator → Zonal flow, $|\phi_q|^2$

$$\frac{\partial}{\partial t}|\phi_q|^2 = \Gamma_q\left[\frac{\partial\langle N\rangle}{\partial k_r}\right]|\phi_q|^2 - \gamma_d|\phi_q|^2 - \gamma_{NL}[|\phi_q|^2]|\phi_q|^2$$

Additional Comment (Philosophy/History)

- Historically, plasma community efforts have benefited from, but trailed, GFD community

- Some evidence for equalization in recent years:

  i.e. “zonostrophic turbulence” B. Galperin, et. al. 2007 →

  akin to coupled wave packets + Z.F. system

Feedback Loops II

- Recovering the 'dual cascade':
  - Prey → <N> ~ <Ω> ⇒ induced diffusion to high k_r
    - ⇒ Analogous → forward potential enstrophy cascade; PV transport
  - Predator → $|\phi_q|^2 \sim \langle V_{E,\theta}^2 \rangle$
    - ⇒ growth of *n=0, m=0* Z.F. by turbulent Reynolds work
    - ⇒ Analogous → inverse energy cascade

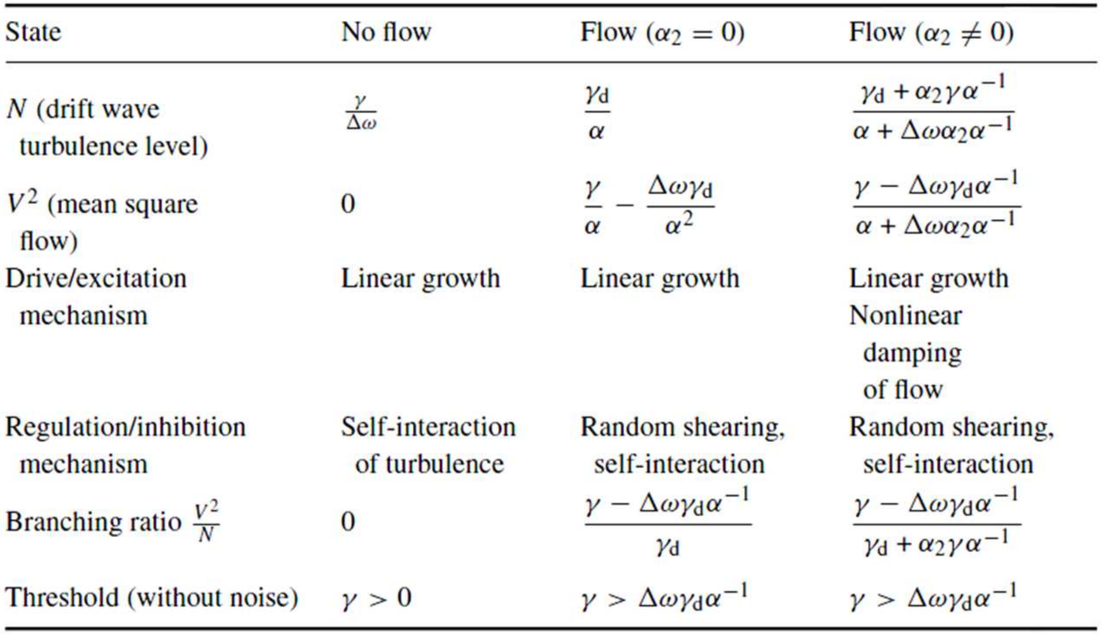
- Mean Field Predator-Prey Model

  (P.D. et. al. '94, DI^{2}H '05)

$$\frac{\partial}{\partial t} N = \gamma N - \alpha V^2 N - \Delta\omega N^2$$

$$\frac{\partial}{\partial t} V^2 = \alpha N V^2 - \gamma_d V^2 - \gamma_{NL}(V^2) V^2$$

System Status

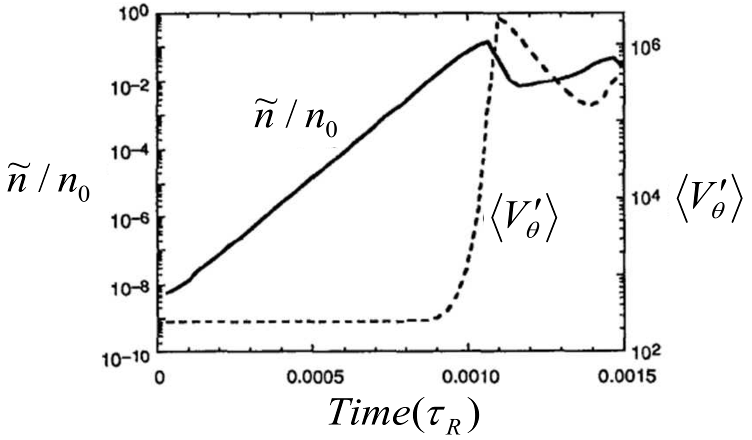


State	No flow	Flow ($\alpha_2 = 0$)	Flow ($\alpha_2 \neq 0$)
N (drift wave turbulence level)	$\frac{\gamma}{\Delta\omega}$	$\frac{\gamma_d}{\alpha}$	$\frac{\gamma_d + \alpha_2\gamma\alpha^{-1}}{\alpha + \Delta\omega\alpha_2\alpha^{-1}}$
V^2 (mean square flow)	0	$\frac{\gamma}{\alpha} - \frac{\Delta\omega\gamma_d}{\alpha^2}$	$\frac{\gamma - \Delta\omega\gamma_d\alpha^{-1}}{\alpha + \Delta\omega\alpha_2\alpha^{-1}}$
Drive/excitation mechanism	Linear growth	Linear growth	Linear growth Nonlinear damping of flow
Regulation/inhibition mechanism	Self-interaction of turbulence	Random shearing, self-interaction	Random shearing, self-interaction
Branching ratio $\frac{V^2}{N}$	0	$\frac{\gamma - \Delta\omega\gamma_d\alpha^{-1}}{\gamma_d}$	$\frac{\gamma - \Delta\omega\gamma_d\alpha^{-1}}{\gamma_d + \alpha_2\gamma\alpha^{-1}}$
Threshold (without noise)	$\gamma > 0$	$\gamma > \Delta\omega\gamma_d\alpha^{-1}$	$\gamma > \Delta\omega\gamma_d\alpha^{-1}$

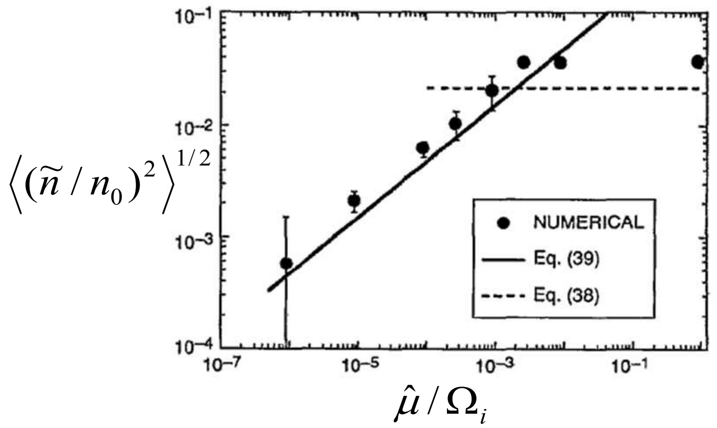
Feedback Loops II

- Early simple simulations confirmed several aspects of modulational predator-prey dynamics

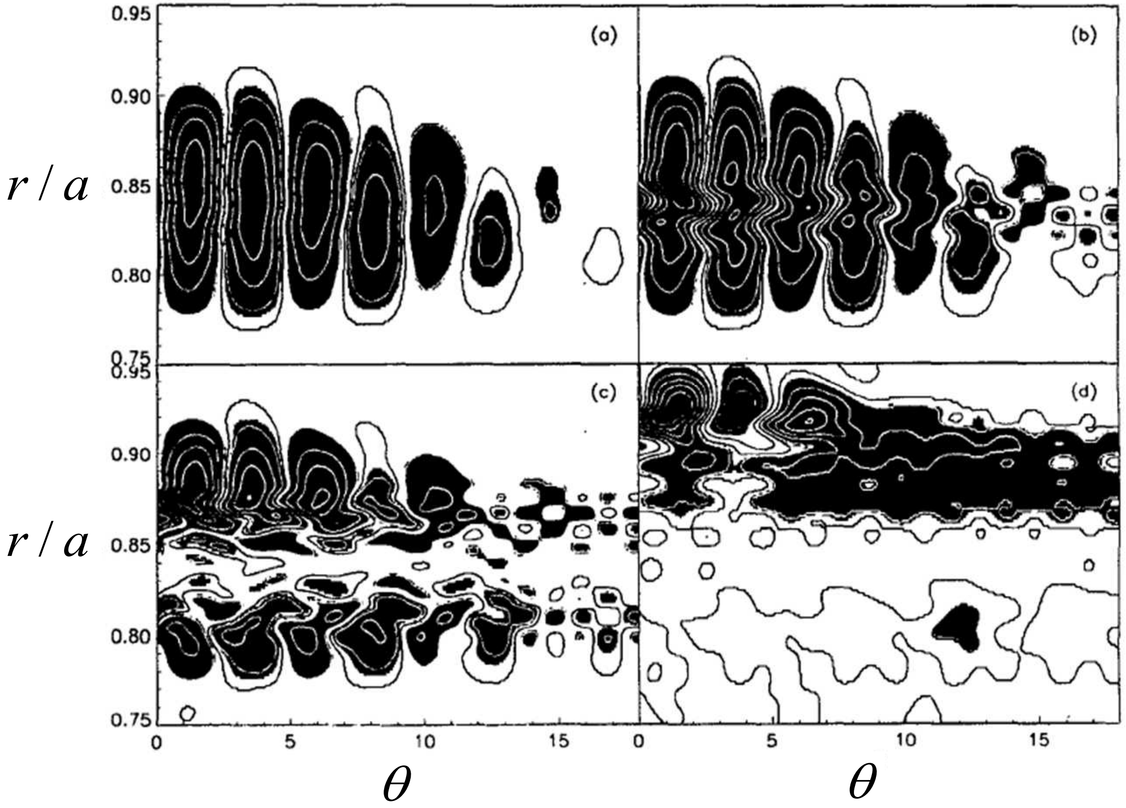
(L. Charlton et. al. '94)



Shear flow grows above critical point



'With Flow' and 'No Flow'.
Scalings of $\left\langle (\widetilde{n}/n_0)^2 \right\rangle$ appear. Role of damping evident



Generic picture of fluctuation scale reduction with flow shear

Additional Comment (Eternal)

- What of collisionless Z.F. damping, saturation?

  Some candidates and comments:

  - instability ↔ (G)KH → magnetic shear → feable (!?)
  - trapping / spectral transition → multi-packets (?)
  - feedback on PV flux → cross-phase in $\left\langle \widetilde{V}_r \widetilde{V}_\theta \right\rangle$

⇒ Can we extract a general lesson?

Feedback Loops III

- ∇P coupling
  - γ_L drive
  - $\langle V_E \rangle'$

$$\partial_t \varepsilon = \overset{\downarrow}{\varepsilon N} - a_1 \varepsilon^2 - a_2 V^2 \varepsilon - a_3 V_{ZF}^2 \varepsilon$$

$$\partial_t V_{ZF} = b_1 \frac{\varepsilon V_{ZF}}{1 + b_2 V^2} - b_3 V_{ZF}$$

$$\partial_t N = -c_1 \varepsilon N - c_2 N + Q$$

$\mathcal{E}$ ≡ *DW energy*

V_{ZF} ≡ *ZF shear*

$N \equiv \nabla\langle P \rangle$ ≡ *pressure gradient*

$V = dN^2$ *(radial force balance)*

- Simplest example of 2 predator + 1 prey problem (E. Kim, P.D., 2003 see also: Malkov, P.D., 2009)

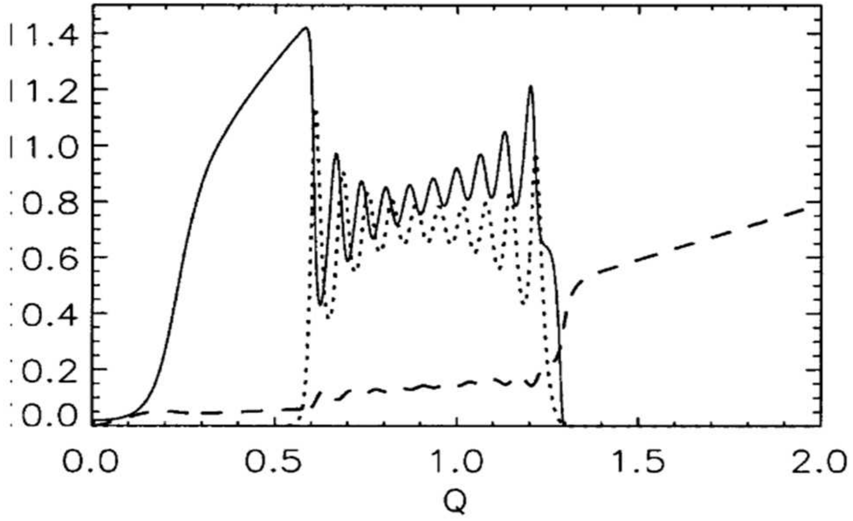
  i.e. prey sustains predators
  predators limit prey } usual feedback

  now:
  - 2 predators (*ZF*, $\nabla\langle P \rangle$) compete
  - $\nabla\langle P \rangle$ as both drive and predator
  - avalanches → multiplicative noise

- Relevance: LH transition, ITB
  - Builds on insights from Itoh's, Hinton
  - ZF ⇒ triggers
  - $\nabla\langle P \rangle$ ⇒ 'locking in'

Multiple predators are possible

Feedback Loops III, cont'd

- 

Solid - $\mathcal{E}$

Dotted - V_{ZF}

Dashed - $\nabla\langle P\rangle$

- Observations:
  - ZF's trigger transition, $\nabla\langle P\rangle$ and $\langle V\rangle$ lock it in
  - Period of dithering, pulsations .... during ZF, $\nabla\langle P\rangle$ oscillation as Q ↑
  - ☼ Phase between $\mathcal{E}$, V_{ZF} , $\nabla\langle P\rangle$ varies as Q increases
  - $\nabla\langle P\rangle \Leftrightarrow$ ZF interaction $\Rightarrow$ effect on wave form
  - Back transition: need not re-visit I-phase

B) A Look Ahead

Current Applications to Selected Problems of Interest

Progress

i) Zonal Flows with RMP

ii) β-plane MHD and the Solar Tachocline

Provocation

i) The PV and ExB Staircase

ii) Zonal flows and spreading: Help or Hinder?

Pinnacle

Dynamics of the L→H Transition

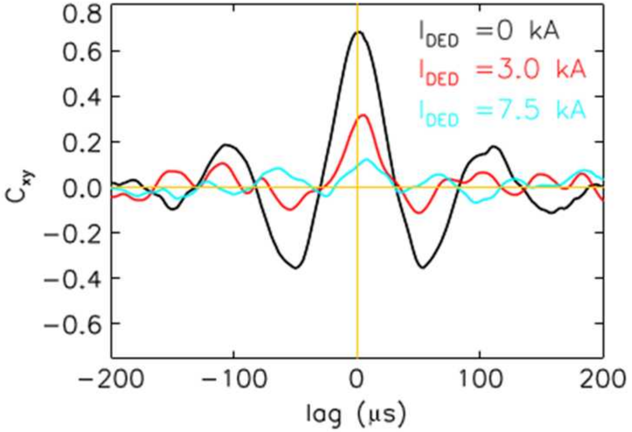
“What bifurcations, made by funksters, like mushrooms sprout both far and wide”

— Vladimir Sorokin, in “Day of the Oprichnik”

Progress I: ZF's with RMP (with M. Leconte)

- ITER 'crisis du jour': ELM Mitigation and Control
- Popular approach: RMP
- ? Impact on Confinement?

Y. Xu '11



⇒ RMP causes drop in fluctuation LRC, suggesting reduced Z.F. shearing

⇒ What is "cost-benefit ratio" of RMP?

- Physics:
  - in simple H-W model, polarization charge in zonal annulus evolves according:

$$\frac{dQ}{dt} = -\int dA\left[\left\langle \tilde{v}_x \tilde{\rho}_{pol}\right\rangle + \left(\frac{\delta B_r}{B_0}\right)^2 D_{\|} \frac{\partial}{\partial x}\left(\langle\phi\rangle - \langle n\rangle\right)\right]\Bigg|_{r_1}^{r_2}$$

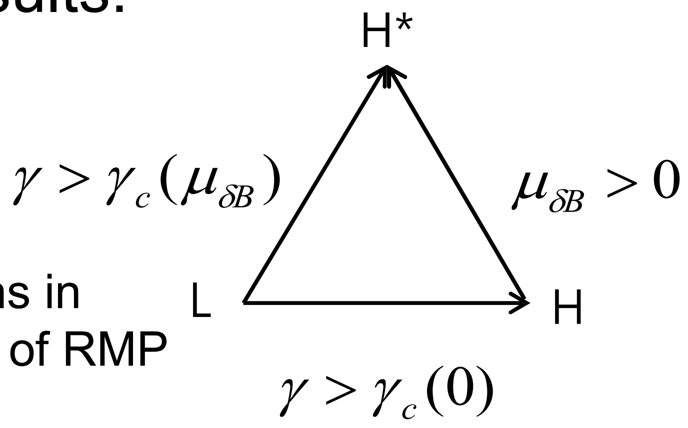
  - Key point: δB_r of RMP induces radial electron current → enters charge balance

Progress I, cont'd

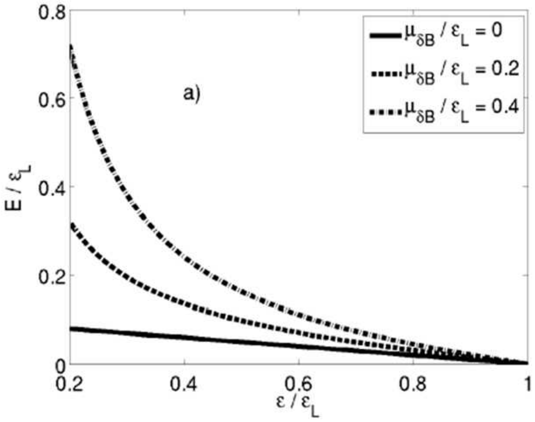
- Implications
  - δB_r linearly couples zonal $\hat{\phi}$ and zonal $\hat{n}$
  - Weak RMP → correction, strong RMP → $\langle E_r \rangle_{ZF} \cong -T_e \partial_r \langle n \rangle / |e|$
- Equations: $\frac{d}{dt}\delta n_q + D_T q^2 \delta n_q + i b_q (\delta\phi_q - (1-c)\delta n_q) - D_{RMP} q^2 (\delta\phi_q - \delta n_q) = 0$

$$\frac{d}{dt}\delta\phi_q + \mu\delta\phi_q - a_q(\delta\phi_q - (1-c)\delta n_q) + \frac{D_{RMP}}{\rho_s^2}(\delta\phi_q - \delta n_q) = 0$$

- Results:



Transitions in presence of RMP



$E_{ZF}/\mathcal{E}_L$ vs $\mathcal{E}/\mathcal{E}_L$ for various RMP coupling strengths

Progress II : β-plane MHD (with S.M. Tobias, D.W. Hughes)

Model

- Thin layer of shallow magneto fluid, i.e. solar tachocline
- β-plane MHD ~ 2D MHD + β-offset i.e. solar tachocline

$$\partial_t \nabla^2 \phi + \nabla \phi \times \hat{z} \cdot \nabla \nabla^2 \phi - \nu \nabla^2 \nabla^2 \phi = \beta \partial_x \phi + B_0 \partial_x \nabla^2 A + \nabla A \times \hat{z} \cdot \nabla \nabla^2 A + \widetilde{f}$$

$$\partial_t A + \nabla \phi \times \hat{z} \cdot \nabla A = B_0 \partial_x \phi + \eta \nabla^2 A \qquad \vec{B}_0 = B_0 \hat{x}$$

- Linear waves: Rossby – Alfven $\omega^2 + \omega\beta \frac{k_x}{k^2} - k_x^2 V_A^2 = 0$ (R. Hide)
- cf P.D., et al; Tachocline volume, CUP (2007)

  S. Tobias, et al: ApJ (2007)

Progress II, cont’d

Observation re: What happens?

- Turbulence → stretch field → $\langle \tilde{B}^2 \rangle >> B_0^2$ i.e. $\langle \tilde{B}^2 \rangle / B_0^2 \sim R_m$

  (ala Zeldovich)

- Cascades : - forward or inverse?

  - MHD or Rossby dynamics dominant !?

- PV transport: $\frac{dQ}{dt} = -\int dA \langle \tilde{v} \tilde{q} \rangle$ ⟶ net change in charge content due PV/polarization charge flux

Now $$\frac{dQ}{dt} = -\int dA \left[\underset{\uparrow}{\langle \tilde{v} \tilde{q} \rangle} - \underset{\uparrow}{\langle \tilde{B}_r \tilde{J}_\parallel \rangle} \right] = -\int dA\, \partial_x \left\{ \langle \tilde{v}_x \tilde{v}_y \rangle - \langle \tilde{B}_x \tilde{B}_y \rangle \right\} \longrightarrow$$ Reynolds mis-match

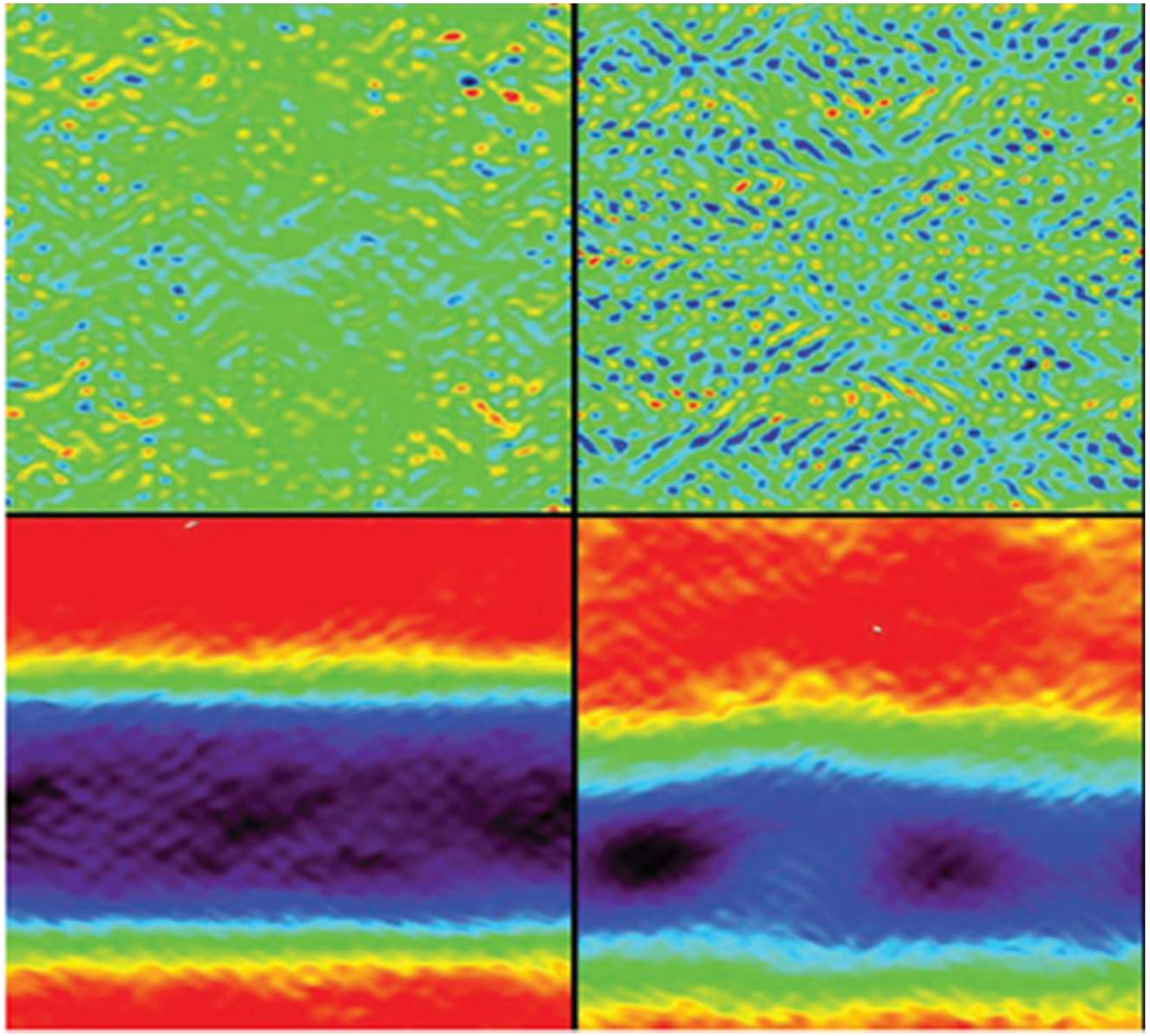
PV flux   current along tilted lines

⟶ vanishes for Alfvenized state

Taylor: $\langle \tilde{B}_x \tilde{J}_\parallel \rangle = -\partial_x \langle \tilde{B}_x \tilde{B}_y \rangle$

Progress II, cont'd

- **With Field**



$B_0 = 10^{-1}$

$B_0 = 10^{-2}$

$B_0 = 0$

$B_0 = 10^{-3}$

Progress II, cont'd

- Control Parameters for $\vec{\tilde{B}}$ enter Z.F. dynamics

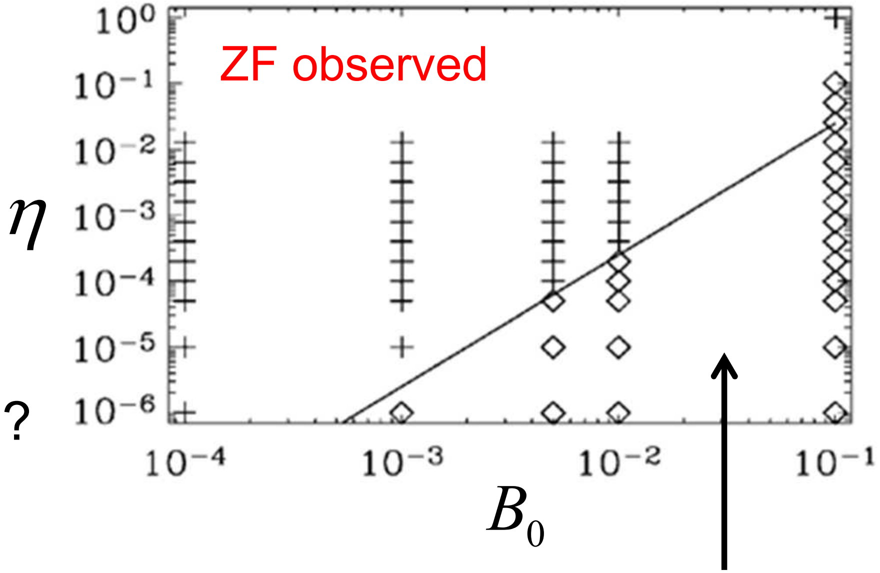
  Like RMP, Ohm's law regulates Z.F.

- Recall
  - $\langle \tilde{v}^2 \rangle$ vs $\langle \tilde{B}^2 \rangle$
  - $\langle \tilde{B}^2 \rangle \sim B_0^2 R_m \longrightarrow$ origin of B_0^2 / η scaling !?

- Further study → differentiate between :
  - cross phase in $\langle \tilde{v}_r \tilde{q} \rangle$ and O.R. vs J.C.M
  - orientation : $\vec{B} \parallel \vec{V}$ vs $\vec{B} \perp \vec{V}$
  - spectral evolution

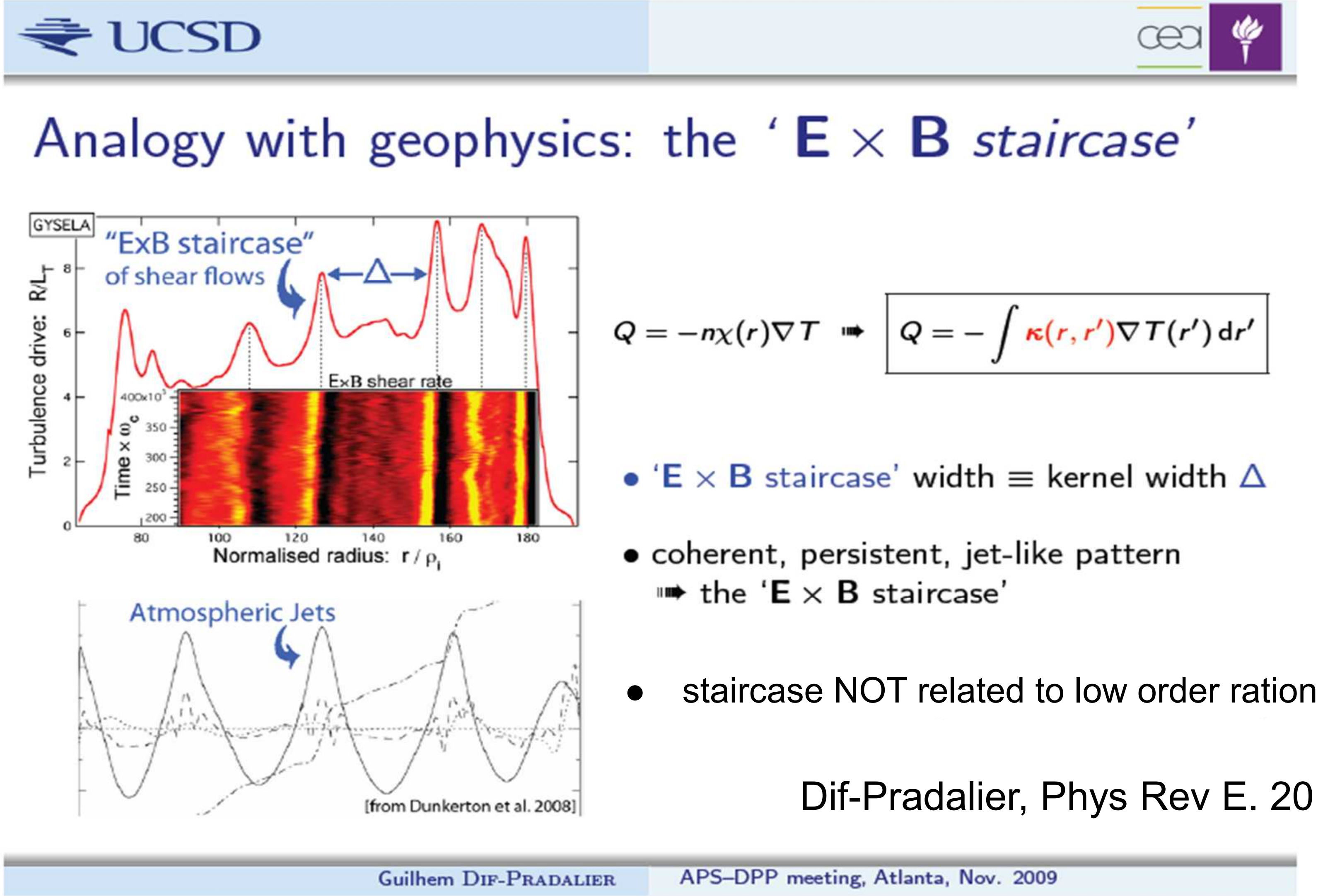
\+ = zonal flow state

◇ = no zonal flow state



No ZF observed

Provocation I: Staircase and Nonlocality
(with G. Dif-Pradalier, et. al.)



UCSD

Analogy with geophysics: the '**E** × **B** *staircase*'

$$Q = -n\chi(r)\nabla T \Rightarrow \boxed{Q = -\int \kappa(r,r')\nabla T(r')\,dr'}$$

- '**E** × **B** staircase' width ≡ kernel width Δ
- coherent, persistent, jet-like pattern
  ➠ the '**E** × **B** staircase'
- staircase NOT related to low order rationals!

Dif-Pradalier, Phys Rev E. 2010

Guilhem Dif-Pradalier APS–DPP meeting, Atlanta, Nov. 2009

Provocation I, cont'd

- The point:
  - fit: $Q = -\int dr' \kappa(r,r') \nabla T(r')$ $\kappa(r,r') \sim \dfrac{S^2}{(r-r')^2 + \Delta^2}$ → some range in exponent
  - $\Delta >> \Delta_c$ i.e. $\Delta \sim$ Avalanche scale >> $\Delta_c \sim$ correlation scale
  - Staircase 'steps' separated by Δ ! → stochastic avalanches produce quasi-regular flow pattern!?

    N.B.

    - The notion of a staircase is not new – especially in systems with natural periodicity (i.e. NL wave breaking…)
    - What IS new is the connection to stochastic avalanches, independent of geometry
  - What is process of self-organization linking avalanche scale to zonal pattern step?

    i.e. How extend predator-prey feedback model to encompass both avalanche and zonal flow staircase? Self-consistency is crucial!

Provocation II: Z.F.'s + Spreading Help or Hinder

- DO ZONAL FLOWS HELP OR HINDER SPREADING? If promote, how effective?
- The conflict:
  - natural expectation re: shearing
  - symmetry breaking effect on wave packet propagation
  - purely non-local interaction (in scale)
  - non-local + local interaction

Provocation II: Z.F.'s + Spreading Help or Hinder

- Zonal spreading
  - MECHANISM is LINEAR GROUP PROPAGATOION
  - i.e. for Rossby waves

$$\omega = -\frac{\beta k_x}{\mathbf{k}^2}, \quad v_g = \frac{2\beta k_x k_y}{(\mathbf{k}^2)^2}$$

for symmetric spectrum $<k_x k_y> = 0 \rightarrow <v_{gy}> = 0$ no propagation

- if zonal shear: $\frac{d}{dt} k_y = -\partial_y \left(k_x \langle v_x \rangle \right)$

$$k_y = k_{y0} - \int k_x \langle v_x \rangle' dt$$

$$\therefore v_{gy} = -2\beta k_x^2 \int \langle v_x \rangle' dt / \left(\mathbf{k}^2\right)^2$$

- shear "correlates" k_y, k_x → no ambiguity in $<k_x k_y>$ but
- inertia k^2 increase in time → efficiency?

Provocation II: Z.F.'s + Spreading Help or Hinder

- Zonal spreading, cont'd
  - n.b. not sufficient to establish propagation, need to establish/quantify:
    a. penetration, i.e. how far does turbulence penetrate into stable/damped region?
    b. efficiency, i.e. how much of initial source is radiated?

- analysis must include: growth/damping profiles and dissipation

- analysis should be non-perturbative, i.e. NLS models will miss enhanced inertia

Provocation II: Z.F.'s + Spreading Help or Hinder

Model and Analysis

- 1D, eikonal → **asymptotic, but non-perturbative**
- w =pseudomomentum → akin to wave momentum density

$$\partial_t w + \underset{\substack{\downarrow \\ \text{group propagation}}}{\partial_y(v_{gr,y} w)} = \left(\underset{\substack{\downarrow \\ \text{growth}}}{\gamma(y)} - \underset{\substack{\downarrow \\ \text{damping}}}{D_0(y) k_\perp^2}\right) w \tag{1}$$

$$v_{gr,y} = \frac{2\beta k_x k_y}{(k_\perp^2)^2}$$

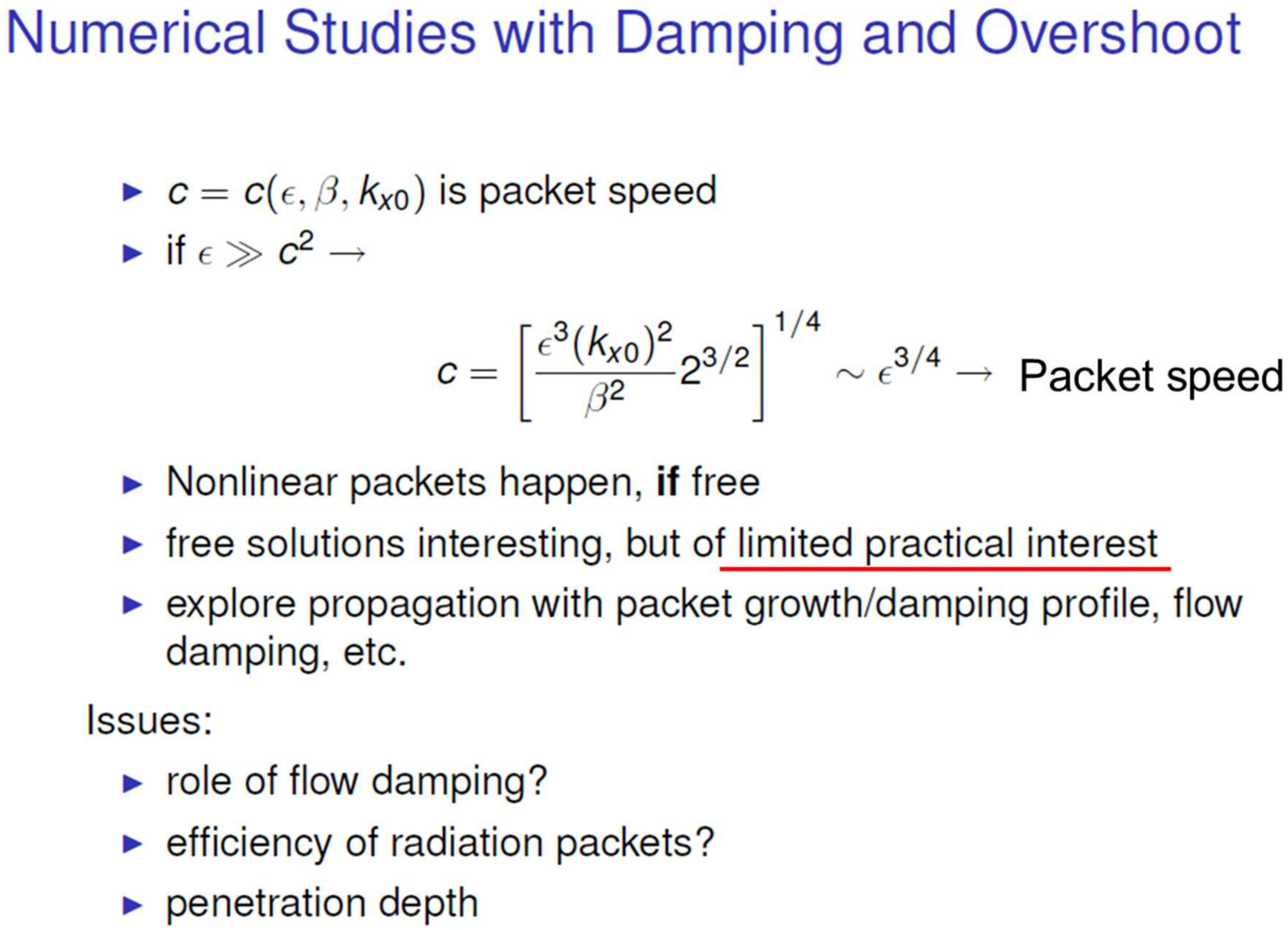
$$\begin{aligned} \partial_t \langle v_x \rangle &= -\partial_y \langle \underset{\updownarrow}{v_y^{'} v_x^{'}} \rangle - \nu \overset{\substack{\text{drag, critical} \\ \downarrow}}{\langle v_x \rangle} \quad \text{Reynolds stress} \quad (2) \\ &\qquad\qquad\qquad\qquad\qquad\qquad\ \updownarrow \\ &= \partial_y (v_{gr,y} w) - \nu \langle v_x \rangle \quad \text{pseudomomentum flux} \end{aligned}$$

- n.b. $\partial_t(\langle v_x \rangle + w)$ =growth/damping → momentum conservation

Provocation II: Z.F.'s + Spreading Help or Hinder

- Model and Analysis II
  - Eikonal equation → straining : $\frac{dk_y}{dt} = -k_{x0}\partial_y\langle v_x\rangle + D\nabla^2 k_y$
  - Free solutions – fronts and propagating nonlinear wave packets
    - take: $D_0, \gamma, \nu, D, etc \rightarrow 0$
    - look for solutions of the form: *f(y-ct)* → nonlinear packets
    - $\frac{c^2}{2k_{x0}^2}\frac{(2\epsilon - c^2)^{1/2}\beta}{\epsilon^2} = 1$ → **exact** speed-amplitude relation

Provocation II: Z.F.'s + Spreading Help or Hinder



Numerical Studies with Damping and Overshoot

- $c = c(\epsilon, \beta, k_{x0})$ is packet speed
- if $\epsilon \gg c^2 \rightarrow$

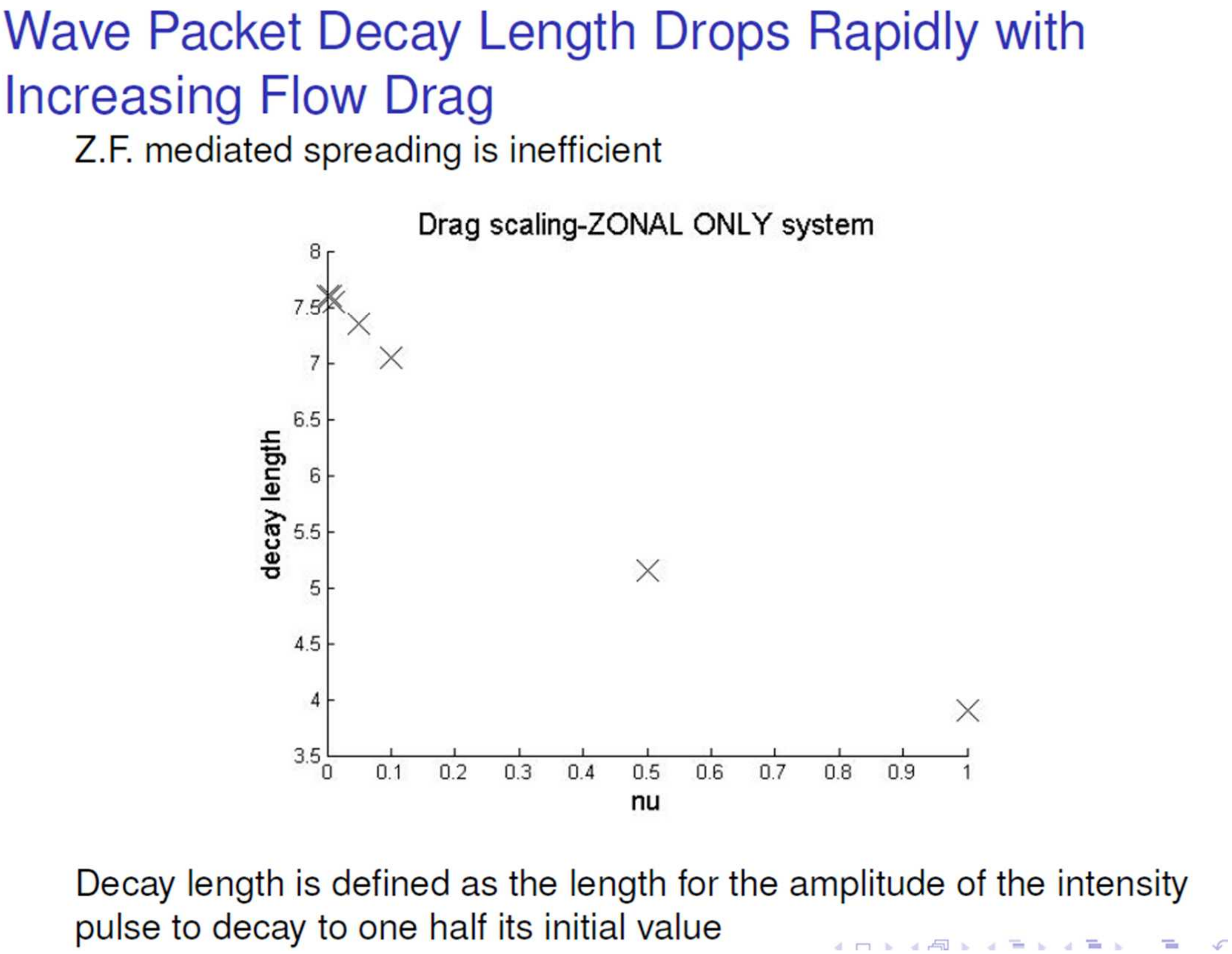
$$c = \left[\frac{\epsilon^3 (k_{x0})^2}{\beta^2} 2^{3/2}\right]^{1/4} \sim \epsilon^{3/4} \rightarrow \text{ Packet speed}$$

- Nonlinear packets happen, **if** free
- free solutions interesting, but of limited practical interest
- explore propagation with packet growth/damping profile, flow damping, etc.

Issues:

- role of flow damping?
- efficiency of radiation packets?
- penetration depth

Provocation II: Z.F.'s + Spreading Help or Hinder



Wave Packet Decay Length Drops Rapidly with Increasing Flow Drag

Z.F. mediated spreading is inefficient

Decay length is defined as the length for the amplitude of the intensity pulse to decay to one half its initial value

Provocation II: Z.F.'s + Spreading Help or Hinder

Local and Zonal Evolution

Comparison Point: Local and Zonal Model

- Recall local scattering/mixing → propagating fronts

$$\partial_t \epsilon - \partial_x D_0 \epsilon \partial_x \epsilon = \gamma \epsilon - \alpha \epsilon^2$$

- Fisher equation with nonlinear diffusion
- resembles $k - \epsilon$ models
- derived via Fokker-Planck theory
- since $\epsilon = \frac{\omega_k}{k_x} w$, can combine local, zonal interactions in w equation

$$\partial_t w + \partial_y \left(v_{gr,y} w \right) - \partial_y \frac{D_0 \beta}{k^2} w \partial_y w + \alpha \frac{\beta}{k^2} w w = \left(\gamma - D_0 k^2 \right) w$$

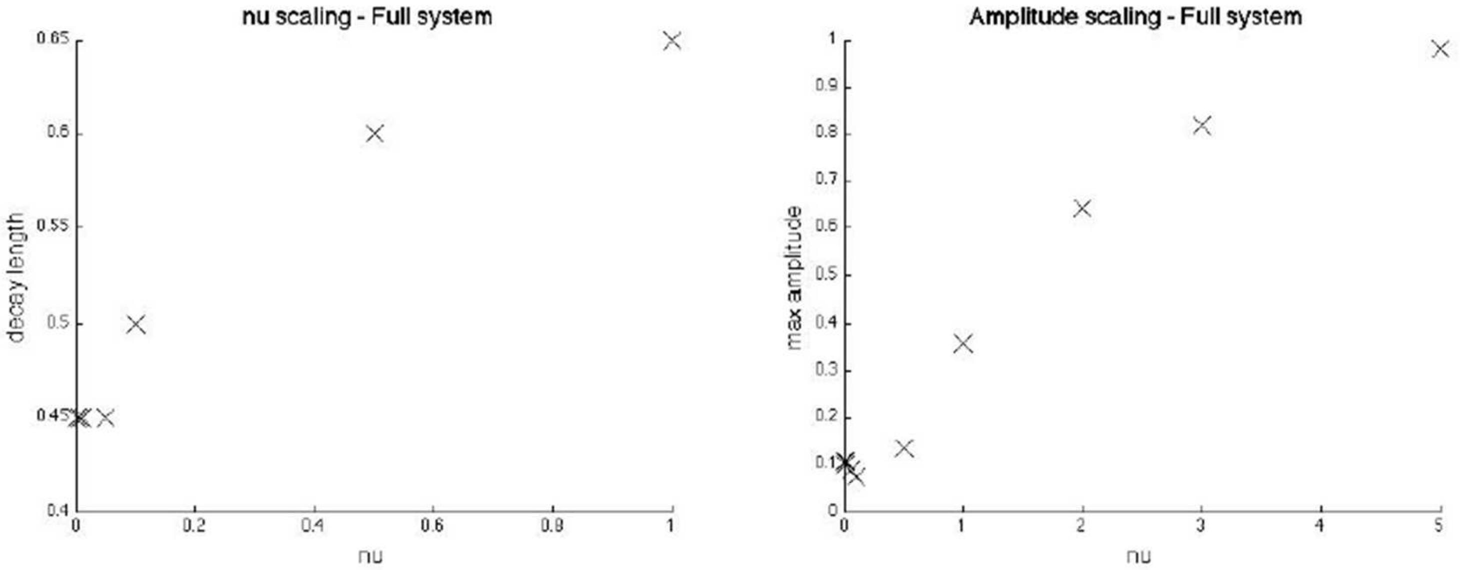
- $\langle v_x \rangle$,k_y equations as before

Note:

- in combined model, energy can propagate by:
  1. zonal coupling → $v_{gr,y} w$
  2. local scattering → $\partial_y \frac{Dw}{k^2} \partial_y w$
- but: local scattering robust, insensitive to zonal flow dissipation, phase relations
- naturally, explore synergy/complementarity

Provocation II: Z.F.'s + Spreading Help or Hinder

Scaling with Flow Drag in combined system



- In contrast to zonal-only system, decay length **increases** with ν. Maximum Envelope Amplitude **increases** with ν
- Local couplings robust to Z.F. damping

Provocation II: Z.F.'s + Spreading Help or Hinder

Bottom Line:

Zonal Flows may help spreading, but only a little…

C) Pinnacle

Z.F.'s and the Dynamics of the L→H Transition

- L→H transition (F. Wagner '82) has driven considerable research on shear flows
- Tremendous progress in recent experiments:

  G. Conway, T. Estrada and C. Hidalgo,

  L. Schmitz, G. McKee and Z. Yan,

  K. Kamiya and K. Ida, G.S. Xu,

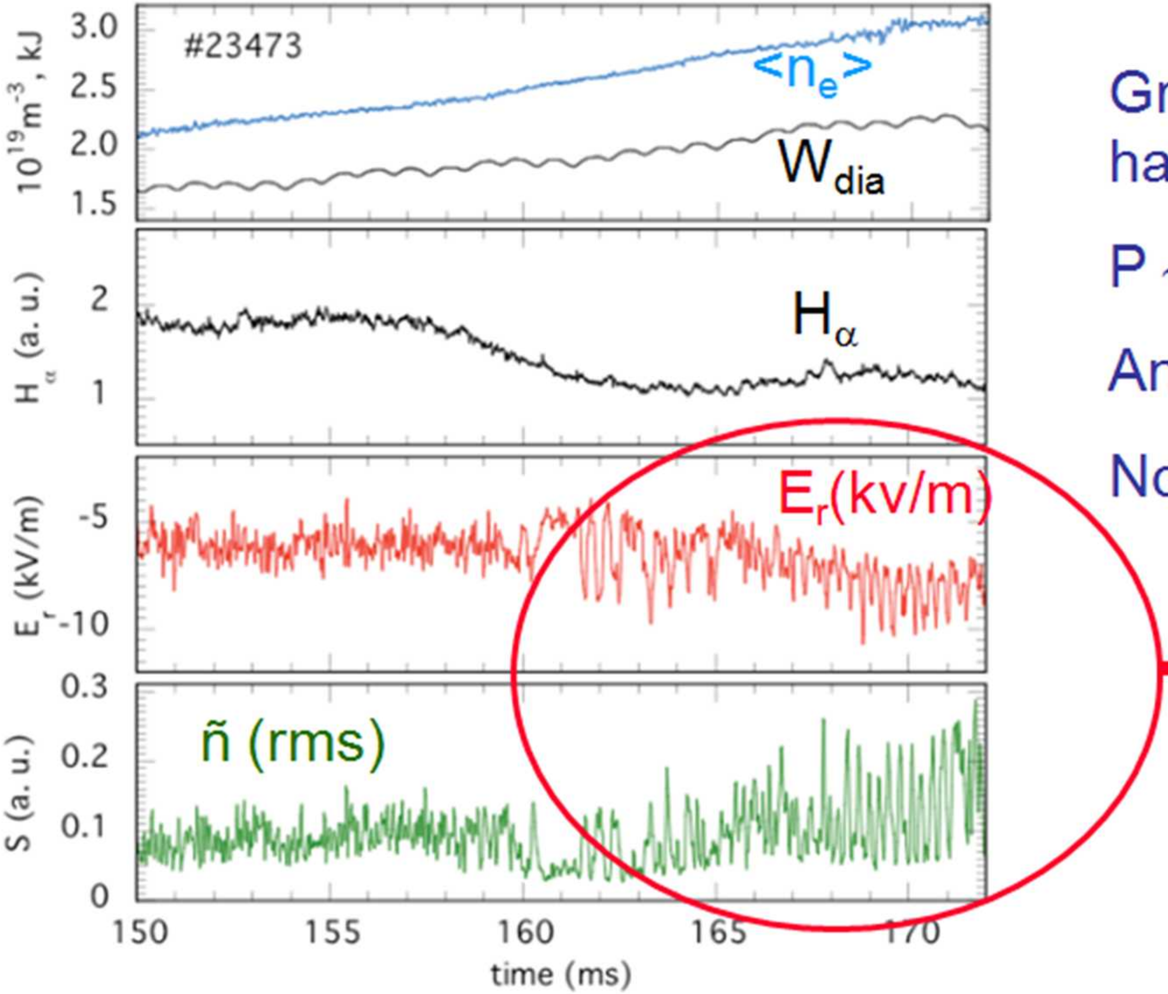
  A. Hubbard, S. J. Zweben

- Seems like we are almost there …

BUT: "It ain't over till its over" – an eastern (division) Yogi

Flows and turbulence dynamics, Gradual L-H transitions

Laboratorio Nacional Fusión



Gradual transitions happen for

$P \sim P_{treshold}$

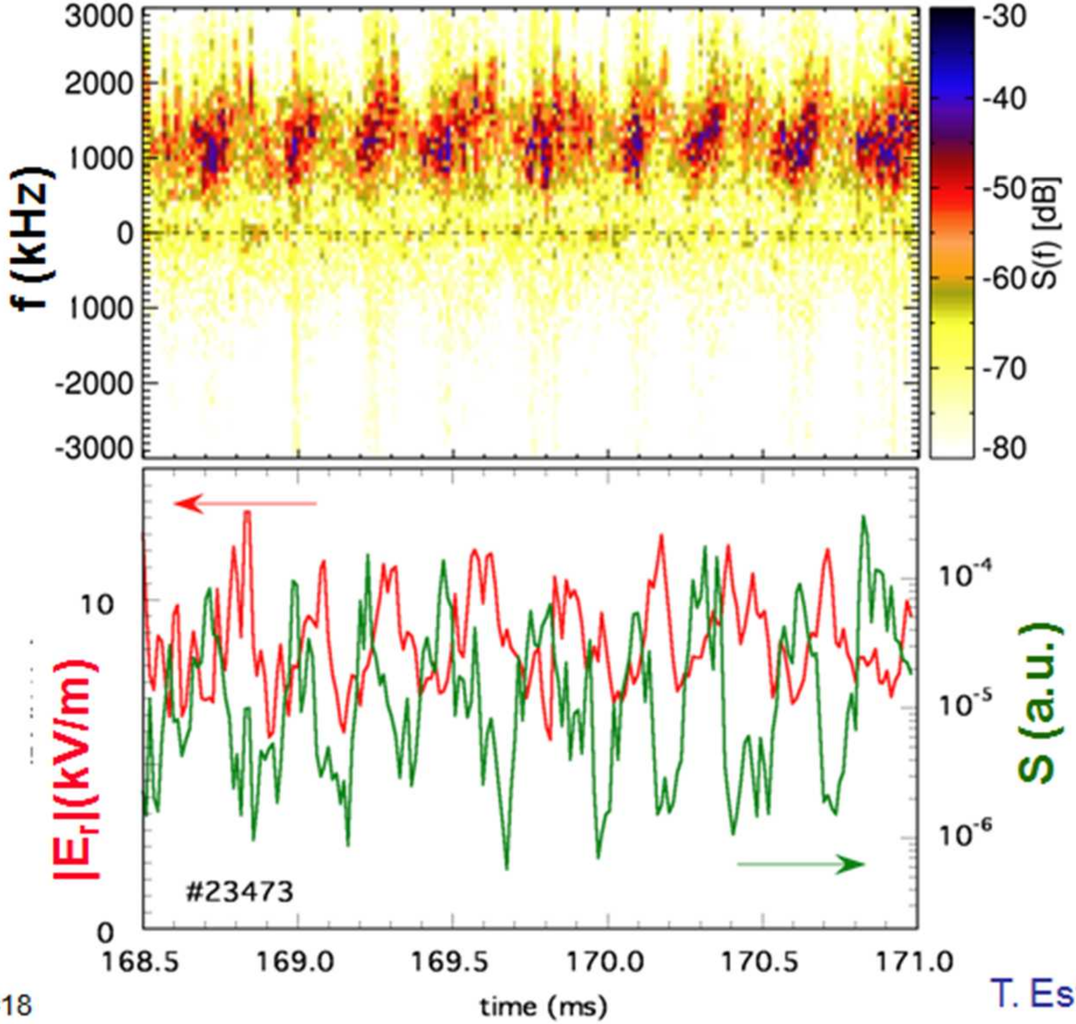
And/or

Non optimal ι range

T. Estrada, et. al. (2009)

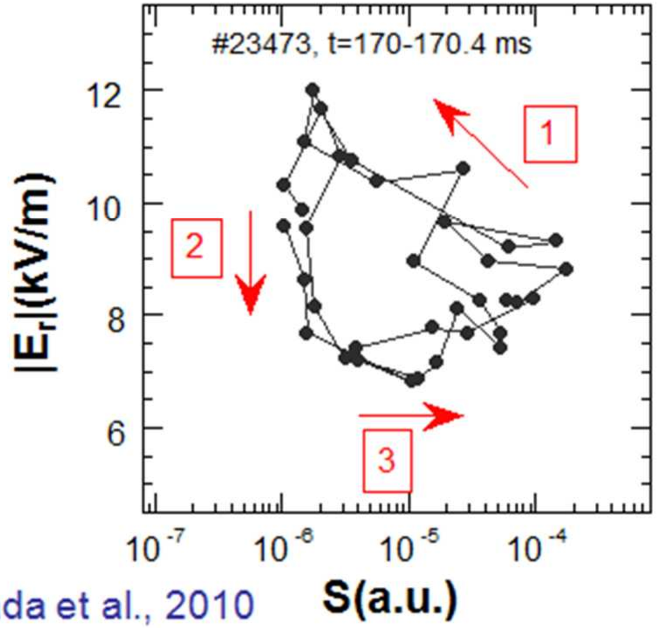
Flows and turbulence dynamics

Doppler reflectometry



The time evolution shows a predator-prey behaviour:

Periodic evolution of E_r and ñ with the E_r following ñ with a phase delay of 90º.



#18

T. Estrada et al., 2010

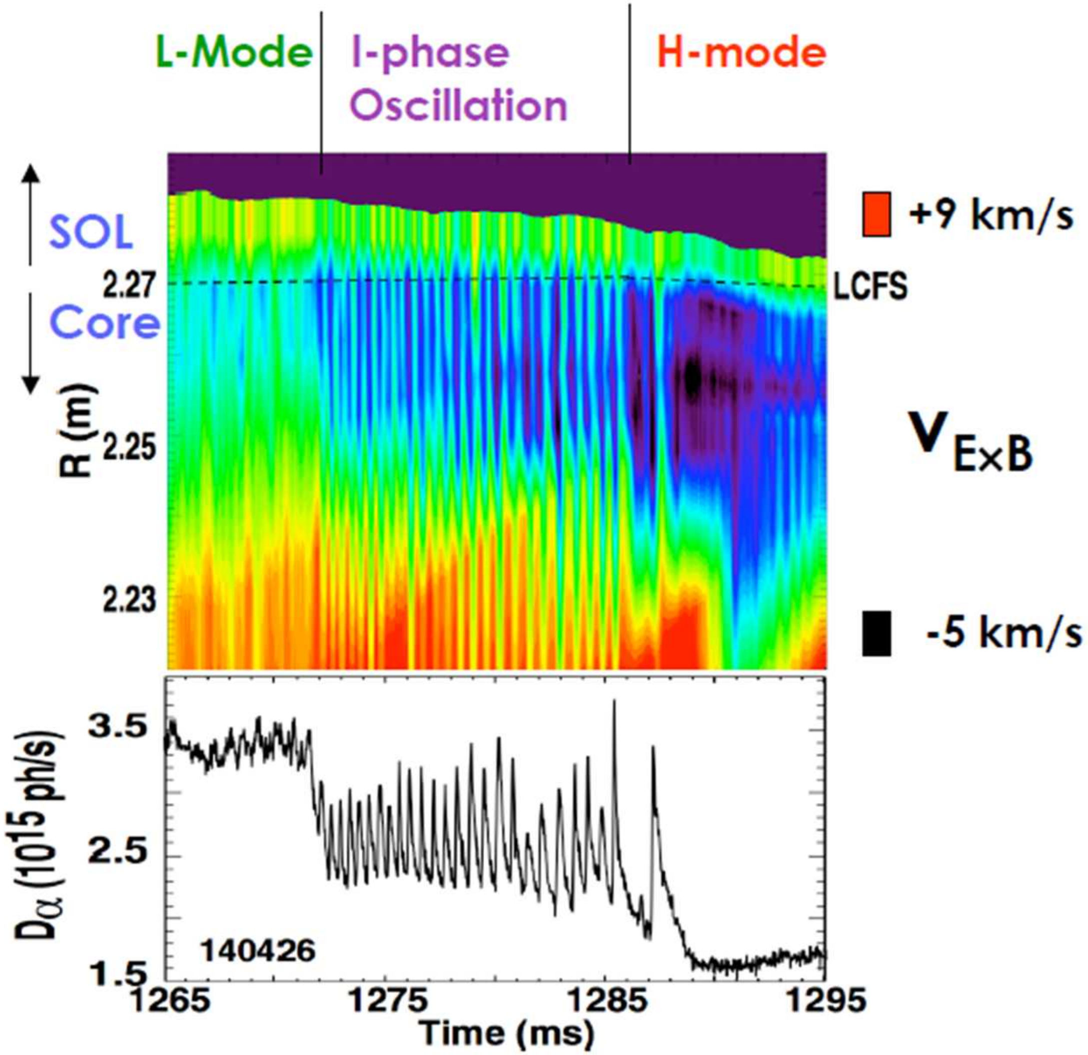
s

The Oscillating Flow Layer Widens Radially (Frequency Decreases) - Steady Flow after Final H-Mode Transition

A weak E×B flow layer exists in L-mode (L-mode shear layer)

At the I-phase transition, the E×B flow becomes more negative first near the separatrix, flow layer then propagates inward

The flow becomes steady at the final H-mode transition (after one final transient)



L. Schmitz TTF’11

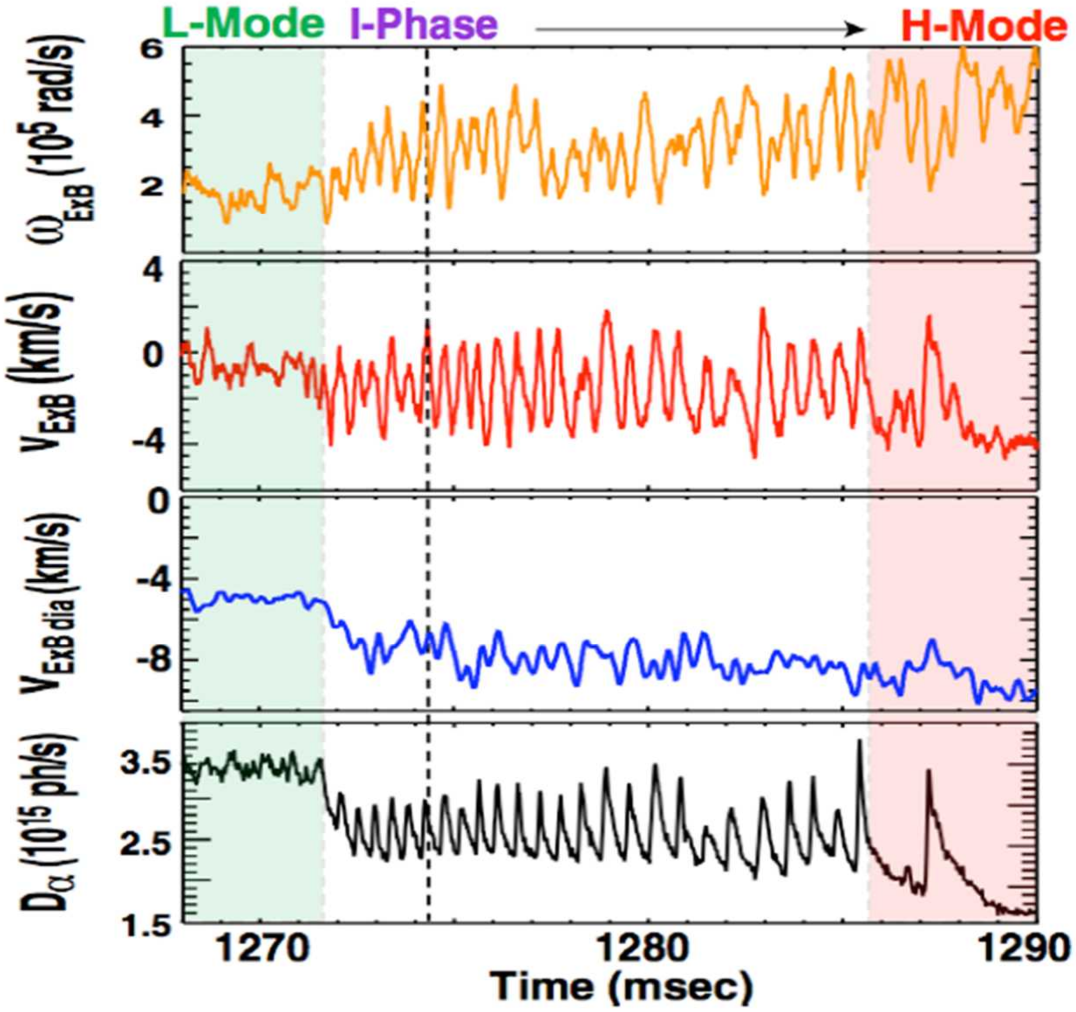
During the I-phase, the Mean Shear $<\omega_{ExB}>$ Increases with Time and Eventually Dominates

Outer layer Shearing Rate (Mean flow+ ZF)

ExB Flow from DBS (includes ZF)

Diamagnetic component of ExB flow (from ion pressure Profile)

R~2.265m



L. Schmitz TTF'11

Pinnacle, cont'd

- For $P \sim P_{th}$, cyclic / dithering oscillations observed in flows, turbulence
- Multi-shear flow competition at work in transition process
- Flow structure evolves as transition progresses
- Many aspects of dynamics well described by multi-predator shearing models ala' K+D
- Variety of results, hints, suggestions, proclamations as to precise trigger… GAM, ZF, Mean ExB_0 Flow, Mean Poloidal Flow…

  Need there be a unique route to transition?

Pinnacle, cont'd

- Facing the Challenge
  - theory should:
    - forsake 0D for 1D minimal models in *r, t*

      (c.f. K. Miki, P.D., APTWG 2011)
    - predict something qualitatively new

      suggestion: ELM-free back transition (EAST !?)
    - link micro-dynamics and macroscopics (i.e. threshold)
  - both theory and experiment should elucidate SOL flow effects on shear profile inside separatrix (B. LaBombard '04)
- Stay Tuned…

1st Asia Pacific Transport Working
Group(APTWG) International Conference,
June 14-17, 2011, Japan
C-O3

Towards a 1D model of L→I→ H evolution dynamics

[1)]K. Miki and [1,2)]P. H. Diamond

[1)]WCI Center for Fusion Theory, NFRI, Korea

[2)] CMTFO and CASS, UCSD, USA

Towards the Model for L-I-H transition

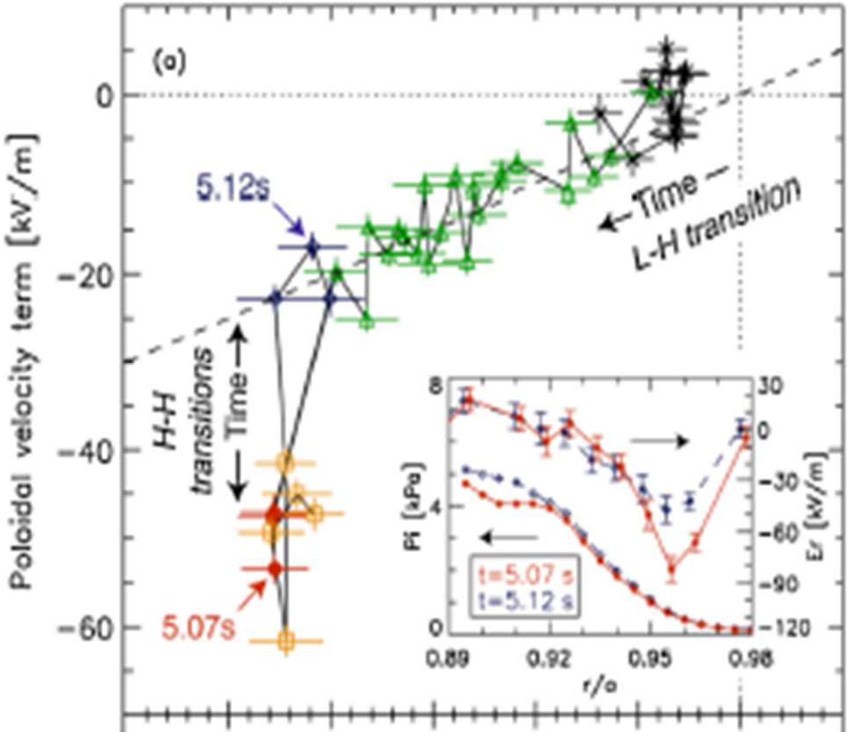
From 0D *in vitro* to 1D *in vivo*

- Identification of habitation
- Stability of states

Interaction of micro-scales with macro-scales

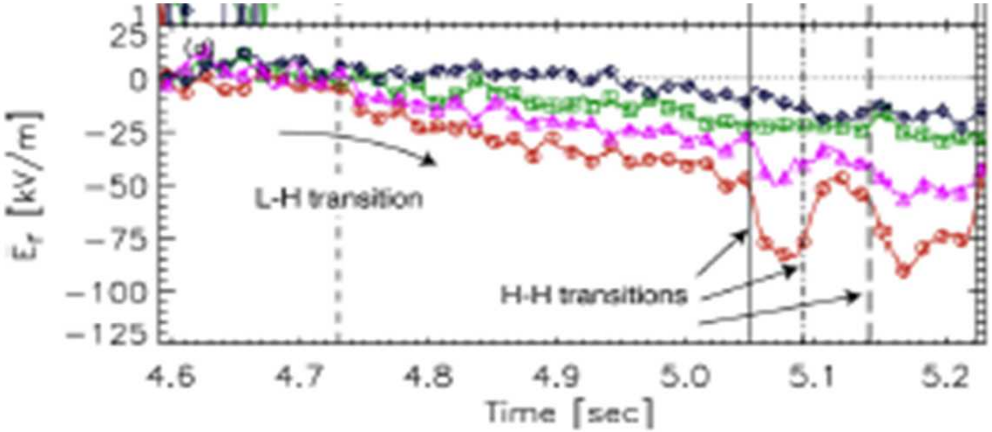
- Non-locality
- Profile evolution
- Mean flow dynamics

Multiple states of transport barrier(2) in JT-60U[Kamiya PRL 2010]



Diamagnetic term

0

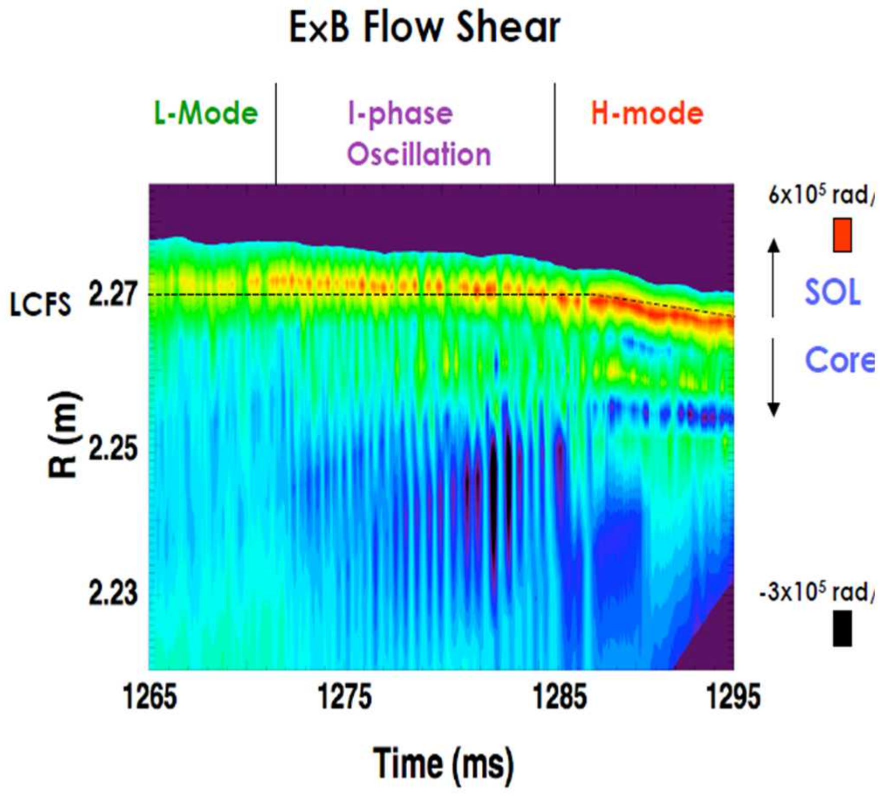


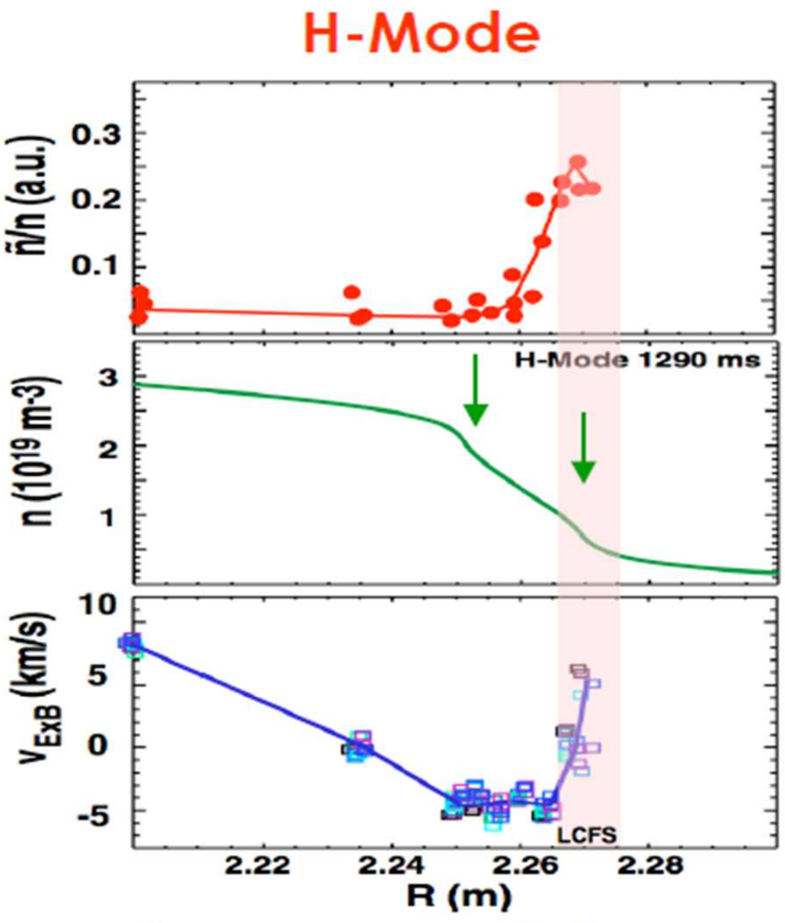
- Two stages of H-modes
  - with similar toroidal velocities and diamagnetic flow velocities
  - **with different poloidal velocities**
- Duration is MUCH LONGER than that of Limit cycles.

Multiple states of transport barrier(1): dual shear layers are observed in DIII-D [Schmitz US-TTF 2011]

Narrow outer layer

Widening inner layer





Inner Shear layer

Outer Shear layer

I(Intermediate)-phase ≠ I-mode (distinct from C-Mod results)

- Characterized by the gradual evolution of mean flow shear and dynamical interaction between E_r and turbulence, i.e. limit-cycle

DIII-D results → Indicating 1 space – 1 time is the minimum system.

Towards the Model for L-I-H transition

Hinton's 1D 1-field model (density(*n*)) **[Hinton PFB '91], [Levedev PoP '96]**

Treating 1D profile evolution associated with ExB mean flow shearing(V'_E)

→Malkov-Diamond 1D 2-field model (p,n)

[Malkov and Diamond 2008 PoP]

density $$\frac{\partial n}{\partial t} - \frac{\partial}{\partial x}\left[D_0 + \boxed{\frac{D_1}{1+\alpha V_E'^2}}\right]\frac{\partial n}{\partial x} = S(x)$$

Pressure $$\frac{3}{2}\frac{\partial p}{\partial t} - \frac{\partial}{\partial x}\left[\chi_0 + \boxed{\frac{\chi_1}{1+\alpha V_E'^2}}\right]\frac{\partial p}{\partial x} = H(x)$$

(neoclassical transport) + (turbulent transport)

Kim-Diamond predator-prey model

(Turbulence, ZF, mean flow)

MF:Radial force balance

V'_E=(diamagnetic drift) + (poloidal rotation) + ~~(toroidal rotation)~~

Neoclassical poloidal spin-up

[McDevitt 2010 PoP]

Assumption of steady state $<v_\theta>$

Basic equations

Neoclassical transport Correlation time of turbulence

Pressure profile

$$\frac{\partial \bar{p}}{\partial t}-\frac{\partial}{\partial x}\left((\chi_{neo}+\underbrace{\tau_c I}_{})\frac{\partial \bar{p}}{\partial x}\right)=H_x$$

coefficient of anomalous diffusion=turbulence.

Heat source

$$H_x=\frac{\partial}{\partial x}\left[2q_a\frac{x}{a}\left(1-\frac{x^2}{2a^2}\right)\right]\equiv\frac{\partial H}{\partial x}$$

Density profile

$$\frac{\partial \bar{n}}{\partial t}-\frac{\partial}{\partial x}\left((D_{neo}+\tau_c I)\frac{\partial \bar{n}}{\partial x}\right)=S_x$$

Particle source

$$S_x=\frac{\partial}{\partial x}[\gamma_a e^{-\eta(n_a(a-x)+g_a(a-x)^2/2)}]\equiv\frac{\partial S}{\partial x}$$

Turbulence intensity

$$\frac{\partial I}{\partial t}=[\gamma_L-\Delta\omega I-\alpha_0 E_0-\alpha_V E_V]I+\chi_N\left(I\frac{\partial I}{\partial x}\right)$$

$$E_V=V_E'^2$$

ZF shearing MF shearing

Turbulence spreading *

Zonal Flow energy minimal rep.

$$\frac{\partial E_0}{\partial t}=\alpha_0\left[I\underbrace{(1+\zeta_0 E_V)}_{\text{ZF/MF competition}}{}^{-1}-I_{*0}\right]E_0$$

Collision(=$\gamma_{\text{damp}}/\alpha_0$)

Radial force balance

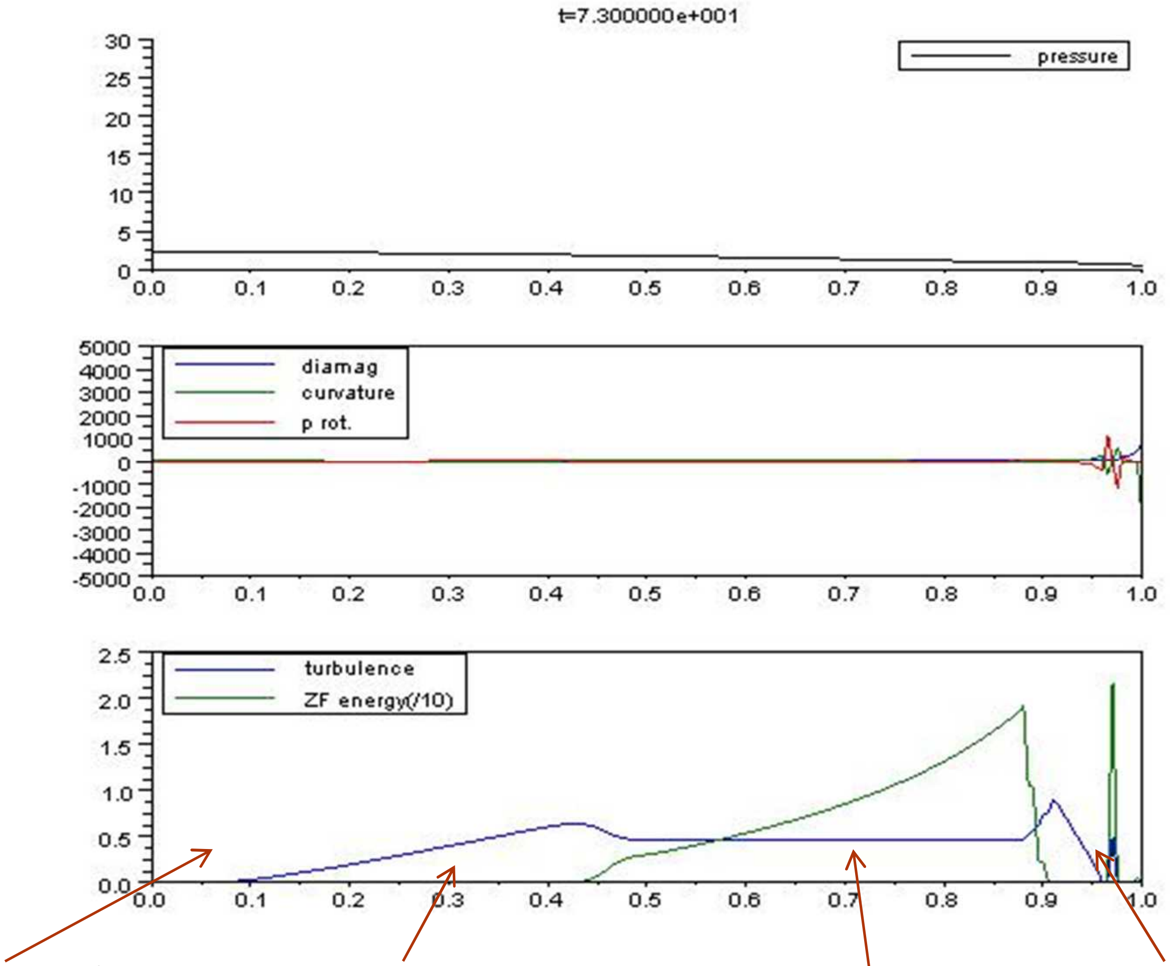
$$V_E'=\frac{1}{eB}\left[-\frac{1}{n}\left(\frac{d\bar{n}}{dx}\frac{d\bar{p}}{dx}\right)+\frac{1}{n^2}\left(\frac{d^2p}{dx^2}\right)\right]-\underbrace{\left[V_{\substack{pol\\neo}}+\frac{S_0}{\gamma_{damp}}\frac{dI}{dx}\right]}_{}$$

Turbulence drive(assume ITG)

$$\gamma_L=\gamma_0(R/L_T-[R/L_T]_{crit})$$
$$=\gamma_{L0}\left[L_p^{-1}-L_n^{-1}-L_{T,crit}^{-1}\right]$$

Poloidal flow driven by neoclassical effects and turbulence drive [McDevitt]

Evolution of profiles (1)
– typical L-mode state



Habitat isolation established.

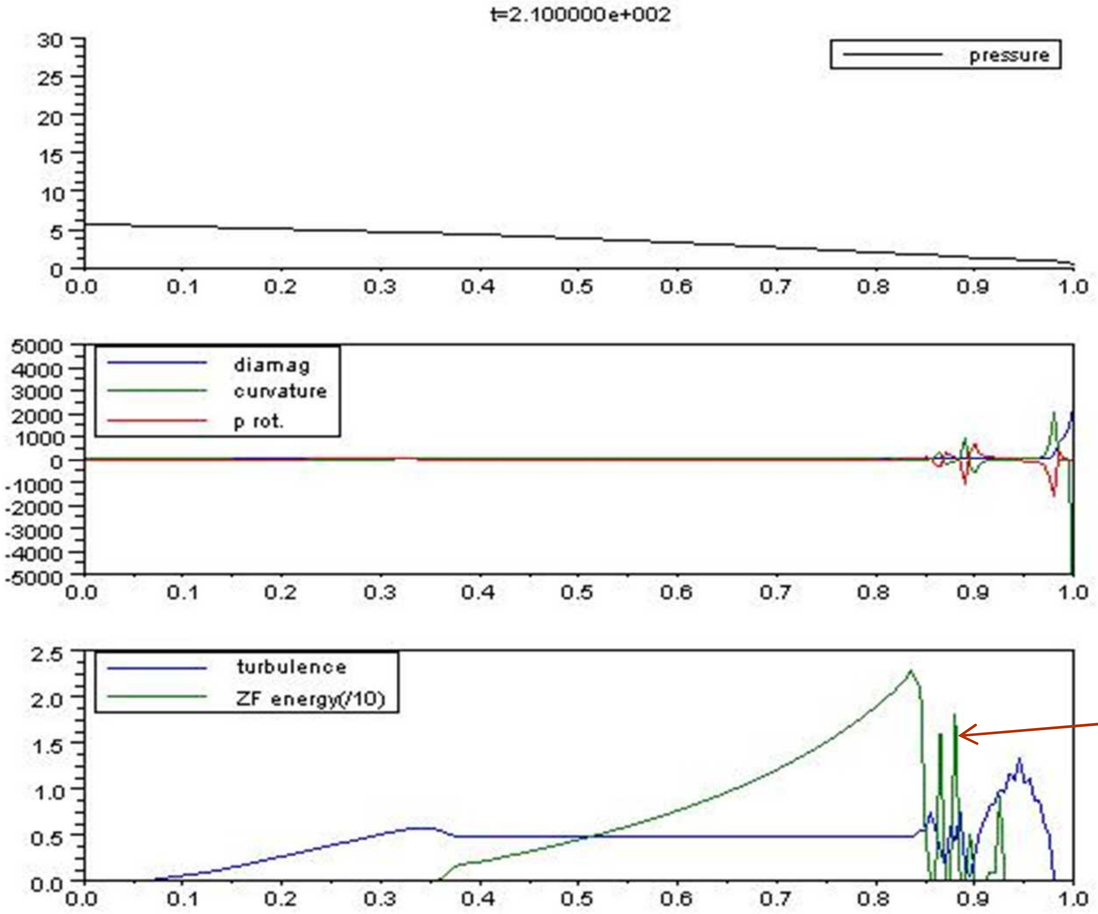
Subcritical region $\gamma_L<0$.

Turbulent dominant region

ZF/turb coexistence

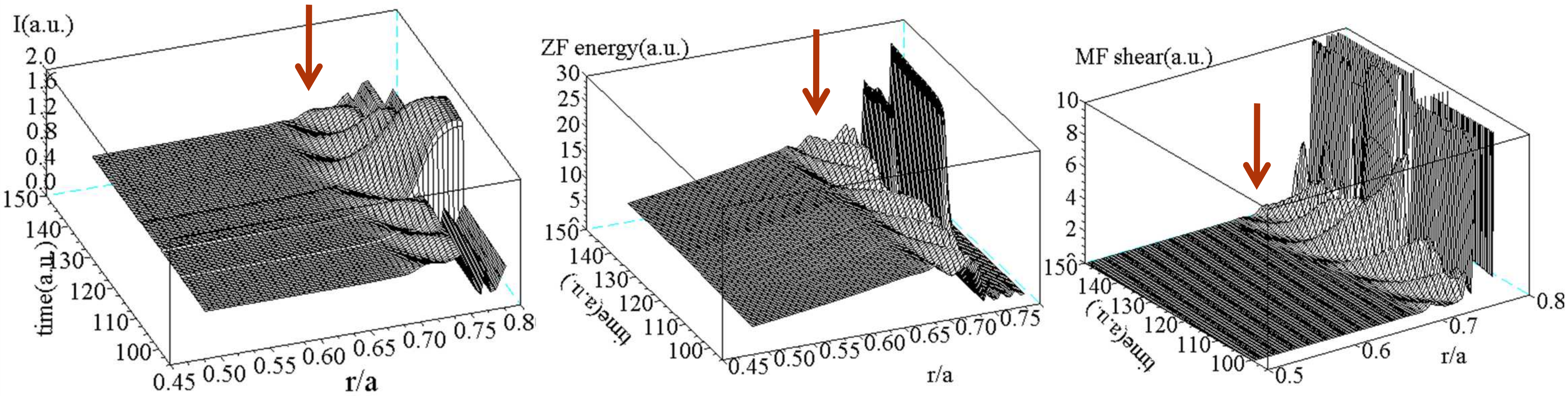
MF dominant region

Evolution of profiles(2)
-- I-phase, i.e. limit-cyclic behavior between turb/ZF coexistence and MF dominant regions.



Limit cyclic behavior of turb/ZF/MF

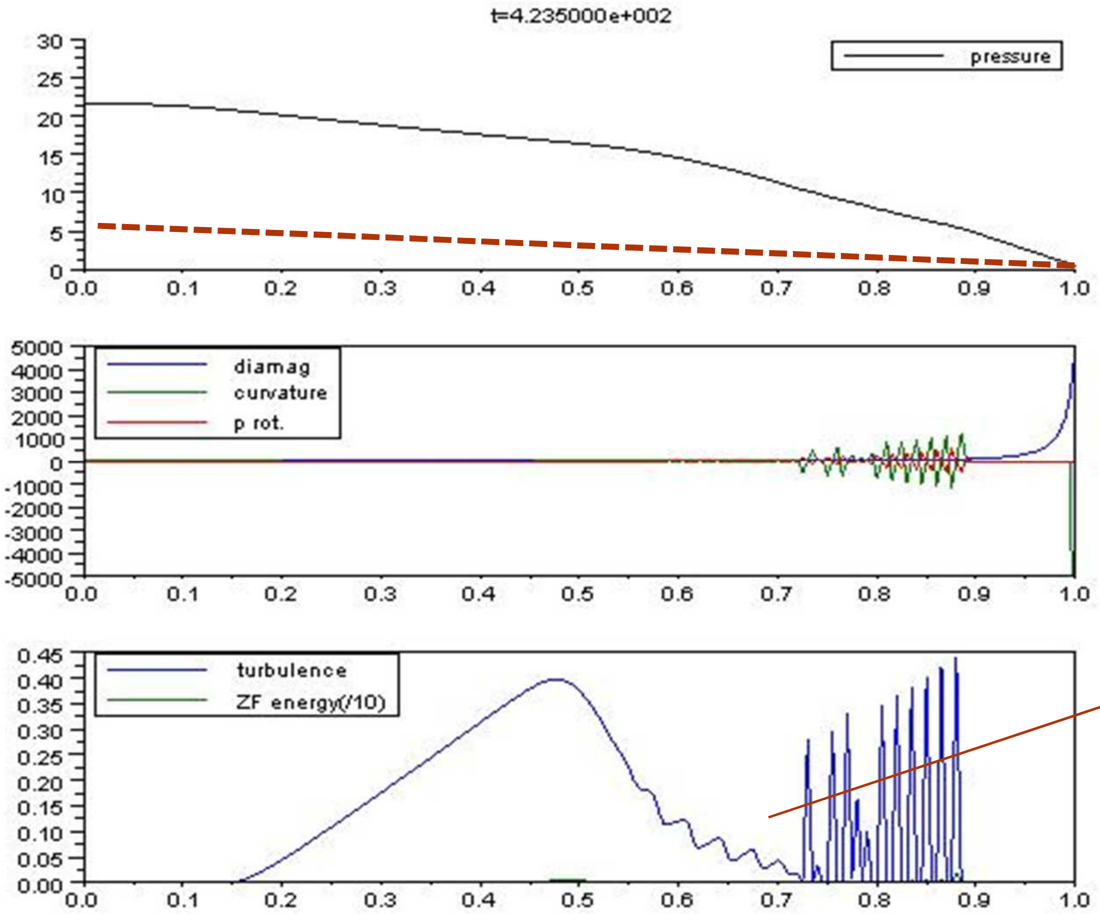
Limit-cycle behavior has a spatio-temporal structure, propagating inward.
- inside the barrier region, due to turbulence spreading



Question:

Phase delay in radial space in experiments?

Evolution of profiles(3)
-- above a threshold, immediate transition to H-mode state

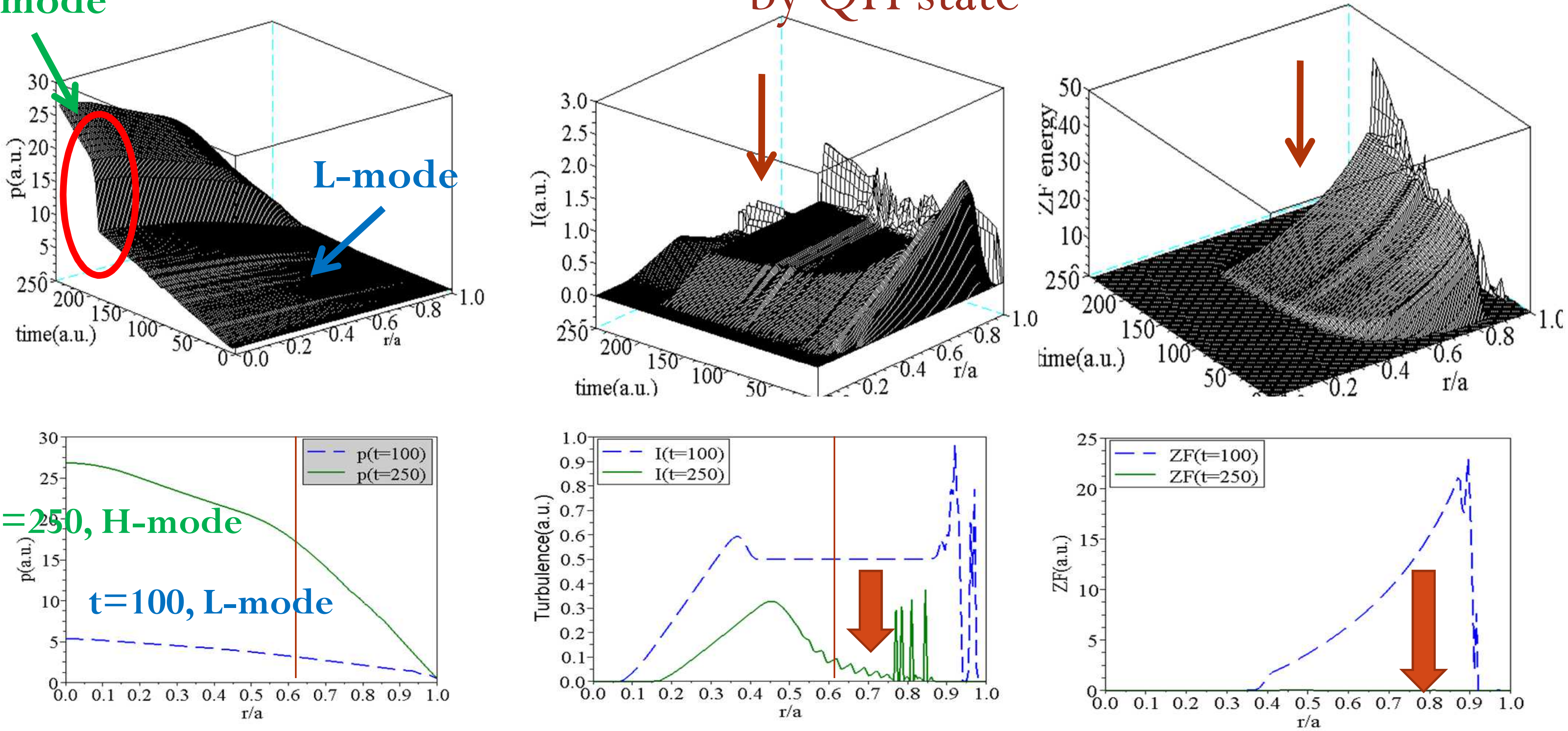


Propagation generally seen, as a kind of ELM after H-mode transition?

Immediately MF dominant region expands from r/a~0.9 to 0.7.

An evolution of transport barrier in power ramp up, corresponding to quench of turbulence and ZF, i.e. T -> QH.

Quench of turb/ZF followed by QH state



H-mode

L-mode

t=250, H-mode

t=100, L-mode

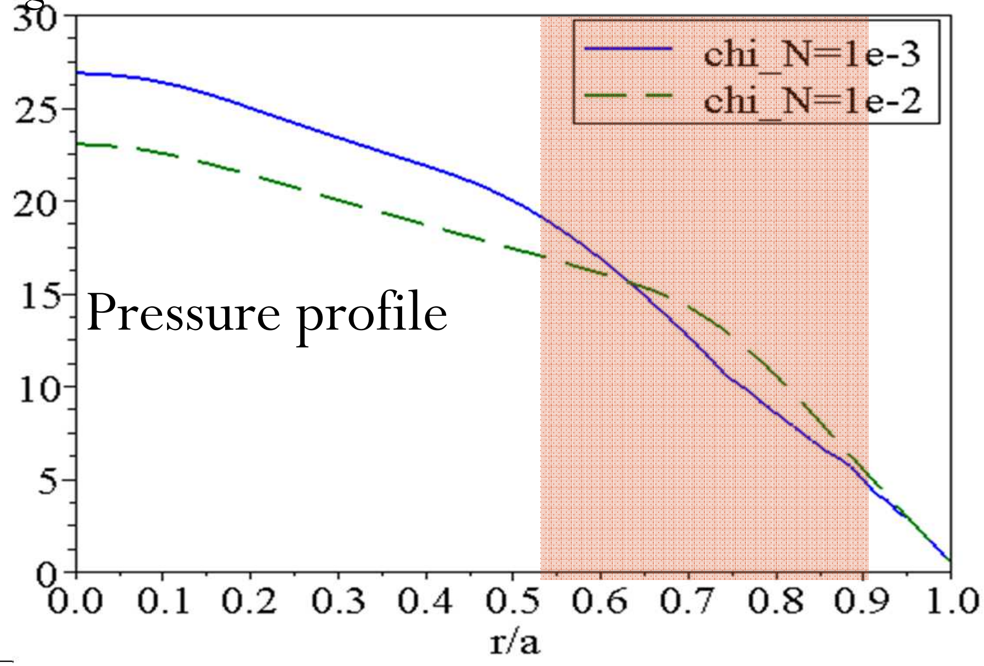
Corners of the pressure profile provides a new type of transport barrier caused by curvature and poloidal rotation.

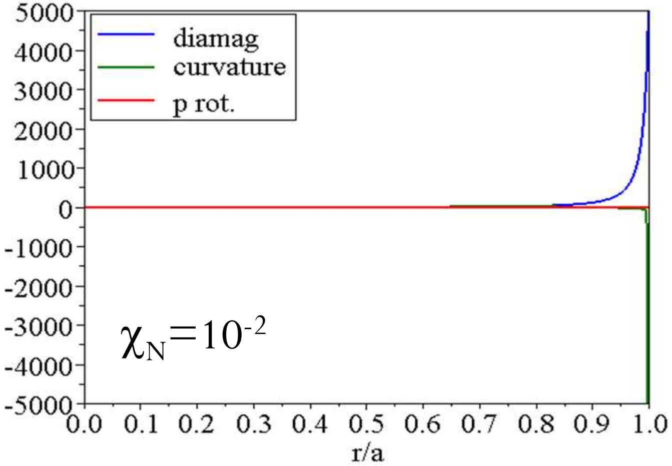
-On the corners, pressure curvature affects significantly on MF shear,

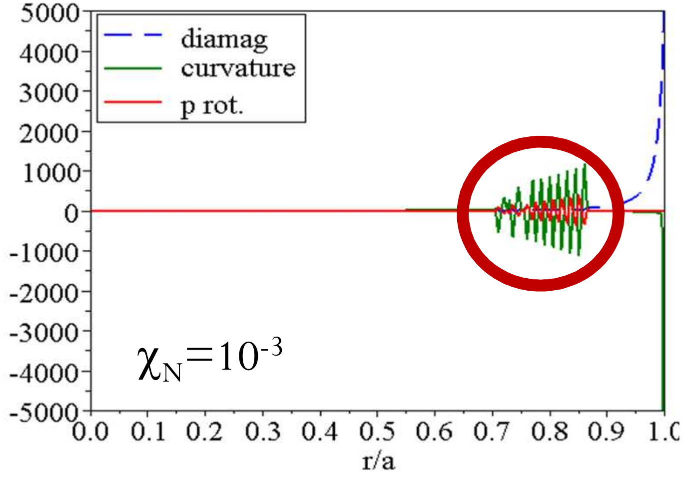
-balancing with poloidal flow driven by turbulence intensity gradient.

-turbulence intensity can couple with turbulence spreading --- turbulence spreading dissipates the corrugation

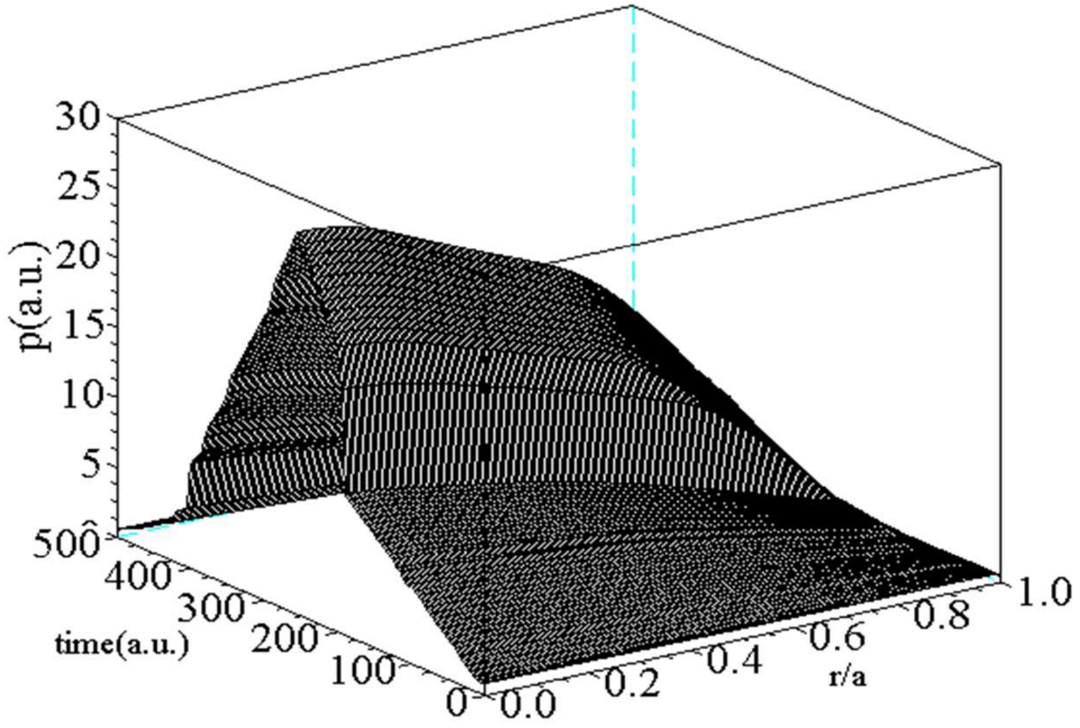
$$V_E' = \frac{1}{eB}\left[\underbrace{-\frac{1}{n}\left(\frac{d\bar{n}}{dx}\frac{d\bar{p}}{dx}\right)}_{\text{Diamagnetic drift}} + \underbrace{\frac{1}{n^2}\left(\frac{d^2p}{dx^2}\right)}_{\text{Curvature}}\right] - \underbrace{\left[V_{\substack{pol\\neo}} + \frac{S_0}{\gamma_{damp}}\frac{dI}{dx}\right]}_{\text{Poloidal rotation}}$$

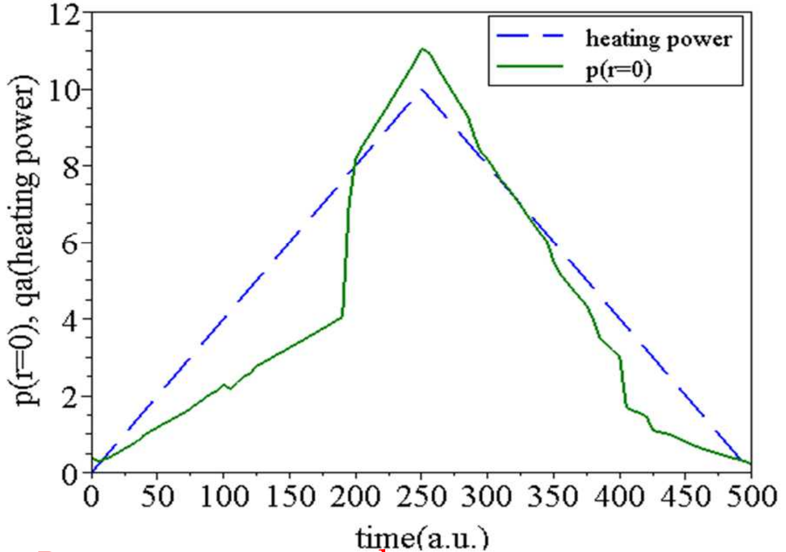




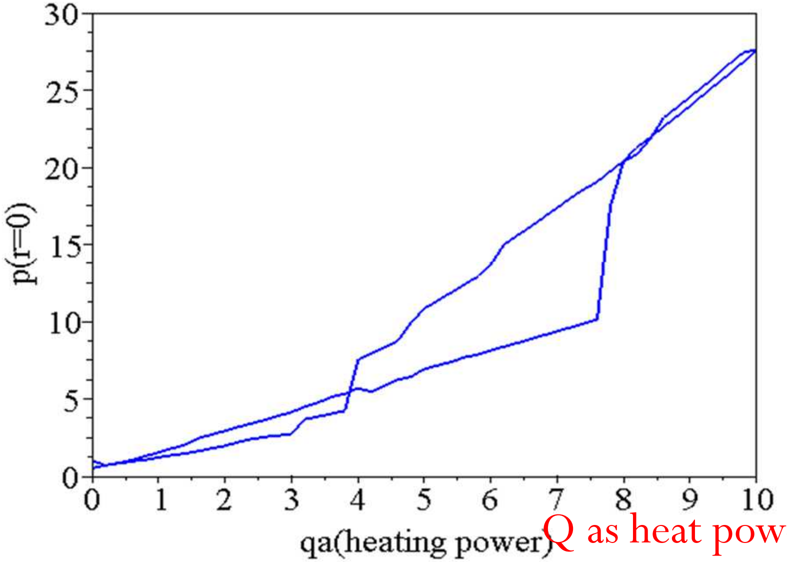


Power ramp up-down exhibits hysteretic behavior

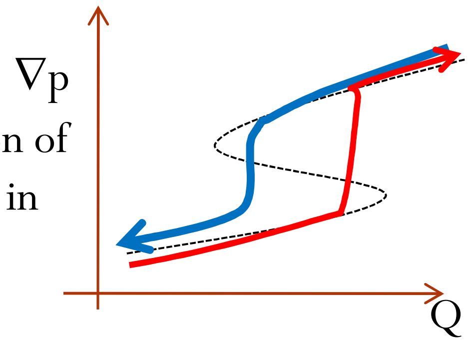




P_0 as corresp. <grad p>



Q as heat power



Partial destruction of
transport barrier in
ramp down?

Conclusion

- 1D Kim-Diamond model reproduces self-consistent radial evolution of transport barrier above a heat power threshold.
- Limit-cycle is reproduced with a radial structure associating with inward/outward turbulence/ZF propagation.
- Dual shearing layer structure is reproduced: one is from the diamagnetic drift shear, the other is from the profile corners coupling with curvature term and poloidal rotation in mean flow shear.
  - May these be relevant to the multistage H-mode in JT-60U, linking to the dual shearing layer in DIII-D?

Yet More

Heat flux

- $(\varepsilon)(\nabla P) \rightarrow \left[\varepsilon Q / (\chi_0 \varepsilon + \chi_{neo})\right]$
  - heat flux variability → footprint on transition dynamics
  - variability
    - Sawteeth
    - Heating non-stationarity
    - Avalanches (1/f)

How parametrize

PDF(Q)?

  - non-locality: $\Delta_{avalanches}$ zone at edge?

Kernel width

- SOL flow impact on V_E' ?
- Apart MHD, what limits inward pedestal penetration? i.e. match L→H to pedestal dynamics?

Conclusion

- There are no conclusions. This topic is alive and well, and will evolve dynamically.
- Cross-disciplinary dialogue with GFD/AFD communities has been very beneficial and should continue!
- Prediction: This will not be the last prize awarded for the theory of drift wave-zonal flow turbulence.

“All true genius is unrecognized.”

- Friedrich Dürrenmatt, “The Physicists”

N.B.: “The physicists” is a satiric play set in an insane asylum. It features three protagonists, one who thinks he is Newton, one who thinks he is Einstein, and one who thinks he hears the voice of the wise King Solomon.

"人不知，而不愠；不亦君子乎！"

～孔子

" Not recognized by others, and yet not upset;
What a noble person! "

– Confucius
(Kongtze)

— courtesy of L. Chen, Alfven Prize Lecture, 2008

The Evolution of Reaction to Progress in Theoretical Physics:

Stage1 : “Its wrong!”

Stage2 : “Its trivial!”

Stage3 : “I did it first !!”

\- Anonymous

“I didn’t really say everything I said”

- Yogi Berra

“You will get the most attention from those who hate you. No friend, no admirer, and no partner will flatter you with as much curiosity.”

- N.N. Taleb “The Bed of Procrustes”