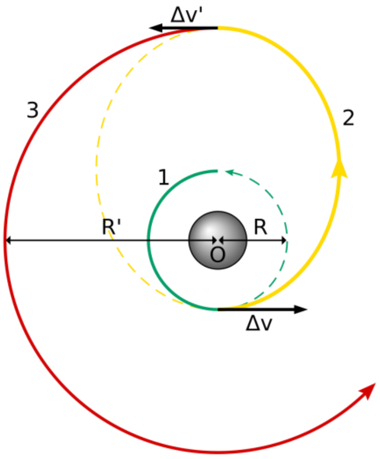
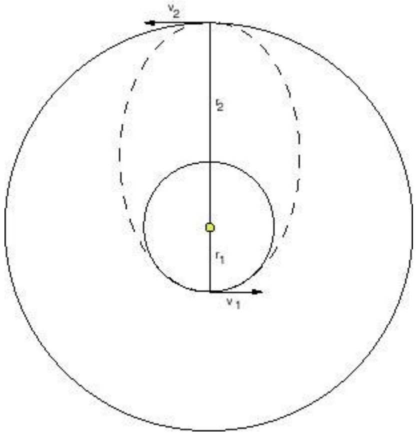
Hohmann Transfer Orbit





For a small body orbiting another very much larger body, such as a satellite orbiting the earth, the total energy of the smaller body is

$$E = \tfrac{1}{2}mv^2 - \frac{GMm}{r} = \frac{-GMm}{2a}.$$

Velocity equation

$$v^2 = \mu\left(\frac{2}{r} - \frac{1}{a}\right)$$

where:

- v is the speed of an orbiting body
- $\mu = GM$ is the standard gravitational parameter of the primary body,
- r is the distance of the orbiting body from the primary focus
- a is the semi-major axis of the body's orbit.

Therefore the delta-*v* required for the Hohmann transfer can be computed as follows, under the assumption of instantaneous impulses:

$$\Delta v_1 = \sqrt{\frac{\mu}{r_1}}\left(\sqrt{\frac{2r_2}{r_1 + r_2}} - 1\right),$$

to enter the elliptical orbit at $r = r_1$ from the r_1 circular orbit

$$\Delta v_2 = \sqrt{\frac{\mu}{r_2}} \left(1 - \sqrt{\frac{2r_1}{r_1 + r_2}} \right),$$

to leave the elliptical orbit at $r = r_2$ to the r_2 circular orbit

* r_1 and r_2 are, respectively, the radii of the departure and arrival circular orbits; the smaller (greater) of r_1 and r_2 corresponds to the periapsis distance (apoapsis distance) of the Hohmann elliptical transfer orbit.

Whether moving into a higher or lower orbit, by Kepler's third law, the time taken to transfer between the orbits is:

$$t_H = \frac{1}{2}\sqrt{\frac{4\pi^2 a_H^3}{\mu}} = \pi\sqrt{\frac{(r_1 + r_2)^3}{8\mu}}$$

(one half of the orbital period for the whole ellipse), where a_H is length

of semi-major axis of the Hohmann transfer orbit.

In application to traveling from one celestial body to another it is crucial to start maneuver at the time when the two bodies are properly aligned. Considering the target angular velocity being

$$\omega_2 = \sqrt{\frac{\mu}{r_2^3}}$$

angular alignment α (in radians) at the time of start between the source object and the target object shall be

$$\alpha = \pi - \omega_2 t_H = \pi \left(1 - \frac{1}{2\sqrt{2}} \sqrt{\left(\frac{r_1}{r_2} + 1 \right)^3} \right)$$