Maneuvering In Space

4.1.5

In This Section You'll Learn to...

- Explain the basic method for moving a satellite from one orbit to another
- Determine the velocity change (ΔV) needed to complete a Hohmann Transfer between two orbits
- Explain plane changes and how to determine the required ΔV to do them (enrichment topic)
- Explain orbital rendezvous and how to determine the required ΔV and wait time needed to start one (enrichment topic)

Outline

A spacecraft seldom stays very long in its assigned orbit. On nearly every space mission, there's a need to change one or more of the classic orbital elements at least once. Communication satellites, for instance, never go directly into their geostationary positions. They first go into a low-perigee (300 km or so) "parking orbit" before transferring to geosynchronous altitude (about 35,780 km). While this large change in semimajor axis occurs, another maneuver reduces their inclinations from that of the parking orbit to 0°. Even after they arrive at their mission orbit, they regularly have to adjust it to stay in place. On other missions, spacecraft maneuver to rendezvous with another spacecraft, as when the Space Shuttle rendezvoused with the Hubble Space Telescope to repair it (Figure 4.1.5-1).

Space Mission Architecture. This chapter deals with the Trajectories and Orbits segment of the Space Mission Architecture.

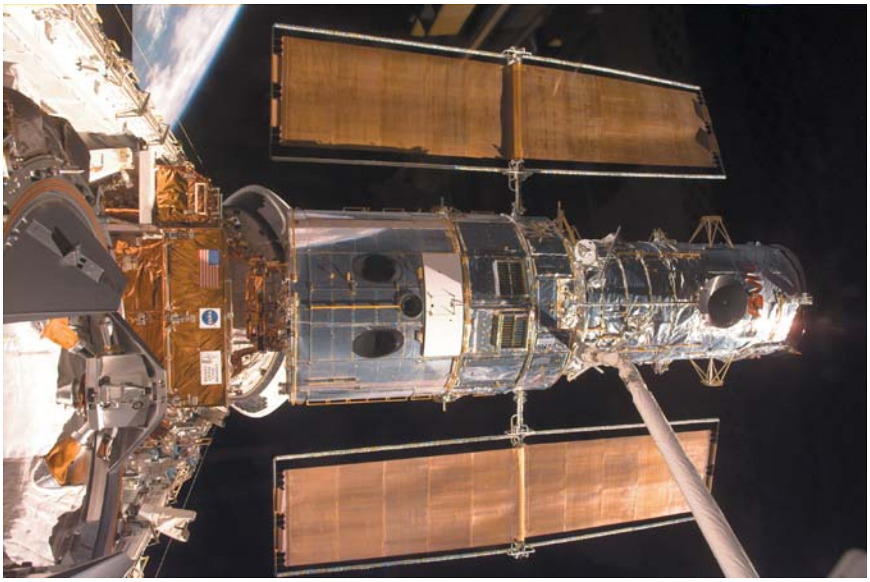


Figure 4.1.5-1. Shuttle Rendezvous with Hubble Space Telescope. In 1995 and again in 1999, the Space Shuttle launched into the same orbital plane as the Hubble Space Telescope. After some maneuvering, the Shuttle rendezvoused with and captured the telescope to make repairs. *(Courtesy of NASA/Johnson Space Center)*

As we'll see in this chapter, these orbital maneuvers aren't as simple as "motor boating" from one point to another. Because a spacecraft is always in the gravitational field of some central body (such as Earth or the Sun), it has to follow orbital-motion laws in getting from one place to another. In this chapter we'll use our understanding of the two-body problem to learn about maneuvering in space. We'll explain the most economical way to move from one orbit to another, find how and when to change a spacecraft's orbital plane, and finally, describe the intricate ballet needed to bring two spacecraft together safely in an orbit.

4.1.5.1 Simple Orbit Changes

In This Section You'll Learn to...

- Describe the steps needed to move a satellite from one orbit to another in the same plane

One of the first problems space-mission designers faced was figuring how to go from one orbit to another. Refining this process for eventual missions to the Moon was one of the objectives of the Gemini program in the 1960s, shown in Figure 4.1.5-2. Let's say we're in one orbit and we want to go to another. To keep things simple, we'll assume that the initial and final orbits are in the same plane. We often use such maneuvers to move spacecraft from their initial parking orbits to their final mission orbits. Because fuel is critical for all orbital maneuvers, let's look at the most fuel-efficient method: the Hohmann Transfer.

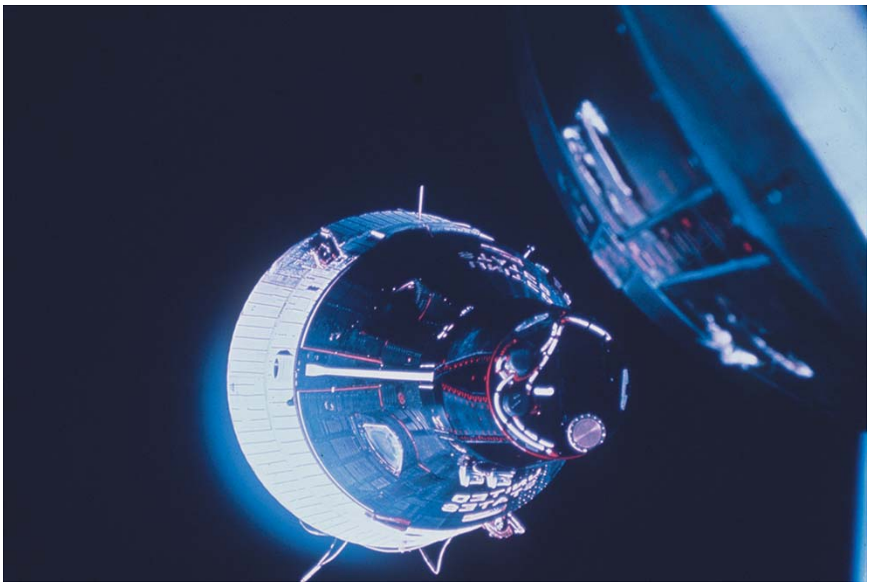


Figure 4.1.5-2. The Gemini Program. During the Gemini program in the 1960s, NASA engineers and astronauts developed the procedures for all orbital maneuvers needed for the complex Lunar missions. Here the Gemini 6A command module is rendezvousing with the Gemini 7 command module. *(Courtesy of NASA/Johnson Space Center)*

In 1925 a German engineer, Walter Hohmann, thought of a fuel-efficient way to transfer between orbits. (It's amazing someone was thinking about this, considering artificial satellites didn't exist at the time.) This method, called the *Hohmann Transfer*, uses an elliptical transfer orbit tangent to the initial and final orbits.

To better understand this idea, let's imagine you're driving a fast car around a racetrack, as shown in Figure 4.1.5-3. The effort needed to exit the track depends on the off-ramp's location and orientation. For instance, if the off-ramp is tangent to the track, your exit is easy—you just straighten the wheel. But if the off-ramp is perpendicular to the track, you have to slow down a lot, and maybe even stop, to negotiate the turn. Why the difference? With the tangential exit you have to change only the magnitude of your velocity, so you just hit the brakes. With the perpendicular exit, you quickly must change the *magnitude* and *direction* of your velocity. This is hard to do at high speed without rolling your car!

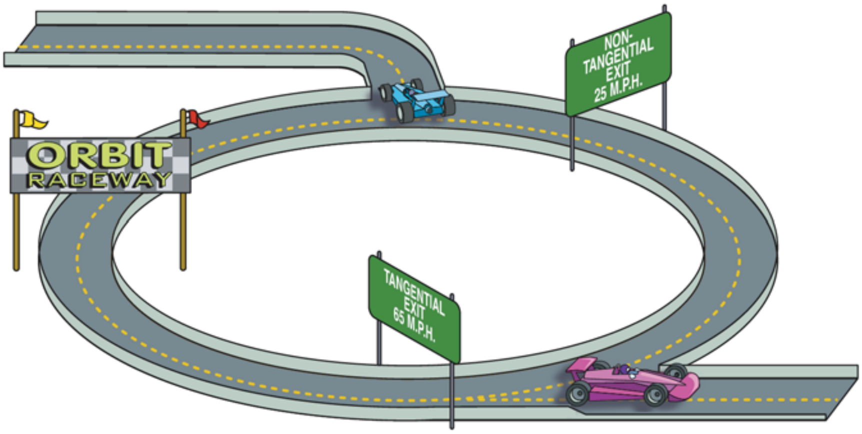


Figure 4.1.5-3. Maneuvering. One way to think about maneuvering in space is to imagine driving around a racetrack. Exiting at a sharp turn takes more effort than exiting tangentially.

The Hohmann Transfer applies this simple racetrack example to orbits. By using "on/off-ramps" tangent to our initial and final orbits, we change orbits using as little energy as possible. For rocket scientists, saving energy means saving fuel, which is precious for space missions. How can a spacecraft change its energy? It increases or decreases its velocity by firing rocket engines. For a Hohmann Transfer, these velocity changes (*delta-V*s or ΔV) are aimed to make sure they are tangent to the initial and final orbits. Remember that velocity, $\vec{V}$, is a vector. This means it has both magnitude (or speed) and direction. To change velocity tangentially, you must fire the spacecraft's rocket

- parallel to the direction of travel (point the rocket behind you) in order to increase velocity
- against the direction of travel (point the rocket in front of you) to slow down

These tangential ΔVs are the real secret to the Hohmann Transfer's energy savings.

Now let's look at what these velocity changes are doing to the orbit. Whenever we add or subtract velocity, we change the orbit's specific mechanical energy, ε, and hence its size, or semimajor axis, a.

Important Concept

The specific mechanical energy of a spacecraft in orbit depends only on the body's gravitational parameter and the orbit's semimajor axis (size).

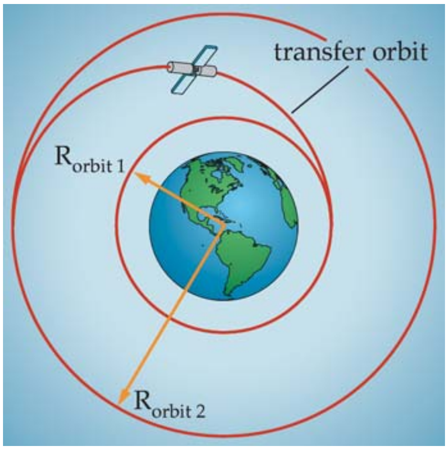


Figure 4.1.5-4. Getting From One Orbit to Another. The problem in orbital maneuvering is getting from orbit 1 to orbit 2. Here we see a spacecraft moving from a lower orbit to a higher one in a transfer orbit. If it doesn't perform the second ΔV when it reaches orbit 2, it will remain in the transfer orbit.

Note: Recall from Chapter 4 that we use the Greek symbol, Δ, as shorthand for the change in a quantity.

Equation (4.1.5-1) shows this relationship in shorthand form. By convention, the bigger the orbit the less negative (bigger) the energy. This is like temperature: –10 deg is "hotter" (less negative) than –20 deg.

$$\varepsilon = -\frac{\mu}{2a} \tag{4.1.5-1}$$

where

ε = specific mechanical energy (km^2/s^2)

μ = gravitational parameter = 3.986×10^5 (km^3/s^2) for Earth

a = semimajor axis (km)

If we want to move a spacecraft to a higher orbit, we have to increase the semimajor axis (adding energy to the orbit) by increasing velocity. On the other hand, to move the spacecraft to a lower orbit, we decrease the semimajor axis (and the energy) by decreasing the velocity.

Let's look at an example of where these velocity changes might be necessary. Imagine a communications satellite in a low-Earth orbit (orbit 1) that needs to go into a higher orbit (orbit 2) so it can serve more customers. To get from orbit 1 to orbit 2, the satellite must travel along an intermediate orbit called a *transfer orbit*, as shown in Figure 4.1.5-4.

This process takes two steps, as shown in Figure 4.1.5-5. To get from orbit 1 to the transfer orbit, we change the orbit's energy by changing the spacecraft's velocity by an amount ΔV_1. Then, when the spacecraft gets to orbit 2, we must change its energy again (by changing its velocity by an amount ΔV_2). If we don't, the spacecraft will remain in the transfer orbit, indefinitely, returning to where it started in orbit 1, then back to orbit 2, etc. Thus, the complete maneuver requires two separate energy changes, accomplished by changing the orbital velocities (using ΔV_1 and ΔV_2). For mission planning, we simply add the ΔV from each burn to find the total ΔV needed for the trip from orbit 1 to orbit 2.

Now that we've gone through the Hohmann Transfer, let's step back to see what went on here. In the example, the spacecraft went from a low orbit to a higher orbit. To do this, it had to increase velocity twice: ΔV_1 and ΔV_2. But notice the velocity in the higher circular orbit is less than in the lower circular orbit. Thus, the spacecraft increased velocity twice, yet ended up in a slower orbit! How does this make sense?

ΔV_1 increases the spacecraft's velocity, taking the spacecraft out of orbit 1 and putting it into the transfer orbit. In the transfer orbit, its velocity gradually decreases as its radius increases, trading kinetic energy for potential energy, just as a baseball thrown into the air loses vertical velocity as it gets higher. When the spacecraft reaches the radius of orbit 2, it accelerates again, with ΔV_2 putting it into the final orbit. Even though

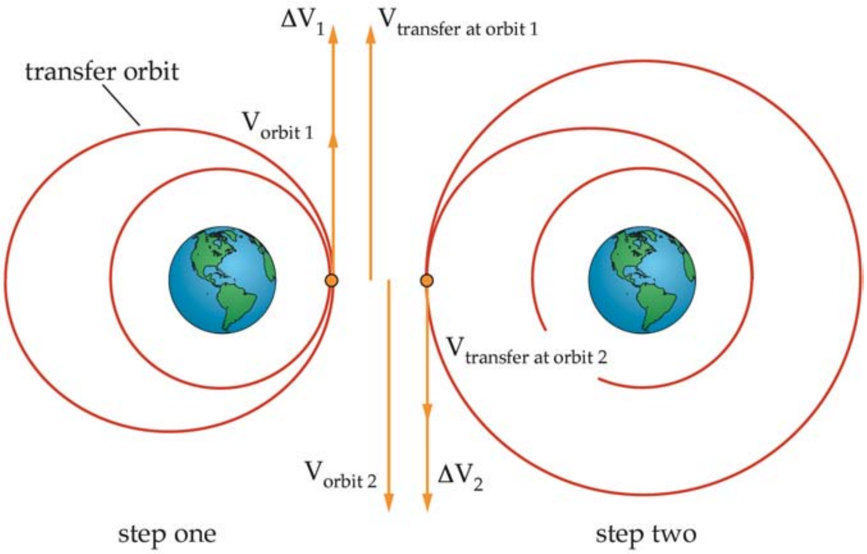


Figure 4.1.5-5. Hohmann Transfer. Step 1: The first burn or ΔV of a Hohmann Transfer takes the spacecraft out of its initial, circular orbit and puts it in an elliptical, transfer orbit. Step 2: The second burn takes it from the transfer orbit and puts it in the final, circular orbit.

the velocity in orbit 2 is lower than in orbit 1, the *total energy* is higher because it's at a larger radius. Remember, energy is the sum of kinetic plus potential energy. Thus, we use the spacecraft's rockets to add kinetic energy, which makes it gain potential energy as it moves out toward orbit 2. Once it reaches orbit 2, it has higher total energy.

Hohmann transfers provide the basis for all types of operational orbital maneuvers. A Hohmann transfer is generally assumed to take place between two orbits in the same plane, but its possible to use a variation of these maneuvers to move between orbit planes as well. Another application of Hohmann transfers is the problem of bringing two spacecraft together at the same point at the same time called a *rendezvous*.

Section Review

Key Concepts

➤ The Hohmann Transfer moves a spacecraft from one orbit to another in the same plane. It's the simplest kind of orbital maneuver because it focuses only on changing the spacecraft's specific mechanical energy, ε.

➤ The Hohmann Transfer consists of two separate ΔVs

- The first, ΔV_1, accelerates the spacecraft from its initial orbit into an elliptical transfer orbit
- The second, ΔV_2, accelerates the spacecraft from the elliptical transfer orbit into the final orbit

4.1.5.2 Orbital Changes in Action

In This Section You'll Learn to...

- Determine the velocity change (ΔV) needed to complete a Hohmann Transfer
- Explain when to use a simple plane change and how a simple plane change can modify an orbital plane
- Determine the ΔV needed for simple plane changes

In Section 6.1 we learned the basic process for moving from one orbit to another using a Hohmann Transfer. In this section we'll pull out our calculators to figure out how much ΔV we need to move from one specific orbit to another. We'll start with the basic Hohmann Transfer and then look briefly at the problem of changing the orbital plane.

Hohmann Transfers in Action

In the last section we learned that to move from one orbit to another using a Hohmann Transfer takes two steps, as shown again in Figure 4.1.5-6. In the first step, the satellite in orbit 1 fires its rocket engines once to increase velocity (ΔV_1) and enter an elliptical transfer orbit that will take it out toward orbit 2. When the satellite reaches apogee in the transfer orbit, it fires its engines a second time to increase velocity again (ΔV_2).

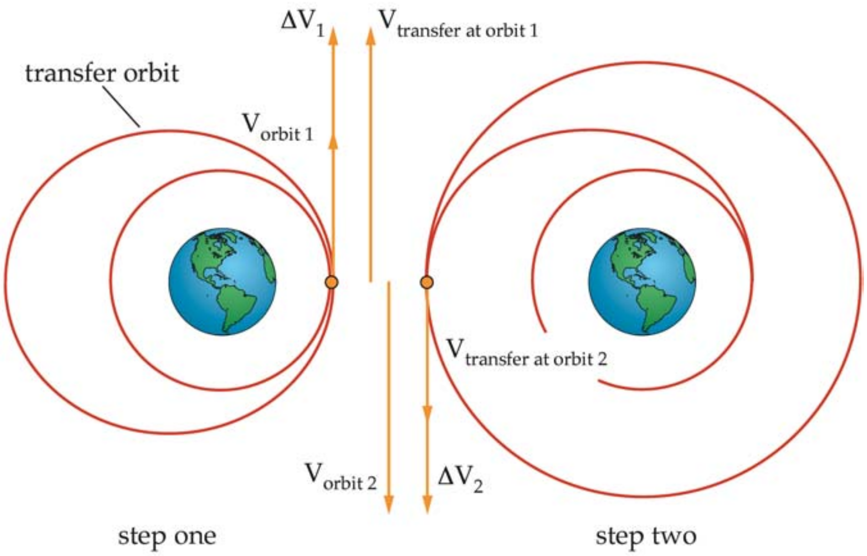


Figure 4.1.5-6. Hohmann Transfer. Step 1: The first burn or ΔV of a Hohmann Transfer takes the spacecraft out of its initial, circular orbit and puts it in an elliptical, transfer orbit. Step 2: The second burn takes it from the transfer orbit and puts it in the final, circular orbit.

Any ΔV represents a change from the present velocity to a selected velocity. For a tangential burn, we can write this as

$$\Delta V = |V_{selected} - V_{present}|$$

Notice we normally take the absolute value of this difference because we want to know the amount of velocity change, so we can calculate the energy and thus, the fuel needed. We're not concerned with the sign of the ΔV because we must burn fuel whether the spacecraft accelerates to reach a higher orbit or decelerates to drop into a lower orbit. If ΔV_1 is the change in velocity that takes the spacecraft from orbit 1 into the transfer orbit, then,

$$\Delta V_1 = |V_{\text{transfer at orbit 1}} - V_{\text{orbit 1}}|$$

where

ΔV_1 = velocity change to go from orbit 1 into the transfer orbit (km/s)

$V_{\text{transfer at orbit 1}}$ = velocity in the transfer orbit at orbit 1 radius (km/s)

$V_{\text{orbit 1}}$ = velocity in orbit 1 (km/s)

ΔV_2 is the change to get the spacecraft from the transfer orbit into orbit 2. Both of these ΔVs are shown in Figure 4.1.5-6.

$$\Delta V_2 = |V_{\text{orbit 2}} - V_{\text{transfer at orbit 2}}|$$

where

ΔV_2 = velocity change to move from the transfer orbit into orbit 2 (km/s)

We add the ΔV from each burn to find the total ΔV needed for the trip from orbit 1 to orbit 2.

$$\Delta V_{total} = \Delta V_1 + \Delta V_2 \qquad (4.1.5\text{-}2)$$

where

ΔV_{total} = total velocity change needed for the transfer (km/s)

When we cover the rocket equation in Chapter 14, we'll see how to convert this number into the amount of fuel required.

To compute ΔV_{total}, we use the energy equations from orbital mechanics. Everything we need to know to solve an orbital-maneuvering problem comes from these two valuable relationships, as you'll see later, in Example 6-1. First, we need the specific mechanical energy, ε

$$\varepsilon = \frac{V^2}{2} - \frac{\mu}{R} \qquad (4.1.5\text{-}3)$$

where

ε = spacecraft's specific mechanical energy (km^2/s^2)

V = magnitude of the spacecraft's velocity vector (km/s)

μ = gravitational parameter (km^3/s^2) = 3.986×10^5 km^3/s^2 for Earth

R = magnitude of the spacecraft's position vector (km)

Then, we need the alternate form of the specific mechanical energy equation

$$\varepsilon = -\frac{\mu}{2a} \qquad (4.1.5\text{-}4)$$

where

ε = spacecraft's specific mechanical energy (km^2/s^2)

μ = gravitational parameter (km^3/s^2) = 3.986×10^5 km^3/s^2 for Earth

a = semimajor axis (km)

Let's review the steps in the transfer process to see how all this fits together. Referring to Figure 4.1.5-6,

- Step 1: ΔV_1 takes a spacecraft from orbit 1 and puts it into the transfer orbit
- Step 2: ΔV_2 puts the spacecraft into orbit 2 from the transfer orbit

To solve for these ΔVs, we need to find the energy in each orbit. If we know the sizes of orbits 1 and 2, then we know their semimajor axes ($a_{orbit\ 1}$ and $a_{orbit\ 2}$). The transfer orbit's major axis equals the sum of the two orbital radii, as shown in Figure 4.1.5-7.

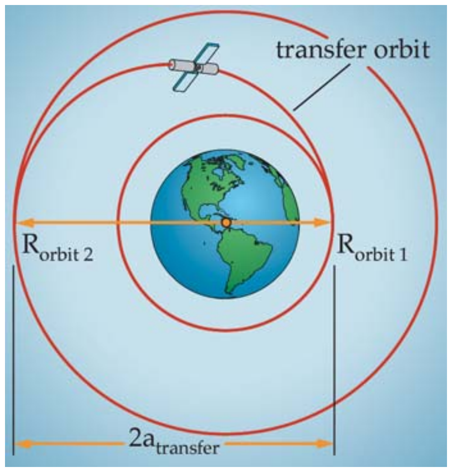


Figure 4.1.5-7. Size of the Transfer Orbit. The major axis of the transfer orbit equals the sum of the radii of the initial and final orbits.

$$2a_{transfer} = R_{orbit\ 1} + R_{orbit\ 2} \qquad (4.1.5\text{-}5)$$

Using the alternate equation for specific mechanical energy, we determine the energy for each orbit

$$\varepsilon_{orbit\ 1} = -\frac{\mu}{2a_{orbit\ 1}} \qquad (4.1.5\text{-}6)$$

$$\varepsilon_{orbit\ 2} = -\frac{\mu}{2a_{orbit\ 2}} \qquad (4.1.5\text{-}7)$$

$$\varepsilon_{transfer} = -\frac{\mu}{2a_{transfer}} \qquad (4.1.5\text{-}8)$$

With the energies in hand, we use the main equation for specific mechanical energy, rearranged to calculate the orbits' velocities

$$V_{orbit\ 1} = \sqrt{2\left(\frac{\mu}{R_{orbit\ 1}} + \varepsilon_{orbit\ 1}\right)}$$

$$V_{orbit\ 2} = \sqrt{2\left(\frac{\mu}{R_{orbit\ 2}} + \varepsilon_{orbit\ 2}\right)}$$

$$V_{transfer\ at\ orbit\ 1} = \sqrt{2\left(\frac{\mu}{R_{orbit\ 1}} + \varepsilon_{transfer}\right)}$$

$$V_{\text{transfer at orbit 2}} = \sqrt{2\left(\frac{\mu}{R_{\text{orbit 2}}} + \varepsilon_{\text{transfer}}\right)}$$

Finally, we take the velocity differences to find ΔV_1 and ΔV_2, then add these values to get ΔV_{total}

$$\Delta V_1 = |V_{\text{transfer at orbit 1}} - V_{\text{orbit 1}}|$$

$$\Delta V_2 = |V_{\text{orbit 2}} - V_{\text{transfer at orbit 2}}|$$

$$\Delta V_{\text{total}} = \Delta V_1 + \Delta V_2$$

The Hohmann Transfer is energy efficient, but it can take a long time. To find the time of flight, look at the diagram of the maneuver. The transfer covers exactly one half of an ellipse. Recall that we find the total period for any closed orbit by

$$P = 2\pi\sqrt{\frac{a^3}{\mu}} \qquad (4.1.5\text{-}9)$$

So, the transfer orbit's time of flight (TOF) is half of the period

$$\text{TOF} = \frac{P}{2} = \pi\sqrt{\frac{a^3_{\text{transfer}}}{\mu}} \qquad (4.1.5\text{-}10)$$

where

TOF = spacecraft's time of flight (s)

P = orbital period (s)

a = semimajor axis of the transfer orbit (km)

μ = gravitational parameter (km^3/s^2) = 3.986×10^5 km^3/s^2 for Earth

Example 4.1.5-1 shows how to find time of flight for a Hohmann Transfer.

Plane Changes

So far we've seen how to change an orbit's size using a Hohmann Transfer. However, we restricted this transfer to coplanar orbits. As you'd expect, to change its orbital plane, a spacecraft must point its velocity change (ΔV) out of its current plane. By changing the orbital plane, it also alters the orbit's tilt (inclination, i) or its swivel (right ascension of the ascending node, Ω), depending on where in the orbit it does the ΔV burn. For plane changes, we must consider the direction and magnitude of the spacecraft's initial and final velocities.

To understand plane changes, imagine you're on a racetrack with off-ramps such as those on a freeway. If you want to exit from the track, you not only must change your velocity *within* its plane but also must go *above* or *below* the level of the track. This "out-of-plane" maneuver causes you to use even more energy than a level exit because you now have to accelerate to make it up the ramp or brake as you go down. Thus, out-of-plane maneuvers typically require much more energy than in-plane maneuvers.

With a *simple plane change*, only the direction of the orbital velocity changes. The velocity's magnitude (orbital speed) stays the same. Because the simple plane change is the most important one to understand, we'll concentrate on it.

Let's imagine we have a spacecraft in an orbit with an inclination, i, of 28.5° (the inclination we'd get if we launched it due east from the Kennedy Space Center, as the Shuttle often does.) Assume we want to change it into an equatorial orbit (i = 0°). We must change the spacecraft's velocity, but we want to change only the orbit's orientation, not its size. This means the velocity vector's magnitude stays the same $|\vec{V}_{initial}| = |\vec{V}_{final}|$, but its direction changes. (As noted in Appendix A, the vertical lines on each side of a vector quantity show we're looking only at its magnitude.)

How do we change just the direction of the velocity vector? Look at the situation in Figure 4.1.5-8. You can see we initially have an inclined orbit with a velocity $\vec{V}_{initial}$, and we want to rotate the orbit by an angle θ to reach a final velocity, $\vec{V}_{final}$. The vector triangle shown in Figure 4.1.5-8 summarizes this problem. It's an isosceles triangle (meaning it has two sides of equal length). Using plane geometry, we get a relationship for ΔV_{simple}—the change in velocity needed to rotate the plane

$$\Delta V_{simple} = 2\ V_{initial} \sin\left(\frac{\theta}{2}\right) \qquad (4.1.5\text{-}11)$$

where

ΔV_{simple}	= velocity change for a simple plane change (km/s)
$V_{initial} = V_{final}$	= velocities in the initial and final orbits (km/s)
θ	= plane-change angle (deg or rad)

If we want to change only the orbit's inclination, we must change the velocity at either the ascending node or the descending node. When the ΔV occurs at one of these nodes, the orbit pivots about a line connecting the two nodes, thus changing only the inclination.

We also can use a plane change to change the right ascension of the ascending node, Ω. This might be useful if we want a remote-sensing satellite to pass over a certain point on Earth at a certain time of day. When we consider a polar orbit (i = 90°), we see that a ΔV_{simple} at the North or South Pole changes just the right ascension of the ascending node, as illustrated in Figure 4.1.5-9. We can also change Ω alone for inclinations other than 90°. The trick is to perform the ΔV_{simple} where the initial and final orbits intersect. (Think of this maneuver as pivoting around a line connecting the burn point to Earth's center.) We won't go into the details of these cases because the spherical trigonometry gets a bit complicated for our discussion here.

The amount of velocity change a spacecraft needs to re-orient its orbital plane depends on two things—the angle it is turning through and its initial velocity. As the angle it's turning through increases, so does ΔV_{simple}. For example, when this angle is 60°, the vector triangle becomes

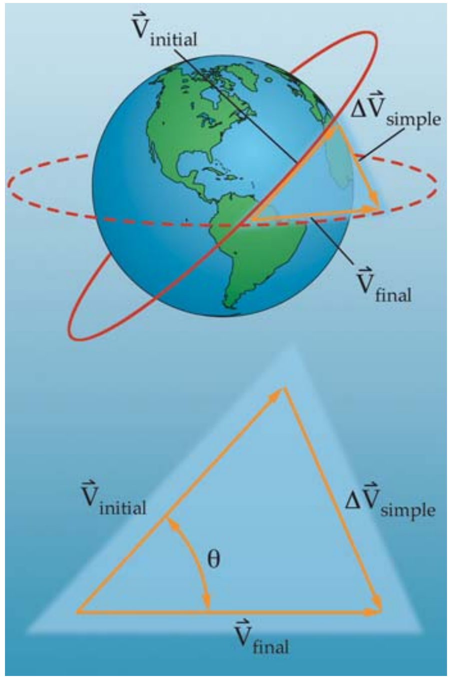


Figure 4.1.5-8. Simple Plane Change. A simple plane change affects only the direction and not the magnitude of the original velocity

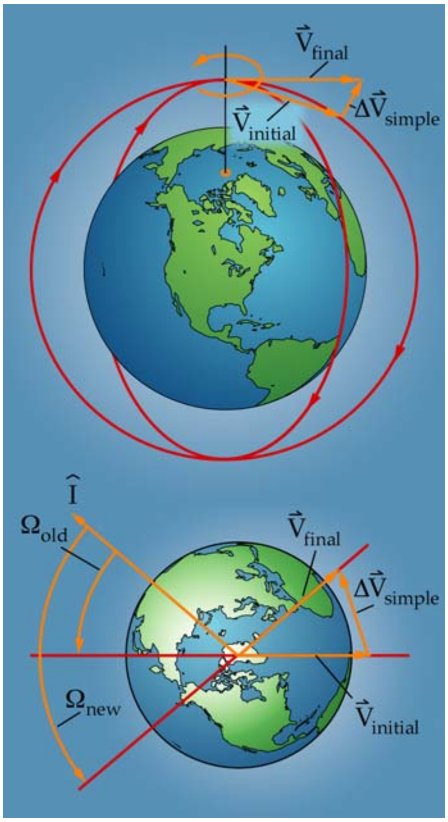


Figure 4.1.5-9. Changing Ω. A simple plane change as a spacecraft crosses the pole in a polar orbit (i = 90°) will change only the right ascension of the ascending node, Ω. Imagine the orbital plane pivoting about Earth's poles.

equilateral (all sides equal). In this case, ΔV_{simple} equals the initial velocity, which is the amount of velocity it needed to get into the orbit in the first place! That's why we'd like the initial parking orbit to have an inclination as close as possible to the final mission orbit.

Also notice that ΔV_{simple} increases as the initial velocity increases. Therefore, we can lower ΔV_{simple} by reducing the initial velocity. The velocity is constant throughout a circular orbit, but we know a spacecraft in an elliptical orbit slows down as it approaches apogee. Thus, if we can choose where to do a simple plane change in an elliptical orbit, we should do it at apogee, where the spacecraft's velocity is slowest. Remember our earlier analogy about changing speeds and directions on a racetrack. It's easier to change direction when we're going slower (even for a stunt driver). Example 4.1.5-2 (at the end of this section) demonstrates a simple plane change.

You've now seen two types of orbit maneuvers in action: the Hohmann transfer and the simple plane change. In the next enrichment section, we'll look at another special application of orbital maneuvering—rendezvous.

Section Review

Key Concepts

- We use plane-change maneuvers to move a spacecraft from one orbital plane to another
  - Simple plane changes alter only the direction, not the magnitude, of the velocity vector for the original orbit

    $|\vec{V}_{initial}| = |\vec{V}_{final}|$
  - A simple plane change at either the ascending or descending node changes only the orbit's inclination.
  - On a polar orbit a simple plane change made over the North or South Pole changes only the right ascension of the ascending node.
  - A simple plane change made anywhere else changes inclination and right ascension of the ascending node.
- It's always cheaper (in terms of ΔV) to change planes when the orbital velocity is *slowest*, which is at apogee for elliptical transfer orbits

Example 4.1.5-1

Problem Statement

Imagine NASA wants to place a communications satellite into a geosynchronous orbit from a low-Earth, parking orbit.

$R_{orbit\ 1} = 6570$ km

$R_{orbit\ 2} = 42{,}160$ km

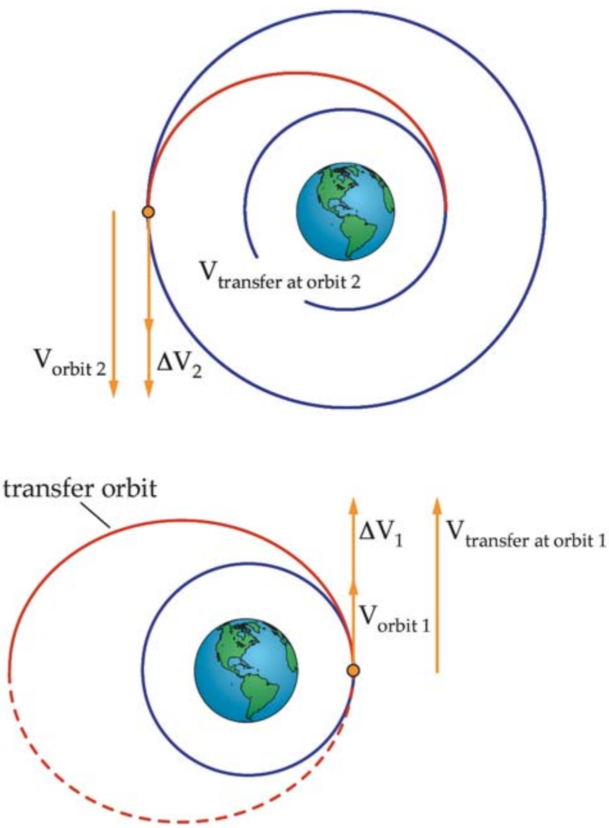
What is the ΔV_{total} for this transfer and how long will it take?

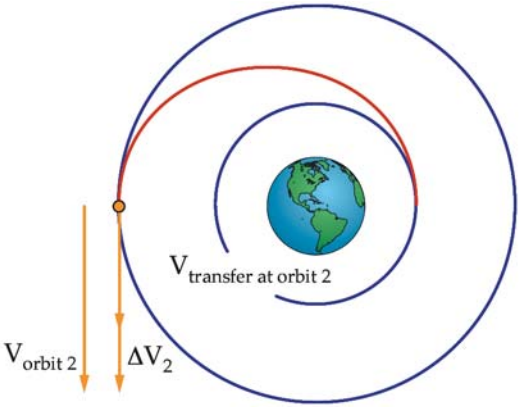
Problem Summary

Given: $R_{orbit\ 1} = 6570$ km
$R_{orbit\ 2} = 42{,}160$ km

Find: ΔV_{total} and TOF

Problem Diagram





Conceptual Solution

1) Compute the semimajor axis of the transfer orbit

$$a_{\text{transfer}} = \frac{R_{\text{orbit 1}} + R_{\text{orbit 2}}}{2}$$

2) Solve for the specific mechanical energy of the transfer orbit

$$\varepsilon_{\text{transfer}} = -\frac{\mu}{2a_{\text{transfer}}}$$

3) Solve for the energy and velocity in orbit 1

$$\varepsilon_{\text{orbit 1}} = -\frac{\mu}{2a_{\text{orbit 1}}}$$

$a_{\text{orbit 1}} = R_{\text{orbit 1}}$ (circular orbit)

$$\varepsilon = \frac{V^2}{2} - \frac{\mu}{R}$$

$$\therefore V_{\text{orbit 1}} = \sqrt{2\left(\frac{\mu}{R_{\text{orbit 1}}} + \varepsilon_{\text{orbit 1}}\right)}$$

4) Solve for $V_{\text{transfer at orbit 1}}$

$$V_{\text{transfer at orbit 1}} = \sqrt{2\left(\frac{\mu}{R_{\text{orbit 1}}} + \varepsilon_{\text{transfer}}\right)}$$

5) Find ΔV_1

$$\Delta V_1 = \left|V_{\text{transfer at orbit 1}} - V_{\text{orbit 1}}\right|$$

6) Solve for $V_{\text{transfer at orbit 2}}$

$$V_{\text{transfer at orbit 2}} = \sqrt{2\left(\frac{\mu}{R_{\text{orbit 2}}} + \varepsilon_{\text{transfer}}\right)}$$

7) Solve for the energy and velocity in orbit 2

$$\varepsilon_{orbit\ 2} = -\frac{\mu}{2a_{orbit\ 2}}$$

$a_{orbit\ 2} = R_{orbit\ 2}$ (circular orbit)

$$\varepsilon = \frac{V^2}{2} - \frac{\mu}{R}$$

$$\therefore V_{orbit\ 2} = \sqrt{2\left(\frac{\mu}{R_{orbit\ 2}} + \varepsilon_{orbit\ 2}\right)}$$

8) Find ΔV_2

$$\Delta V_2 = \left|V_{orbit\ 2} - V_{transfer\ at\ orbit\ 2}\right|$$

9) Solve for ΔV_{total}

$$\Delta V_{total} = \Delta V_1 + \Delta V_2$$

10) Compute TOF

$$TOF = \pi\sqrt{\frac{a^3_{transfer}}{\mu}}$$

Analytical Solution

1) Compute the semimajor axis of the transfer orbit

$$a_{transfer} = \frac{R_{orbit\ 1} + R_{orbit\ 2}}{2} = \frac{6570\ \text{km} + 42{,}160\ \text{km}}{2}$$

$a_{transfer} = 24{,}365$ km

2) Solve for the specific mechanical energy of the transfer orbit

$$\varepsilon_{transfer} = -\frac{\mu}{2a_{transfer}} = -\frac{3.986 \times 10^5 \frac{\text{km}^3}{\text{s}^2}}{2(24{,}365\ \text{km})}$$

$$\varepsilon_{transfer} = -8.1798\frac{\text{km}^2}{\text{s}^2}$$

(Note the energy is negative, which implies the transfer orbit is an ellipse; as we'd expect.)

3) Solve for energy and velocity of orbit 1

$$\varepsilon_{orbit\ 1} = -\frac{\mu}{2a_{orbit\ 1}}$$

$$= -\frac{3.986 \times 10^5 \frac{\text{km}^3}{\text{s}^2}}{2(6570\ \text{km})} = -30.33\frac{\text{km}^2}{\text{s}^2}$$

$$V_{\text{orbit 1}} = \sqrt{2\left(\frac{\mu}{R_{\text{orbit 1}}} + \varepsilon_{\text{orbit 1}}\right)}$$

$$\sqrt{2\left(\frac{3.986 \times 10^5 \frac{\text{km}^3}{\text{s}^2}}{6570\ \text{km}} - 30.33\frac{\text{km}^2}{\text{s}^2}\right)} = 7.789\frac{\text{km}}{\text{s}}$$

4) Solve for $V_{\text{transfer at orbit 1}}$

$$V_{\text{transfer at orbit 1}} = \sqrt{2\left(\frac{\mu}{R_{\text{orbit 1}}} + \varepsilon_{\text{transfer}}\right)}$$

$$\sqrt{2\left(\frac{3.986 \times 10^5 \frac{\text{km}^3}{\text{s}^2}}{6570\text{km}} - 8.1798\frac{\text{km}^2}{\text{s}^2}\right)}$$

$$V_{\text{transfer at orbit 1}} = 10.246\ \frac{\text{km}}{\text{s}}$$

5) Find ΔV_1

$$\Delta V_1 = \left|V_{\text{transfer at orbit 1}} - V_{\text{orbit 1}}\right|$$

$$\left|10.246\frac{\text{km}}{\text{s}} - 7.789\frac{\text{km}}{\text{s}}\right|$$

$$\Delta V_1 = 2.457\frac{\text{km}}{\text{s}}$$

6) Solve for $V_{\text{transfer at orbit 2}}$

$$V_{\text{transfer at orbit 2}} = \sqrt{2\left(\frac{\mu}{R_{\text{orbit 2}}} + \varepsilon_{\text{transfer}}\right)}$$

$$\sqrt{2\left(\frac{3.986 \times 10^5 \frac{\text{km}^3}{\text{s}^2}}{42{,}160\ \text{km}} - 8.1798\frac{\text{km}^2}{\text{s}^2}\right)}$$

$$V_{\text{transfer at orbit 2}} = 1.597\frac{\text{km}}{\text{s}}$$

7) Solve for energy and velocity in orbit 2

$$\varepsilon_{\text{orbit 2}} = -\frac{\mu}{2a_{\text{orbit 2}}}$$

$$= -\frac{3.986 \times 10^5 \frac{km^3}{s^2}}{2(42,160\ km)} = -4.727 \frac{km^2}{s^2}$$

$$V_{orbit\ 2} = \sqrt{2\left(\frac{\mu}{R_{orbit\ 2}} + \varepsilon_{orbit\ 2}\right)}$$

$$\sqrt{2\left(\frac{3.986 \times 10^5 \frac{km^3}{s^2}}{42,160\ km} - 4.727 \frac{km^2}{s^2}\right)} = 3.075 \frac{km}{s}$$

8) Find ΔV_2

$$\Delta V_2 = \left|V_{orbit\ 2} - V_{transfer\ at\ orbit\ 2}\right|$$

$$\left|3.075 \frac{km}{s} - 1.597 \frac{km}{s}\right|$$

$$\Delta V_2 = 1.478 \frac{km}{s}$$

9) Solve for ΔV_{total}

$$\Delta V_{total} = \Delta V_1 + \Delta V_2 = 2.457 \frac{km}{s} + 1.478 \frac{km}{s}$$

$$= 3.935 \frac{km}{s}$$

10) Compute TOF

$$TOF = \pi \sqrt{\frac{a^3_{transfer}}{\mu}}$$

$$= \pi \sqrt{\frac{(24,365\ km)^3}{3.986 \times 10^5 \frac{km^3}{s^2}}}$$

TOF = 18,925 s ≅ 315 min = 5 hrs 15 min

Interpreting the Results

To move the communication satellite from its low-altitude (192 km) parking orbit to geosynchronous altitude, the engines must provide a total velocity change of about 3.9 km/s (about 8720 m.p.h.). The transfer will take five and a quarter hours to complete.

Example 4.1.5-2

Problem Statement

Suppose a satellite is in a circular orbit at an altitude of 250 km. It needs to move from its current inclination of 28° to an inclination of 57°. What ΔV does this transfer require?

Problem Summary

Given: Altitude = 250 km

$i_{initial} = 28.0°$

$i_{final} = 57.0°$

Find: ΔV_{simple}

Conceptual Solution

1) Solve for the orbit's energy and velocity

$$\varepsilon = -\frac{\mu}{2a}$$

$$= -\frac{\mu}{2R} \quad \text{(circular orbit)}$$

$$\varepsilon = \frac{V^2}{2} - \frac{\mu}{R}$$

$$V = \sqrt{2\left(\frac{\mu}{R} + \varepsilon\right)}$$

2) Solve for the inclination change

$$\theta = |i_{final} - i_{initial}|$$

3) Find the change in velocity for a simple plane change

$$\Delta V_{simple} = 2\ V_{initial} \sin\frac{\theta}{2}$$

Analytical Solution

1) Solve for the energy and velocity of the orbit

$$\varepsilon = -\frac{\mu}{2R} = -\frac{3.986 \times 10^5 \frac{km^3}{s^2}}{2(6378 + 250\ km)}$$

$$= -30.069\ \frac{km^2}{s^2}$$

$$V_{initial} = \sqrt{2\left(\frac{\mu}{R} + \varepsilon\right)}$$

$$V_{initial} = \sqrt{2\left(\frac{3.986 \times 10^5 \frac{km^3}{s^2}}{6628\ km} - 30.069\frac{km^2}{s^2}\right)}$$

$$= 7.755\frac{km}{s}$$

2) Solve for the inclination change

$$\theta = |i_{final} - i_{initial}| = |57° - 28°|$$

$\theta = 29°$

Find ΔV for the simple plane change

$$\Delta V_{simple} = 2\ V_{initial} \sin\frac{\theta}{2} = 2\left(7.755\frac{km}{s}\right)\sin\frac{29°}{2}$$

$\Delta V_{simple} = 3.88\ km/s$

Interpreting the Results

To change the inclination of the satellite by 29°, we must apply a ΔV of 3.88 km/s. This is 50% of the velocity we needed to get the satellite into space in the first place. Plane changes are *very expensive* (in terms of ΔV.)

4.1.5.3 Rendezvous

In This Section You'll Learn to...

- Describe orbital rendezvous
- Determine the ΔV and wait time to do a rendezvous

For the Hohmann Transfer and plane-change maneuvers we described earlier in this chapter, we focused on how to move a spacecraft without considering where it is in relation to other spacecraft. However, several types of missions require a spacecraft to meet or *rendezvous* with another one, meaning one spacecraft must arrive in the same place at the same time as a second one. The Gemini program perfected this maneuver in the 1960s, as a prelude to the Apollo missions to the Moon, which depended on a Lunar-orbit rendezvous. Two astronauts returning from the Moon's surface had to rendezvous with their companion in the command module in Lunar orbit for the trip back to Earth. As another example, the Space Shuttle needs to rendezvous routinely with the International Space Station to transfer people and equipment. In this section, we'll examine two simple rendezvous scenarios between co-planar and co-orbital spacecraft.

Coplanar Rendezvous

The simplest type of rendezvous uses a Hohmann Transfer between coplanar orbits. *Coplanar orbits* are two orbits in the same plane. The key to this maneuver is timing. Deciding when to fire the engines, we must calculate how much to lead the target spacecraft, just as a quarterback leads a receiver in a football game. At the snap of the ball, the receiver starts running straight down the field toward the goal line, as Figure 4.1.5-10 shows. The quarterback mentally calculates how fast the receiver is running and how long it will take the ball to get to a certain spot on the field. When the quarterback releases the ball, it will take some time to reach that spot. Over this same period, the receiver goes from where he was when the ball was released to the "rendezvous" point with the ball.

Let's look closer at this football analogy to see how the quarterback decides when to throw the ball so it will "rendezvous" with the receiver. Assume we have a quarterback who throws a 20-yard pass traveling at 10 yd/s and a wide receiver who runs at 4 yd/s. (Ironically, we use English units to describe American football.) Assuming the receiver starts running immediately, how long must the quarterback wait from the snap before throwing the ball? To analyze this problem, let's define the following symbols

$V_{receiver}$ = velocity of the receiver running down the field
= 4 yd/s

V_{ball} = velocity of the ball
= 10 yd/s

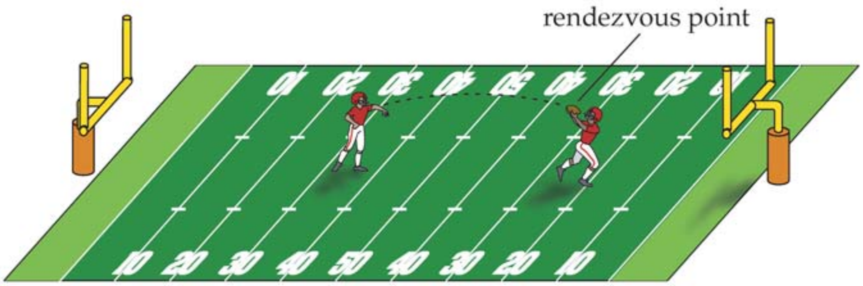


Figure 4.1.5-10. Orbital Rendezvous and Football. The spacecraft-rendezvous problem is similar to the problem a quarterback faces when passing to a running receiver. The quarterback must time the pass just right so the ball and the receiver arrive at the same place at the same time.

We know the quarterback must "lead" the receiver; that is, the receiver will travel some distance while the ball is in the air. But how long will the ball take to travel the 20 yards from the quarterback to the receiver? Let's define

$$\begin{aligned} TOF_{ball} &= \text{time of flight of the ball} \\ &= \text{distance the ball travels}/V_{ball} \\ &= 20\ \text{yd}/(10\ \text{yd/s}) \\ &= 2\ \text{s} \end{aligned}$$

The lead distance is then the receiver's velocity times the ball's time of flight.

$$\begin{aligned} \alpha &= \text{lead distance} \\ &= V_{receiver} \times TOF_{ball} \\ &= (4\ \text{yd/s}) \times 2\ \text{s} \\ &= 8\ \text{yd} \end{aligned}$$

This means the receiver runs an additional 8 yards down the field while the ball is in the air. From this we can figure out how much of a head start the receiver needs before the quarterback throws the ball. If the receiver runs 8 yards while the ball is in the air, and the ball is being thrown 20 yards, the receiver then needs a head start of

$$\begin{aligned} \phi_{head\ start} &= \text{head start distance needed by the receiver} \\ &= 20\ \text{yd} - \alpha \\ &= 20\ \text{yd} - 8\ \text{yd} \\ &= 12\ \text{yd} \end{aligned}$$

So before the quarterback throws the ball, the receiver should be 12 yards down the field. We can now determine how long it will take the receiver to go 12 yards down field.

$$\begin{aligned} W.T. &= \text{wait time} \\ &= \phi_{head\ start}/V_{receiver} \\ &= 12\ \text{yd}/(4\ \text{yd/s}) \\ &= 3\ \text{s} \end{aligned}$$

This is the time the quarterback must wait before throwing the ball to ensure the receiver will be at the rendezvous point when the ball arrives.

That's all well and good for footballs, but what about spacecraft trying to rendezvous in space? It turns out that the approach is the same as in the football problem. Let's look at the geometry of the rendezvous problem shown in Figure 4.1.5-11. We have a target spacecraft (say a disabled communication satellite that the crew of the Shuttle plans to fix) and an interceptor (the Space Shuttle). In this example, the target spacecraft is in a higher orbit than the Shuttle, but we'd take a similar approach if it were in a lower orbit. To rendezvous, the Shuttle crew must start a ΔV to transfer to the rendezvous point. But they must do this ΔV at just the right moment to ensure the target spacecraft arrives at the same point at the same time.

To see how to solve this problem, remember that the quarterback first had to know the velocities of the interceptor (the ball) and the target (the receiver). Because footballs move in nearly straight lines, their velocities are easy to see. Velocities aren't so straightforward for spacecraft in orbits. Instead of using a straight-line velocity (in meters per second or miles per hour), we use rotational velocity measured in radians per second or degrees per hour. We call this rotational velocity "angular velocity" and use the Greek letter small omega, ω, to represent it (not to be confused with the classic orbital element, argument of perigee, ω). Because spacecraft move through 360° (or 2π radians) in one orbital period, we find their angular velocity from

$$\omega = \frac{2\pi(\text{radians})}{2\pi\sqrt{\frac{a^3}{\mu}}}$$

$$\omega = \sqrt{\frac{\mu}{a^3}} \qquad (4.1.5\text{-}12)$$

where

ω = spacecraft's angular velocity (rad/s)

μ = gravitational parameter (km^3/s^2) = $3.986 \times 10^5\ km^3/s^2$ for Earth

a = semimajor axis (km)

For circular orbits, a = R (radius), so this angular velocity is constant.

To solve the football problem, we had to find the ball's time of flight. For rendezvous in orbit, the time of flight is the same as the Hohmann Transfer's time of flight, which we found earlier to be

$$TOF = \pi\sqrt{\frac{a_{transfer}^3}{\mu}} \qquad (4.1.5\text{-}13)$$

where

TOF = interceptor spacecraft's time of flight (s)

π = 3.14159 . . . (unitless)

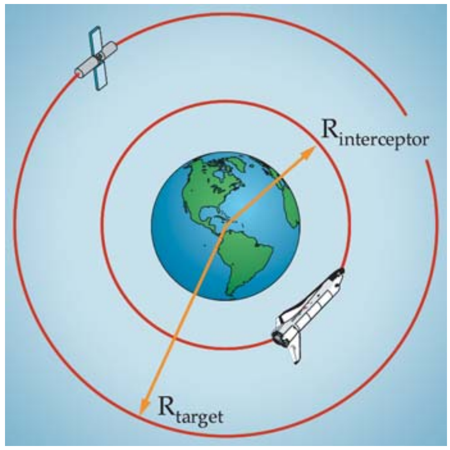


Figure 4.1.5-11. The Rendezvous Problem. The Space Shuttle commander must do a Hohmann Transfer at precisely the right moment to rendezvous with another spacecraft.

$a_{transfer}$ = semimajor axis of the transfer orbit (km)
μ = gravitational parameter (km^3/s^2) = 3.986×10^5 km^3/s^2 for Earth

Finally, we need to get the timing right. In football, the quarterback must lead a receiver by a certain amount to get a pass to the right point for a completion. In rendezvous, the interceptor must lead the target by an amount called the *lead angle, α_{lead}*, when the interceptor starts its Hohmann Transfer. This lead angle, shown in Figure 4.1.5-12, represents the angular distance covered by the target during the interceptor's time of flight. We find it by multiplying the target's angular velocity by the interceptor's time of flight.

$$\alpha_{lead} = \omega_{target} TOF \tag{4.1.5-14}$$

where

α_{lead} = amount by which the interceptor must lead the target (rad)
ω_{target} = target's angular velocity (rad/s)
TOF = time of flight (s)

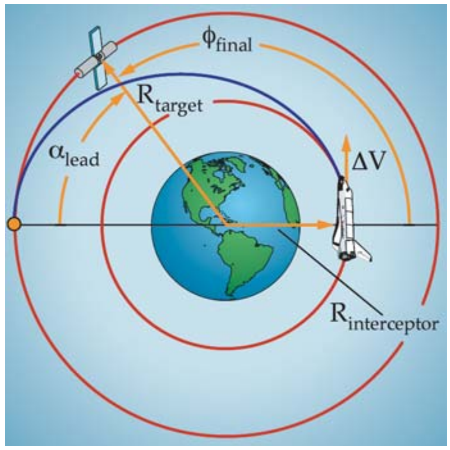


Figure 4.1.5-12. ΔV at the Right Time. The first ΔV of the rendezvous Hohmann Transfer starts when the interceptor is at an angle, ϕ_{final}, from the target.

We can now determine how big of a head start to give the target, just as a quarterback must give a receiver a head start before releasing the ball to complete a pass. For spacecraft, we call this the *phase angle, ϕ*, (Greek letter, small phi) measured from the interceptor's radius vector to the target's radius vector in the direction of the interceptor's motion. The interceptor travels 180° (π radians) during a Hohmann Transfer, so we can easily compute the needed phase angle, ϕ_{final}, if we know the lead angle.

$$\phi_{final} = \pi - \alpha_{lead} \tag{4.1.5-15}$$

where

ϕ_{final} = phase angle between the interceptor and target as the transfer begins (rad)
α_{lead} = angle by which the interceptor must lead the target (rad)

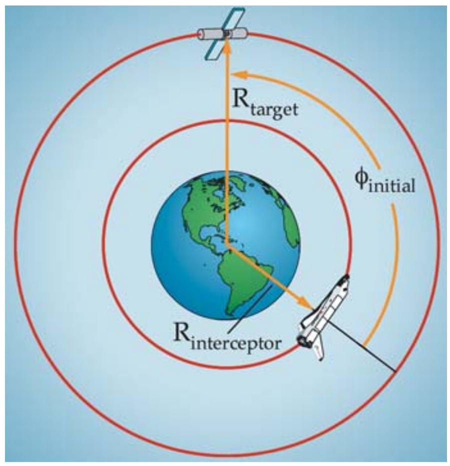


Figure 4.1.5-13. Rendezvous Initial Condition. At the start of the rendezvous problem, the target is some angle, $\phi_{initial}$, away from the interceptor.

Chances are, when the interceptor is ready to start the rendezvous, the target won't be in the correct position, as seen in Figure 4.1.5-13. So what do we do? Just as a quarterback must wait a few seconds before releasing a pass to a receiver, the interceptor must wait until its position relative to the target is correct, as in Figure 4.1.5-12. But how long does it wait? To answer this we have to relate where the target is initially (relative to the interceptor), $\phi_{initial}$, to where the interceptor needs to be, ϕ_{final}, in time to begin the ΔV burn. Because the interceptor and target are moving in circular orbits at constant velocities, $\phi_{initial}$ and ϕ_{final} are related by

$$\phi_{final} = \phi_{initial} + (\omega_{target} - \omega_{interceptor}) \times \text{wait time} \tag{4.1.5-16}$$

Solving for wait time gives us

$$\text{wait time} = \frac{\phi_{final} - \phi_{initial}}{\omega_{target} - \omega_{interceptor}} \tag{4.1.5-17}$$

where

wait time	= time until the interceptor initiates the rendezvous (s)
ϕ_{final}, $\phi_{initial}$	= initial and final phase angles (rad)
ω_{target}, $\omega_{interceptor}$	= target and interceptor angular velocities (rad/s)

So far, so good. But if we look at the wait-time equation, we see that wait time can be less than zero. Does this mean we have to go back in time? Luckily, no. Because the interceptor and the target are going around in circles, the correct angular relationship repeats itself periodically. When the difference between ϕ_{final} and $\phi_{initial}$ changes by 2π radians (360°), the correct initial conditions are repeated. To calculate the next available opportunity to start a rendezvous, we either add 2π to, or subtract it from, the numerator in Equation (4.1.5-17), whichever it takes to make the resulting wait time positive. In fact, we can determine future rendezvous opportunities by adding or subtracting multiples of 2π. Example 4.1.5-3 works through a real-world application of coplanar rendezvous.

Co-orbital Rendezvous

Another twist to the rendezvous problem occurs when the spacecraft are co-orbital, meaning the target and interceptor are in the same orbit, with one ahead of the other. Whenever the target is ahead, as shown in Figure 4.1.5-14, the interceptor must somehow catch the target. To do so, the interceptor needs to move into a waiting or *phasing orbit* that will return it to the same spot one orbit later, in the time it takes the target to move around to that same spot. Notice the target travels less than 360°, while the interceptor travels exactly 360°.

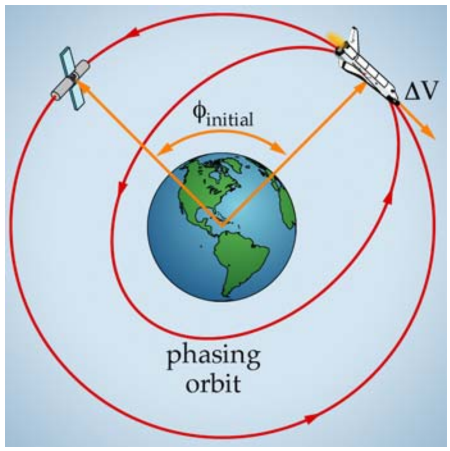


Figure 4.1.5-14. Slow Down to Speed Up. To catch another spacecraft ahead of it in the same orbit, an interceptor slows down, entering a smaller phasing orbit with a shorter period. This allows it to catch the target.

How can one spacecraft catch another one that's ahead of it in the same orbit? By *slowing down*! What?! Does this make sense? Yes, from specific mechanical energy, we know that if an interceptor slows down (decreases energy), it enters a smaller orbit. A smaller orbit has a shorter period, so it completes one full orbit (360°) in less time. If it slows down the correct amount, it will get back to where it started just as the target gets there.

To determine the right amount for an interceptor to slow down, first we find how far the target must travel to get to the interceptor's current position. If the target is ahead of the interceptor by an amount $\phi_{initial}$, it must travel through an angle, ϕ_{travel}, to reach the rendezvous spot, found from

$$\phi_{travel} = 2\pi - \phi_{initial} \tag{4.1.5-18}$$

where

ϕ_{travel}	= angle through which the target travels to reach the rendezvous location (rad)
$\phi_{initial}$	= initial angle between the interceptor and target (rad)

Now, if we know the target's angular velocity, we can find the time it will take to cover this angle, ϕ_{travel}, by using

$$TOF = \frac{\phi_{travel}}{\omega_{target}} \tag{4.1.5-19}$$

Remember, we found the target's angular velocity from Equation (4.1.5-12)

$$\omega_{target} = \sqrt{\frac{\mu}{a_{target}^3}}$$

Because the time of flight equals the period of the phasing orbit, we equate this to our trusty equation for the period of an orbit, producing

$$TOF = \frac{\phi_{travel}}{\omega_{target}} = 2\pi\sqrt{\frac{a_{phasing}^3}{\mu}}$$

We can now solve for the required size of the phasing orbit

$$a_{phasing} = \sqrt[3]{\mu\left(\frac{\phi_{travel}}{2\pi\omega_{target}}\right)^2}$$

where

$a_{phasing}$ = semimajor axis of the phasing orbit (km)

μ = gravitational parameter (km^3/s^2) = 3.986×10^5 km^3/s^2 for Earth

ϕ_{travel} = angular distance the target must travel to get to the rendezvous location (rad)

ω_{target} = target's angular velocity (rad/s)

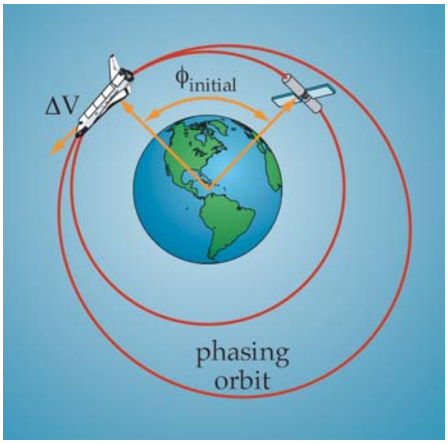


Figure 4.1.5-15. Speed Up to Slow Down. If the target is behind the interceptor in the same orbit, the interceptor must speed up to enter a higher, slower orbit, thereby allowing the target to catch up.

Knowing the size of the phasing orbit, we can compute the necessary ΔVs for the rendezvous. The first ΔV slows the interceptor and puts it into the phasing orbit. The second ΔV returns it to the original orbit, right next to the target. These ΔVs have the same magnitude, so we don't need to calculate the second one.

We must also know how to rendezvous whenever the target is behind the interceptor in the same orbit. In this case, the angular distance the target must cover to get to the rendezvous spot is greater than 360°. Thus, the interceptor's phasing orbit for the interceptor will have a period greater than that of its current circular orbit. To get into this phasing orbit, the interceptor *speeds up*. It then enters a higher, slower orbit, allowing the target to catch up, as Figure 4.1.5-15 illustrates.

Section Review

Key Concepts

- ➤ Rendezvous is the problem of arranging for two or more spacecraft to arrive at the same point in an orbit at the same time
- ➤ The rendezvous problem is very similar to the problem quarterbacks face when they must "lead" a receiver with a pass. But because the interceptor and target spacecraft travel in circular orbits, the proper relative positions for rendezvous repeat periodically.
- ➤ We assume spacecraft rendezvous uses a Hohmann Transfer
- ➤ The lead angle, α_{lead}, is the angular distance the target spacecraft travels during the interceptor's time of flight, TOF
- ➤ The final phase angle, ϕ_{final}, is the "headstart" the target spacecraft needs
- ➤ The wait time is the time between some initial starting time and the time when the geometry is right to begin the Hohmann Transfer for a rendezvous
  - Remember, for negative wait times, we must modify the numerator in the wait time equation by adding or subtracting multiples of 2π radians

Example 4.1.5-3

Problem Statement

Imagine that an automated repair spacecraft in low-Earth orbit needs to rendezvous with a disabled target spacecraft in a geosynchronous orbit. If the initial angle between the two spacecraft is 180°, how long must the interceptor wait before starting the rendezvous?

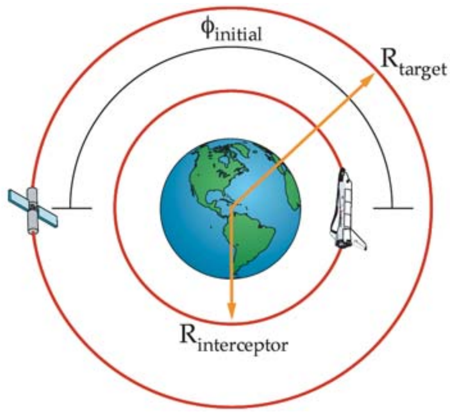
$R_{interceptor} = 6570$ km
$R_{target} = 42{,}160$ km

Problem Summary

Given: $R_{interceptor} = 6570$ km
$R_{target} = 42{,}160$ km
$\phi_{initial} = 180° = \pi$ radians

Find: wait time

Problem Diagram



Conceptual Solution

1) Compute the semimajor axis of the transfer orbit

$$a_{transfer} = \frac{R_{interceptor} + R_{target}}{2}$$

2) Find the time of flight (TOF) of the transfer orbit

$$TOF = \pi\sqrt{\frac{a_{transfer}^3}{\mu}}$$

3) Find the angular velocities of the interceptor and target

$$\omega_{interceptor} = \sqrt{\frac{\mu}{R^3_{interceptor}}}$$

$$\omega_{target} = \sqrt{\frac{\mu}{R^3_{target}}}$$

4) Compute the lead angle

$$\alpha_{lead} = (\omega_{target})(TOF)$$

5) Solve for the final phase angle

$$\phi_{final} = \pi - \alpha_{lead}$$

6) Find the wait time

$$\text{Wait Time} = \frac{\phi_{final} - \phi_{initial}}{\omega_{target} - \omega_{interceptor}}$$

Analytical Solution

1) Compute the semimajor axis of the transfer orbit

$$a_{transfer} = \frac{R_{interceptor} + R_{target}}{2}$$

$$= \frac{6570 \text{ km} + 42{,}160 \text{ km}}{2}$$

$$a_{transfer} = 24{,}365 \text{ km}$$

2) Find the TOF of the transfer orbit

$$TOF = \pi\sqrt{\frac{a^3_{transfer}}{\mu}} = \pi\sqrt{\frac{(24{,}365 \text{ km})^3}{3.986 \times 10^5 \frac{\text{km}^3}{\text{s}^2}}}$$

$$TOF = 18{,}925 \text{ s} = 315 \text{ min } 25 \text{ s}$$

3) Find the angular velocities of the interceptor and target

$$\omega_{interceptor} = \sqrt{\frac{\mu}{R^3_{interceptor}}} = \sqrt{\frac{3.986 \times 10^5 \frac{\text{km}^3}{\text{s}^2}}{(6570 \text{ km})^3}}$$

$$\omega_{interceptor} = 0.0012 \text{ rad/s}$$

$$\omega_{target} = \sqrt{\frac{\mu}{R^3_{target}}} = \sqrt{\frac{3.986 \times 10^5 \frac{\text{km}^3}{\text{s}^2}}{(42{,}160 \text{ km})^3}}$$

$$\omega_{target} = 0.000073 \text{ rad/s}$$

4) Compute the lead angle

$$\alpha_{lead} = (\omega_{target})(TOF)$$

$$= \left(0.000073\frac{\text{rad}}{\text{s}}\right)(18{,}925\ \text{s})$$

$$\alpha_{lead} = 1.38\ \text{rad}$$

5) Solve for the final phase angle

$$\phi_{final} = \pi - \alpha_{lead} = \pi - 1.38\ \text{rad}$$

$$\phi_{final} = 1.76\ \text{rad}$$

6) Find the wait time

$$\text{wait time} = \frac{\phi_{final} - \phi_{initial}}{\omega_{target} - \omega_{interceptor}}$$

$$\text{wait time} = \frac{1.76\ \text{rad} - \pi}{0.000073\frac{\text{rad}}{\text{s}} - 0.0012\frac{\text{rad}}{\text{s}}}$$

$$\text{wait time} = 1225.9\ \text{s} = 20.4\ \text{min}$$

Interpreting the Results

From the initial separation of 180°, the interceptor must wait 20.4 minutes before starting the Hohmann Transfer to rendezvous with the target.

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