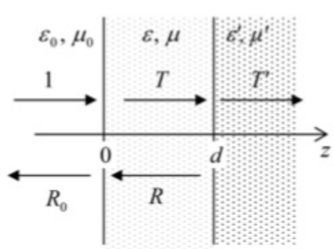
PHYS 203A - Winter 2020

Hw 6:

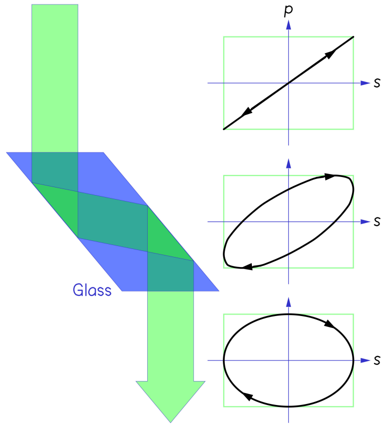
1. The electric field component of an electromagnetic plane wave is given by (the real part of)
$$\vec{E}(\vec{x},t) = E_0\big(\vec{\epsilon}_+ f_+(t) + \vec{\epsilon}_- f_-(t)\big)e^{ikz}$$
where $\vec{\epsilon}_\pm$ are circular polarization vectors normalized to unity. The functions $f_\pm(t)$ are given by
$$f_\pm(t) = \int_{-\infty}^{\infty} d\omega' e^{-i\omega' t} e^{-(\omega'-\omega\pm\Delta\omega)^2/2\sigma^2},$$
where σ and $\Delta\omega$ are constants, and $\sigma \ll \Delta\omega$.
   (a) What is the appropriate time scale T to use in determining average intensity and polarization, as measured by an observer at fixed z? Justify your answer.
   (b) Compute the polarization tensor ρ_{ij}, the Stoke parameters a_1, a_3 and a_3, and the degree of polarization P.
   (c) Discuss the opposite limit, $\Delta\omega \ll \sigma$, including the limit $\Delta\omega = 0$.

2. Suppose that we have a rectangular shape wave guide with height a and width b ($a > b$), placed in air.
   (a) What is the condition on the wave frequency ω such that the wave will propagate in the rectangular pipe?
   (b) If $a = 10$ cm, $b = 5$ cm, how many propagation modes exist for a wave of frequency 1 GHz? (c) What about 3.5 GHz?

3. A plane wave of frequency ω is normally incident, from free space, on a planar surface of a material with real values of the electric permittivity ϵ' and magnetic permeability μ'. To minimize the wave reflection from the surface, you may cover it with a layer of thickness d of another transparent material — see the figure below. Calculate the optimal values of ϵ, μ, and d.



4. The figure shows a schematic representation of a Fresnel rhombus. As indicated, plane polarized light with polarization at 45° to the plane of incidence (and hence also at 45° to the normal is fully circularly polarized after two internal reflections.



(figure by Gavin R Putland)

(a) Find the phase difference between the reflected waves with polarization in the plane of incidence and normal to the plane of incidence ($\phi_{\|} - \phi_{\perp}$) after a total internal reflection in the non-permeable medium of index of refraction n as a function of n and the angle of incidence θ.

(b) Determine the sharp angle of the rhombus, as a function on n, so that the outgoing wave is circularly polarized.

(c) Is it right or left-circularly polarized ($i.e.$, $\vec{\epsilon}_+$ or $\vec{\epsilon}_-$)?

5. It is reasonable to expect that a laser beam is well approximated by plane EM waves only if the wavelength is very small compared to the transverse dimensions of the beam, say D, and we only consider a length of beam not much larger than D. To formalize the statement consider the Eikonal approximation, $\vec{A}(\vec{x},t) = \vec{a}(\vec{x}) \exp\big(ik(\psi(\vec{x}) - x^0)\big)$ as a systematic expansion in powers of $|\vec{\nabla}|/k$:

$$\psi(\vec{x}) = \sum_{n=0}^{\infty} \psi^{(n)}(\vec{x}), \qquad \text{and} \qquad \vec{a}(\vec{x}) = \sum_{n=0}^{\infty} \vec{a}^{\,(n)}(\vec{x}),$$

We have seen in class that $|\vec{\nabla}\psi^{(0)}| = 1$ and $\vec{\nabla}\psi^{(0)} \cdot \vec{a}^{\,(0)} = 0$. Choose $\psi^{(0)}(\vec{x}) = z$ so that the light rays are long z and the wave-fronts are parallel to the xy plane, and choose $\vec{a}^{(0)}$ to be transverse to $\hat{z}$, that is, $\vec{a}^{\,(0)} = \vec{a}_{\perp}^{\,(0)}$ has polarization in the xy plane. There is nothing in this order that constraints the $\vec{x}$ dependence of $\vec{a}_{\perp}^{\,(0)}$, so choose a simple smooth an slowly changing envelope to model the finite width laser beam, $\vec{a}_{\perp}^{\,(0)}(\vec{x}) = a_0 \vec{\epsilon} f(\rho)$, where a_0 is a constant, $\vec{\epsilon}$ is a normalized polarization vector, $\hat{z} \cdot \vec{\epsilon} = 0$, $\vec{\epsilon}^2 = 1$, and $f(\rho) = \exp(-\rho^2/2D^2)$ where $\rho^2 = x^2 + y^2$.

(a) Carry out the perturbative expansion up to order $n = 2$, that is, write the equations that give the $n = 1$ terms in terms of $n = 0$ terms, and $n = 2$ uin terms of $n = 0, 1$.

(b) Use these equations using the zeroth order terms above to infer as much as you may about the finite width laser beam of gaussian cross sections. Describe the solution, physically (perhaps by drawing some pictures and explaining them).

(c) What is qualitatively different in using a sharp cutoff for the beam cross section, eg,

$$f(\rho) = \begin{cases} 1 & \text{for } \rho \leq D \\ 0 & \text{for } \rho > D \end{cases}$$