PHYSICS 239.c : CONDENSED MATTER PHYSICS
HW ASSIGNMENT #4

[1] Consider the one-dimensional Bloch oscillations discussed in §4.5.3 of the Lecture Notes. Setting $k(0) = 0$, show that a Taylor expansion of the motion $x(t)$ agrees with the ballistic result to order t^2, if the ballistic mass is taken to be the effective mass m^* at $k = 0$.

[2] Make a sketch of the extended Brillouin zones like in Fig. 5.2.2 of the Lecture Notes, but for the triangular lattice. Then make plots the free electron Fermi surface for valences $Z = 2$ and $Z = 3$, such as in Fig. 2.3.

[3] Suppose, in the vicinity of the Γ point for some material, the electron dispersion is of the form $E(\boldsymbol{k}) = \frac{1}{2}(m^*)^{-1}_{\mu\nu} k^\mu k^\nu$, where m^* is the effective mass tensor. Find an expression for the low-temperature molar heat capacity in terms of density n and temperature T. Your expression should involve the tensor m^* in some way.

[4] *Cyclotron resonance in Si and Ge* – Both Si and Ge are indirect gap semiconductors with anisotropic conduction band minima and doubly degenerate valence band maxima. In Si, the conduction band minima occur along the $\langle 100 \rangle$ ($\langle \Gamma X \rangle$) directions, and are six-fold degenerate. The equal energy surfaces are cigar-shaped, and the effective mass along the $\langle \Gamma X \rangle$ principal axes (the 'longitudinal' effective mass) is $m^*_l \simeq 1.0\,m_e$, while the effective mass in the plane perpendicular to this axis (the 'transverse' effective mass) is $m^*_t \simeq 0.20\,m_e$. The valence band maximum occurs at the unique Γ point, and there are two isotropic hole branches: a 'heavy' hole with $m^*_{hh} \simeq 0.49\,m_e$, and a 'light' hole with $m^*_{lh} \simeq 0.16\,m_e$.

In Ge, the conduction band minima occur at the fourfold degenerate L point (along the eight $\langle 111 \rangle$ directions) with effective masses $m^*_l \simeq 1.6\,m_e$ and $m^*_t \simeq 0.08\,m_e$. The valence band maximum again occurs at the Γ point, where the hole masses are $m^*_{hh} \simeq 0.34\,m_e$ and $m^*_{lh} \simeq 0.044\,m_e$. Use the following figures to interpret the cyclotron resonance data shown below. Verify whether the data corroborate the quoted values of the effective masses in Si and Ge.

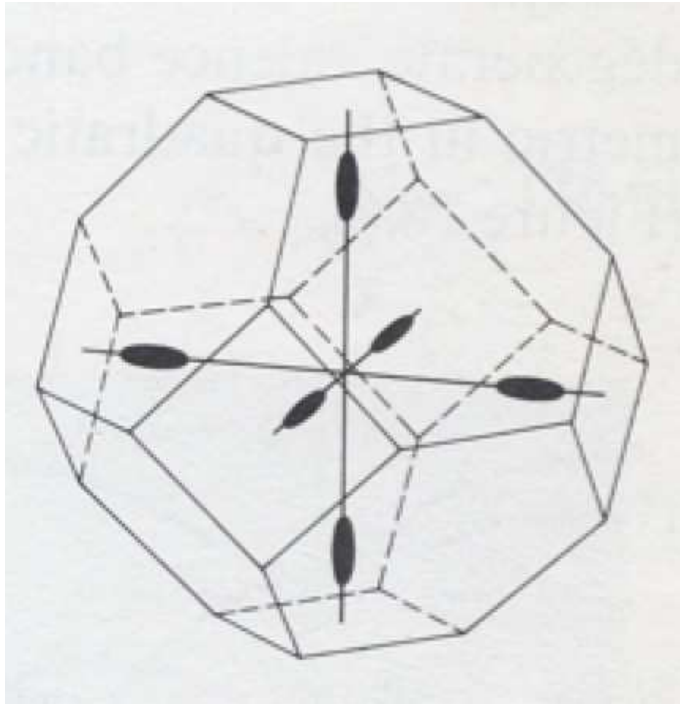(See the following four figures.)

Figure 1: Constant energy surfaces near the conduction band minima in silicon. There are six symmetry-related ellipsoidal pockets whose long axes run along the ⟨100⟩ directions.
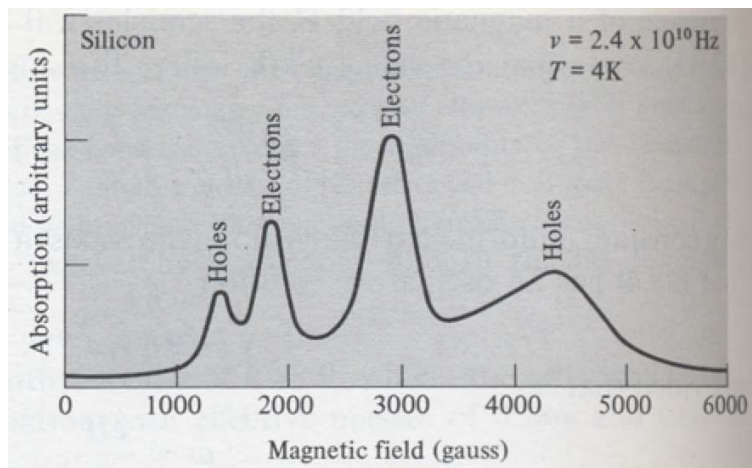

Figure 2: Cyclotron resonance data in Si (G. Dresselhaus *et al.*, *Phys, Rev,* **98**, 368 (1955).) The field lies in a (110) plane and makes an angle of 30° with the [001] axis.
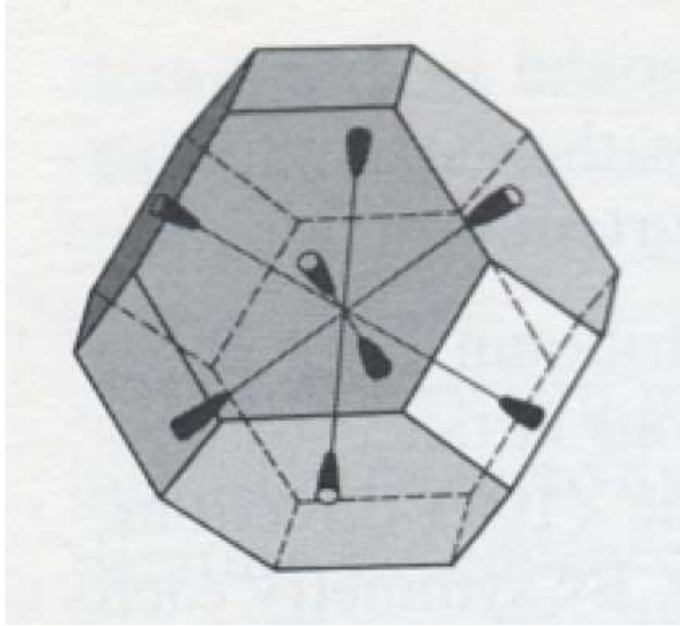
Figure 3: Constant energy surfaces near the conduction band minima in germanium. There are eight symmetry-related half-ellipsoids whose long axes run along the $\langle 111 \rangle$ directions, and are centered on the midpoints of the hexagonal zone faces. With a suitable choice of primitive cell in $\boldsymbol{k}$-space, these can be represented as four ellipsoids, the half-ellipsoids on opposite faces being joined together by translations through suitable reciprocal lattice vectors.
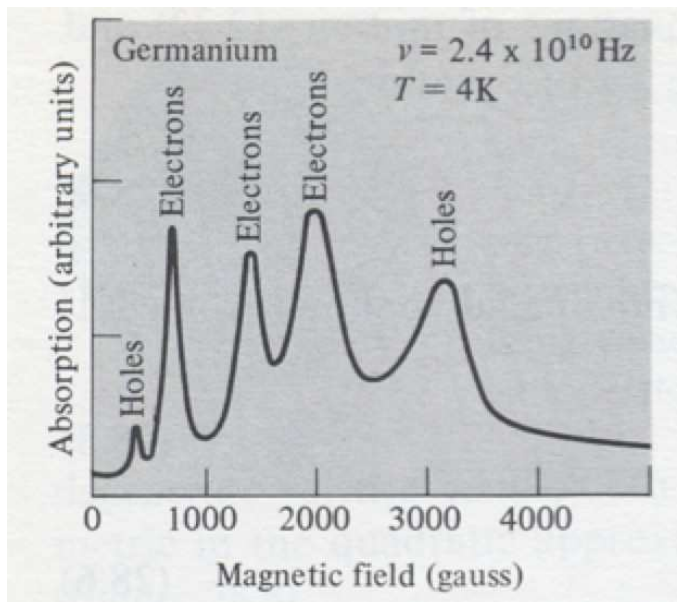

Figure 4: Cyclotron resonance data in Ge (G. Dresselhaus *et al.*, *Phys, Rev,* **98**, 368 (1955).) The field lies in a (110) plane and makes an angle of 60° with the [001] axis.