Rebecca Masline

Abstract

Accretion disks are of great interest to the astrophysical community, but questions of angular momentum, viscosity, turbulence, and other important mechanisms remained unanswered for a considerable amount of time. The key to understanding physics of the accretion disk lies in the magnetorotational instabilty (MRI), a plasma instability that explains many long-standing questions with respect to viscosity and turbulence in protostars, black holes, and many other phenomena observed in astrophysics. The MRI is directly related to the stability of conducting liquids flowing between cylinders [1] in a magnetic field and the stability of a weakly magnetized disc [2], which share many similarities, and provide a robust explanation for the physics of the accretion disk. Here, we will show and discuss this analysis for both the linear and nonlinear treatments, detailing the shortcomings and applicability of the models. Mechanisms of transfer where the MRI fails will be reviewed, including the Hall effect, ambipolar diffusion, and Ohmic diffusion.

Contents

Figure 1: Kubo enjoys his pagoda.

1 Introduction

1.1 What are accretion disks?

Accretion disks are physical structures composed of material in orbit around a central gravitating body. As material spirals inward, angular momentum is redistributed and energy can be extracted, resulting in the formation of a rotating disk around the central body. This process is of particular interest to the astrophysics community, as many phenomena, such as protoplanetary disks and the behavior of black holes, can be modeled by accretion disks [3].

Until recently [2], however, the formation mechanism of such phenomena was not fully understood; while the process of accretion in large planetary bodies has been studied for its relevancy to body formation and black holes, there were shortcomings in the model that could not be fully explained by the mechanisms proposed in earlier works. The seminal picture of modern accretion disk physics was proposed by Shakura and Sunyaev in 1973 [4], where accretion disks are identified as observable characteristics of a black hole: namely, while black holes do not radiate electromagnetic or gravitational waves and therefore cannot be detected via these methods, it is possible to see an observable effect when a matter is accreted by the black hole. Shakura and Sunyaev identified that black holes can be indirectly "observed" by the luminosity and radiation of this accreted disk, and proposed a model to describe this behavior.

The most important takeaways from the so-called Shakura-Sunayev model of the accretion disk were the necessity of the disk to be thin, the parametrization of the kinematic viscosity $\nu = \alpha H c_s$, a temperature profile scaling as $T(r) \propto r^{-3/4}$ (notably independent of mechanisms related to angular momentum), and the idea that the mechanisms governing angular momentum transfer were associated with the magnetic field and likely the result of some unknown turbulent process. These observations provided essential groundwork for future investigations in this field, but introduced a major pitfall, which was irreconcilable with their observations. To match the observed energy emission from these accreted disks, the molecular viscosity would need to be nonphysically high, meaning there had to be some other mechanism to facilitate this large energy generation [5].

1.2 Not easy to Google: The MRI (Magnetorotational Instability)

The question remained unresolved for many years: what was driving the formation of these disks? Hypotheses continued to accrete around the viscosity, but the observed accretion rates were still inconsistent with microscopic values, even with further developments in theory, and ideas of increased viscosity augmented by turbulence failed to provide a definitive explanation for the process. However, in 1991, Balbus and Hawley [2] rediscovered an instability originally proposed by both Velikov (1959) [1] and Chandrasekhar (1961) [6], which seemed to explain the physics of accretion.

The basis of this instability, in its original form, described the stability of Couette flow in an ideally conducting liquid between two cylinders, which was rotating in a magnetic field [1]. Balbus and Hawley derived an analagous result, motivated by explaining the physics of accretion, where they identify the same instability resulting from shear in a magnetized disk [2]. Rather than labeling it with an alphabet soup traditionally awarded to techniques with myriad discoverers, this instability was distinguished as the "magnetorotational instability", or "MRI".

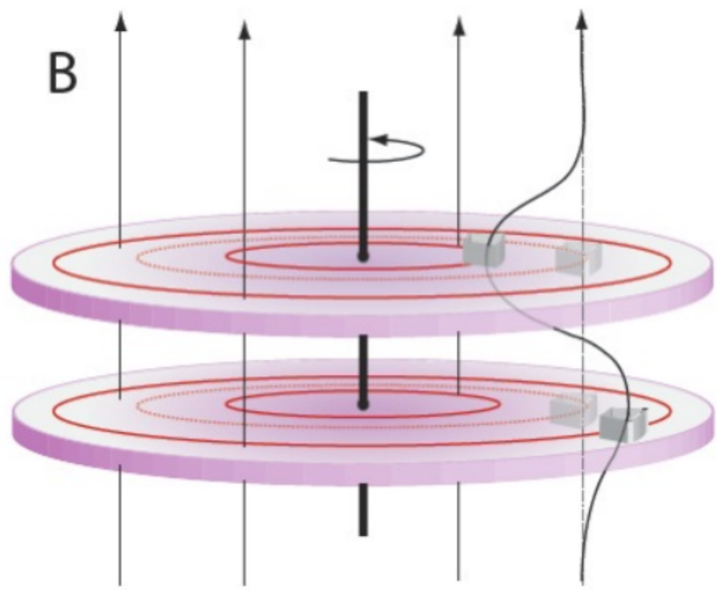

Figure 2: The magnetorotational instability. The fluid element travels radially, while the field acts as a "spring" with a restoring force. Taken from [8].

2 Mechanisms

2.1 What is the magnetorotational instability?

The magnetorotational instability is dependent on shearing of a weak magnetic field, and it is the leading mechanism which drives turbulence and the transfer of momentum in accretion disks. In this system, free energy comes from the negative radial gradient of the angular velocity in the rotating flow, which triggers an instability when exposed to the weak field. In an accretion disk, material towards the center of the disk will move much more quickly than the material towards the outer layer, creating significant shear and enabling the transport of angular momentum outward [4]. Without an external field, stabilizing Coriolis forces will prevent the flow from becoming turbulent. However, this scheme will be destabilized by the magnetic field, as in figure 2: the field lines threading fluid elements will act as springs, providing torques which counteract the Coriolis forces and destabilizing the system [7].

2.2 Stability criterion

Many instabilities can be examined using the energy principle, where stability criterion can be derived based on the potential energy of the system after a perturbation is applied. A simple example is the interchange instability, where a cold plasma in 1D-equilibrium is held by a magnetic field against gravity. If it is perturbed, the stability of the new arrangement is dependent on the amount of mass per unit flux in a tube and position in the gravitational and magnetic field; for example, it is possible to lower the gravitational potential energy in the system without changing the magnetic energy if a flux tube containing less mass situated towards the bottom of a convection cell exchanges position with a flux tube containing more mass situated at the top of a convection cell, and the system is unstable.

Such a treatment is not possible in the magnetorotational instability because the method of energy transfer is opposite to the more "traditional" instabilities (like the interchange instability). Instead of converting magnetic and pressure energy into kinetic energy, as in these "traditional" instabilities, the MRI takes the kinetic energy associated with its rotation and converts it to magnetic energy. Because of this, the MRI cannot be discussed in terms of the energy principle, so MRI stability is approached via a normal mode analysis [5].

3 MHD Equilibrium

3.1 Linear Evolution of the MRI

We have already discussed the "classic" model for evolution in MRI: the garden variety linear analysis. The simple, yet powerful models - directly from Velikov and Chandrasekhar, as well as Balbus and Hawley - provide significant insight into the physics of this instability. For infinite elegant elucidations of this widely studied linear approximation, the reader is invited to descend to the "QC" aisle of Geisel library, approach the "150" call number range section, slam their forehead against their shelf of choice, pick up whatever book lands on the floor, find the appropriate chapter that discusses hydrodynamic stability in a magnetic field, and be amazed. As such, extensive derivation and discussion of such widely-studied material will be omitted here.

However, it is still crucial to understand the importance and value of these models, and discuss their shortcomings. These models explain, *very* well, the underlying physics of the accretion disk. They have immense utility in explaining a widely-studied, long mysterious phenomena in a relatively simple and straightforward way. However, they neglect several critical components which can arise as a result of factors like density, field configuration, and (primarily) ionization.

3.2 Nonlinearities of the MRI

In the MRI, there is a strong coupling between the magnetic field and disk rotation, making this problem highly nonlinear. Most attempts to study the nonlinear evolution of the MRI have been featured in numerous 3D local and numerical studies [**bai2018**]. In these cases, issues can arise based on the inclusion of different effects (such as ambipolar and Ohmic diffusion, as well as the Hall effect, all of which will be discussed in the next section) and problems with numerical viscosity.

4 Nonideal MHD Effects

The MRI has been shown to be the main driver of disk accretion, but the effects of the MRI are strongly modified by non-ideal MHD effects. For the MRI to be possible, the disk must be sufficiently ionized so that the flux is frozen in [9]. The ionization must be sufficient to couple the field to the particles, meaning areas like the midplane of protostellar disks, which are weakly ionized, are not subject to MRI [10]. In this region, the Hall effect, and ambipolar diffusion, and Ohmic resistivity become influential on the dynamics of the disk.

4.1 Hall effect

The case where the gas density is ionized to a point where the electrons are coupled (or partially coupled) with the field, but ion-neutral collisions cause ions to decouple from the magnetic field, is subject to the Hall effect [**w99**]. This effect is driven by the difference between the electron and ion velocity [11]. The Hall effect introduces a handedness with respect to the magnetic field to into the regime, meaning the fluid equations are no longer invariant under reversal of the field. Because of this, oppositely polarized waves can travel at different speeds and damp with different rates.

This effect is also significant because magnetic field strength will affect the conductivity of the medium via Hall parameters; a system in the Hall regime could feasibly experience field strength growth such that the medium will enter the ambipolar diffusion regime. It is important to note that the growth rate of perturbations associated the Hall effect are dependent on the sign of the field, while perturbations associated with ambipolar diffusion aren't, meaning the initial sign of the field can influence the accretion process 99.

4.2 Ambipolar Diffusion

When flux freezing is broken in the weakly ionized regime, the magnetic field is essentially frozen into the limited number of ions, which drift through the neutrals. Fundamentally, the difference between the ion and neutral velocities is what drives ambipolar diffusion [11]. The effects of this drift can dissipate energy through waves

4.3 Ohmic diffusion

As density is increased, collisions between charged particles and neutrals effectively couple the charged particles and neutrals, meaning the charged species will stop drifting and the fluid will become resistive. In cases where Ohmic diffusion is considered, the instability will be damped when the Ohmic diffusion rate is larger than the growth rate of the instability [12].

5 Conclusion

The magnetorotational instability has been investigated, from early developments in developing an understanding of accretion disks to the instability itself, and where it fails. The transport of angular momentum is a critical component to the physics of accretion discs. MRI-driven turbulence solves a longstanding problem with insufficient energy dissipation to match observations: a combination of a negative radial gradient in angular momentum, the Coriolis force, and spring-like effects from weak fields create an instability in these disks that explains the physics and matches experimental observations of accretion disks. Linear models are largely sufficient for analyzing such phenomena, but nonlinearities can be introduced to fully develop the model. Since the MRI is an MHD instability and requires strong ionization to maintain flux freezing, there are several cases where the physics explaining this process fails, which can be accounted for by various diffusive processes in cases where ion and electron coupling differ from the MHD case.

References

[1] E. P. Velikhov, "Stability of an Ideally Conducting Liquid Flowing Between Cylinders Rotating in a Magnetic Field", J. Exptl. Theoret. Phys **36**, 1398–1404 (1959).

[2] S. A. Balbus and J. F. Hawley, "A powerful local shear instability in weakly magnetized disks. I – Linear analysis. II – Nonlinear evolution", Astrophysical Journal **376**, 214–233 (1991).

[3] J. E. Pringle, "Accretion Discs in Astrophysics", Ann. Rev. Astron. Astrophys. **19**, 137–162 (1981).

[4] N. I. Shakura and R. A. Sunyaev, "Black Holes in Binary Systems. Observational Appearance", Astron. Astrophys. **24**, 337–355 (1973).

[5] R. M. Kulsrud, *Plasma Physics for Astrophysics* (Princeton University Press, Princeton, New Jersey, 2005), pp. 186–193.

[6] S. Chandrasekhar, *Hydrodynamic and Hydromagnetic Stability* (Oxford University Press, London, 1961), p. 384.

[7] J. F. Hawley, C. F. Gammie, and S. A. Balbus, "Local Three-dimensional Magnetohydrodynamic Simulations of Accretion Disks", **440**, 742 (1995).

[8] *Envision Radiology*, (2020) `envrad.com/what-to-expect-when-getting-an-mri/` (visited on 03/03/2020).

[9] M. Wardle, "The Balbus-Hawley instability in weakly ionized discs", Monthly Notices of the Royal Astronomical Society **307**, 849–856 (1999).

[10] X.-N. Bai and J. M. Stone, "Wind-driven Accretion in Protoplanetary Disks. I. Suppression of the Magnetorotational Instability and Launching of the Magnetocentrifugal Wind", **769**, 76, 76 (2013).

[11] S. A. Balbus and C. Terquem, "Linear analysis of the hall effect in protostellar disks", The Astrophysical Journal **552**, 235–247 (2001).

[12] L. Jin, "Damping of the Shear Instability in Magnetized Disks by Ohmic Diffusion", **457**, 798 (1996).