Where My Glitches At?
Pulsar Magnetohyrdodynamics and Superfluid Glitches

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Abstract

Strongly magnetized rotating neutron stars, oft-known as pulsars, are a fascinating field of study due to their origins as supernovae, when the progenitor fails to reach sufficient mass to form a black hole. The original observation of pulsars was only recently, in 1967 — and accompanied by the 1974 Nobel Prize — and there has been a plethora of research done since. This discussion will be focused on the various mechanical properties of these pulsars, with a special examination of the effects of superfluidity and quantized magnetic vortices on their astoundingly precise pulse frequency.

Introduction and Pulsar Origins

Pulsars were first observed by a research team at Cambridge on November 28, 1967. They noticed pulses at a constant period from a constant position in the sky which obeyed sidereal time, eliminating the possibility of earth-side interference. The origin of this radio-frequency radiation was unknown, but further observations revealed similar astronomical objects pulsing with different periods with radiation spanning the electromagnetic spectrum.

Just seventeen days earlier, a letter to the editor of Nature sent by Pacini was published in the November 11, 1967 issue, discussing the possibility of a very dense core left behind after a supernova, during which the vast majority of nuclei are converted to neutrons via positron emission, so that only neutrons and electrons remain. This neutron star would be very hot — but quickly dissipate its thermal energy through neutrinos, possess significant angular momentum, and be highly magnetized.

A highly magnetized object which is quickly rotating is a recipe for a strong beam of radiation, rotating much like the lamp of a lighthouse — the radiation observed by the Cambridge team. Over the decades following the categorical discovery of pulsars, thousands of individual pulsars have been observed and catalogued.

The Vela Pulsar, the brightest pulsar observable from Earth, is quite famous and very relevant to our discussion, since it features all the typical phenomena of pulsars in the x-ray and gamma bands, but also exhibits frequent glitches due to the superfluid effects we will discuss in the latter half of this paper.

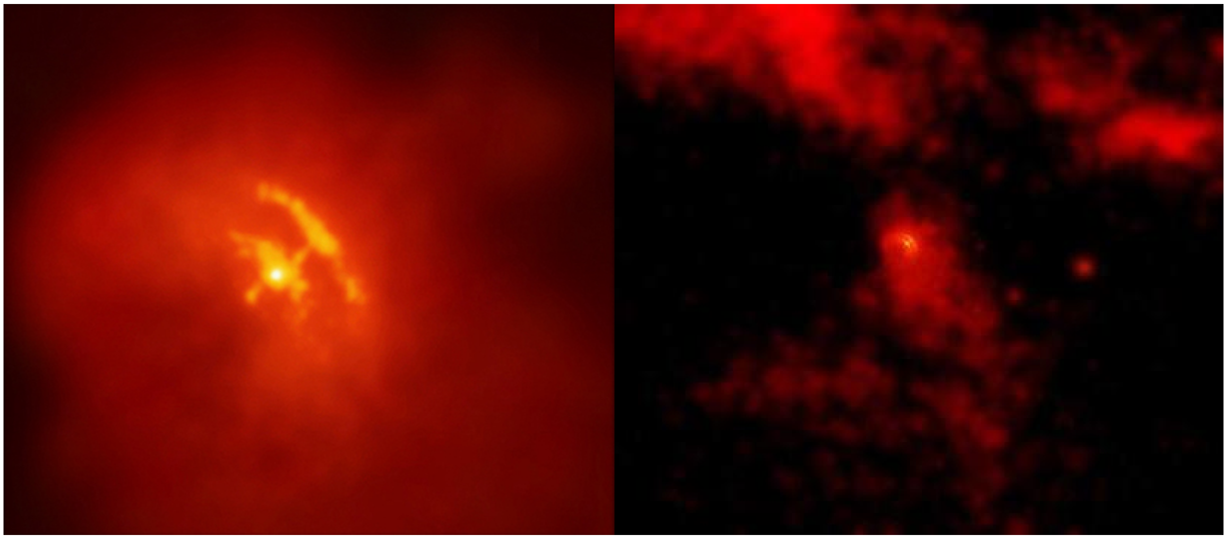

Figure 1: **(a)** Near- and **(b)** wide-field views of the Vela Pulsar, `PSR J0835-4510`, from NASA.

Basic Mechanics of Pulsars

From here, the first questions must be that of the titular feature of a pulsar — the frequency of the pulse itself, and the period with which it happens. These are a pulsar's most basic and prominent identifiers, and where we will commence our study.

First off, note that the radiation frequency of a pulsar is dictated by its rotation frequency, since pulsars constantly emit a beam of electromagnetic radiation in a direction which rotates with the pulsar itself. Ostriker and Gunn (1969) laid out the mechanics for such a pulsar, which we will review here.

To begin, we require that there is a force-free region near the surface of the neutron star, so that an equilibrium region can exist. This field would take the form

$$\big|E\big| = -\big|v \times B\big| \approx \Omega B_s r^{-2} \tag{1}$$

where $B_s \approx 10^{12}$G is the magnetic field on the surface and Ω is simply the rotation rate, i.e. Ω^{-1} is a pulsar's period. The corresponding charge density is

$$\rho = \frac{1}{4\pi}\nabla \cdot E \approx \frac{\Omega B_s}{4\pi r^3}. \tag{2}$$

With plasma frequency $\omega_p \equiv \sqrt{4\pi e\rho/m_e}$ and gyrofrequency $\omega \equiv eB_s/m_e c$, we can reduce the above equation and relate the plasma density of a pulsar to its observed rotation frequency; that is,

$$\omega_p^2 \approx \Omega\omega \Rightarrow \rho \approx \Omega\frac{B_s}{4\pi c}. \tag{3}$$

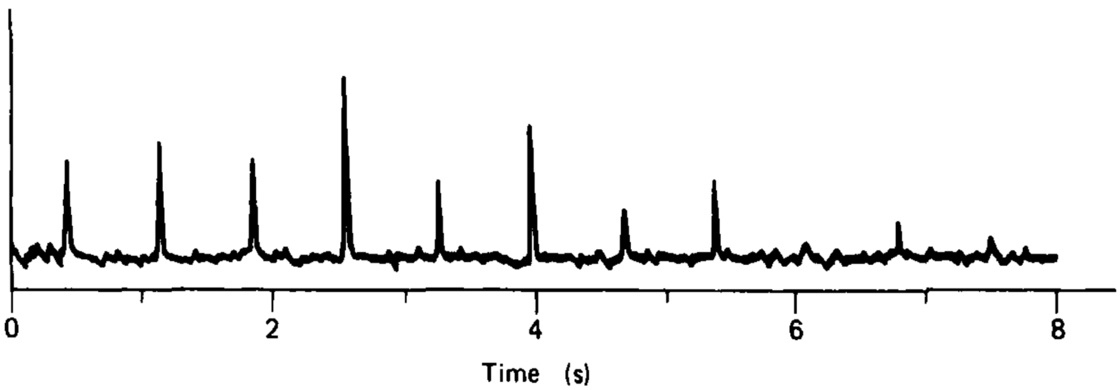

Figure 2: The radiation of `PSR 0329 + 54`, at 410 MHz from Manchester and Taylor (1977). The pulse period (i.e. Ω^{-1}) is about 0.714s.

The next point of discussion is the distance L to an observed pulsar. This may be inferred by its dispersion measure, $DM \equiv \int_0^L n_e d\ell = \langle n_e \rangle L$, so-called due to the dispersion of the pulsar's radiation during interstellar propagation — that is, propagation velocity is directly correlated to frequency — so the temporal width of a received signal implies a total propagation time, and thus a travel distance.

Returning to basic electrodynamics, we can follow Shapiro and Teukolsky (1983) in saying that the motion of an electron forced by a magnetic field is governed by

$$m\ddot{x} = -eE = -eE_0 \exp(i\omega t) \Rightarrow x = \frac{eE}{m\omega^2} \tag{4}$$

for the electric field associated with a propagating electromagnetic wave. The phase and group velocities associated with this propagation are

$$\begin{aligned} v_p &= \frac{\omega}{k} = c\varepsilon^{-1/2} \quad \text{which tells us that} \quad \omega^2 = \omega_p^2 + k^2c^2 \\ v_g &= d_k\omega(k) = c\left(1 - \frac{\omega_p^2}{\omega^2}\right)^{1/2} \approx c\left(1 - \frac{\omega_p^2}{2\omega^2}\right) \quad \text{if} \quad \omega \gg \omega_p \end{aligned} \tag{5}$$

All this lets us calculate the time it takes each individual frequency composing a pulse centered at frequency ω to travel a distance L,

$$t(\omega) = \int_0^L \frac{d\ell}{v_g} \approx \frac{1}{c}\int_0^L \left(1 - \frac{\omega_p^2}{2\omega^2}\right) d\ell = \frac{L}{c} + \frac{2\pi e^2}{mc\omega^2} DM. \tag{6}$$

So, if we know two parameters in the set of $\{\langle n_e \rangle, t(\omega), L\}$, we can find the third. This trinity has been used to find the electron density of pulsars at known distances, which can then be applied to other pulsars to estimate their distance based on their observed dispersion.

Another useful feature for cataloging pulsars is the time evolution of their period, which is as slow as it is predictable, and allows us to determine the age of a pulsar. The oblique rotator model proposed by Pacini (1967) and Ostriker and Gunn (1969) posits that a neutron star can be modeled as a magnetic dipole in a vacuum, rotating with frequency Ω, and having a dipole moment at an angle α to the rotation axis.

The strength of this dipole is given by $m = \frac{1}{2}B_p R^3$, with B_p as the magnetic field at the star's pole, at radius R. Since B_p is not a constant, there is an associated radiation energy

$$\dot{\mathcal{E}} = -\frac{2}{3c^3}|\ddot{m}|^2 \tag{7}$$

which effects as a braking torque. We can break m down into its components,

$$m = \frac{1}{2}B_p R^3 \left(\hat{\imath}\sin\alpha\cos\Omega t + \hat{\jmath}\sin\alpha\sin\Omega t + \hat{k}\cos\alpha\right) \tag{8}$$

letting the rotation axis align with $\hat{k}$. Then, we can rewrite the radiation power,

$$\dot{\mathcal{E}} = -\frac{B_p^2 R^6 \Omega^4 \sin^2\alpha}{6c^3}. \tag{9}$$

This radiated energy goes out in pulses every Ω^{-1} composed of radiation at frequency ω, as discussed earlier. This energy can only come from the rotational energy $\mathcal{E} = \frac{1}{2}I\Omega^2$, so

$$\dot{\mathcal{E}} = -I\Omega\dot{\Omega} \tag{10}$$

which confirms the intuitive sentiment that the pulsar will slow down as it radiates energy. We can use this to find the age of the pulsar by integrating (9) and (10) to get

$$\Omega(t) = \Omega_0 \left(1 + \frac{2\Omega_0^2 B_p^2 R^6 \sin^2\alpha}{6Ic^3}t\right)^{-1/2} \tag{11}$$

where $\Omega_0 = \Omega(t = 0)$. Note that this age is dependent only on observations from the radiation, not observations of the star itself. While this model is accurate for the vast majority of pulsars over the vast majority of times, sometimes glitches have been observed, where the rotation frequency abruptly increases. These glitches are the result of superfluid effects, which is the next subject of our discussion.

Superfluid Magnetohydrodynamics in Pulsars

In 1969, Baym et. al. proposed that the core of these neutron stars is in a superfluid state, and that coupling between the superfluid core and the radiative shell causes small but abrupt changes in the rotation frequency. Before we can discuss the glitches themselves, we must understand their genesis through underlying superfluid magnetohydrodynamic effects.

The basic two-component model lays out the pulsar as two shells: the outer crust with charged particles, and a core of superconducting protons and normal electrons — composing a plasma — on a background of superfluid neutrons. The key difference between this and a traditional magnetohydrodynamic plasma is that the nucleons will form an array of quantized vortices, dominated by the proton vortices due to their energy density.

The proton vortices will each carry one quantum of magnetic flux, $\Phi_0 = hc/2e$, separated by a distance $d_{\rm p} = \sqrt{h/2\sqrt{3}m_{\rm p}\Omega_C}$, where $\Omega_C = eB_0/m_{\rm p}c$ is the cyclotron frequency. The neutron vortices are similarly separated by $d_{\rm n} = \sqrt{h/2\sqrt{3}m_{\rm n}\Omega}$. Since this scale is so small, we will be examining effects averaged over large numbers of vortices.

To get some reference frequencies we will use in our discussion, we examine Kelvin waves traveling parallel to the vortices with frequencies

$$\omega_{\rm vp} = \frac{2m_{\rm p}\varepsilon_{\rm p}k^2}{h\rho_{\rm p}} \quad \text{and} \quad \omega_{\rm vn} = \frac{2m_{\rm n}\varepsilon_{\rm n}k^2}{h\rho_{\rm n}} \tag{12}$$

for protons and neutrons, respectively, with corresponding εs as energy per unit length of the vortices — a constant we will not discuss further for the sake of not losing sight of the problem at hand. For further discussion of these frequencies, refer to Sonin (1987).

From here, to understand the mechanics of the core, we will follow the approach of Mendell (1998) in studying waves in a uniformly rotating pulsar with the vortices we have just described — specifically, showing Alfvén waves cannot exist. For the sake of keeping things sufficiently complicated, we will take the coordinate system where the star is rotating about its z axis. The coordinates can be written in terms of their s-subscripted static versions as

$$\begin{aligned} x &= x_s \cos \Omega t + y_s \sin \Omega t \\ y &= y_s \cos \Omega t - x_s \sin \Omega t \\ z &= z_s \quad \text{and} \quad t = t_s, \end{aligned} \tag{13}$$

as can the convective derivative and fields,

$$\begin{gathered} \partial_t v_s + v_0 \cdot \nabla v_s + v_s \cdot \nabla v_0 = \partial_t v - 2\Omega\left(v \times \hat{z}\right) \\ E = E_s + \frac{v_0}{c} \times B_0 \quad \text{and} \quad B = B_s, \end{gathered} \tag{14}$$

with v_0 as the equilibrium velocity, with associated equilibrium vorticity ω_0. Now, we can write Maxwell's equations, for neutrons and protons, subscripted with n and p, respectively,

$$\begin{aligned} -\frac{\mathrm{i}\omega}{2\Omega}\delta v_{\mathrm{n}} - (\delta v_{\mathrm{n}} \times \hat{z}) &= \frac{\rho_{\mathrm{np}}}{\rho_{\mathrm{n}}}(\delta v_{\mathrm{p}} - \delta v_{\mathrm{n}}) \times \hat{z} + \frac{\mathrm{i}k}{\rho_{\mathrm{n}}}\delta\lambda_{\mathrm{n}} \\ -\frac{\mathrm{i}\omega}{2\Omega}\delta u - (\delta u \times \hat{z}) &= \frac{\rho_{\mathrm{np}}}{\rho_{\mathrm{p}}}(\delta v_{\mathrm{n}} - \delta v_{\mathrm{p}}) \times \hat{z} + \frac{(2\Omega + \Omega_{\mathrm{C}})\,\mathrm{i}k}{2\Omega\rho_{\mathrm{p}}}\delta\lambda_{\mathrm{p}} + \frac{1}{2\Omega\rho_{\mathrm{p}}}\left(\frac{\delta \boldsymbol{J}}{c} \times B_{\mathrm{o}}\hat{z}\right) \end{aligned} \tag{15}$$

where the average charged fluid velocity and weighted combination density are defined as

$$\begin{aligned} u &\equiv \frac{\rho_{\mathrm{p}}}{\tilde{\rho}_{\mathrm{p}}}v_{\mathrm{p}} + \frac{\rho_{\mathrm{e}}}{\tilde{\rho}_{\mathrm{p}}}v_{\mathrm{e}} \\ \rho_{\mathrm{np}} = \rho_{\mathrm{pn}} &= \rho_{\mathrm{p}}\left[1 - \frac{m_{\mathrm{p}}}{m_{\mathrm{p}}^*}\left(1 + F_1^{\mathrm{pp}}/3\right)\right], \end{aligned} \tag{16}$$

respectively, where m_{p}^* is the effective proton mass, and F_1^{pp} is a generalized version of the Landau parameter. Furthermore, we let the descriptive λ vector fields be

$$\begin{aligned} \delta\lambda_{\mathrm{n}} &= \left[\frac{2m_{\mathrm{n}}\varepsilon_{\mathrm{n}}}{2\Omega h} - \left(\frac{\rho_{\mathrm{nn}}}{\rho_{\mathrm{pp}}}\right)^2 \frac{m_{\mathrm{p}}^2c^2}{4\pi e^2}\right](\mathrm{i}k\hat{z} \times \delta v_{\mathrm{n}}) \quad \text{and} \\ \delta\lambda_{\mathrm{p}} &= \left[\frac{2m_{\mathrm{p}}\varepsilon_{\mathrm{p}}}{h\,(2\Omega + \Omega_{\mathrm{C}})} - \frac{m_{\mathrm{p}}^2c^2}{4\pi e^2}\right]\left[\mathrm{i}k\,(\hat{z} \times \delta v_{\mathrm{p}}) + a_{\mathrm{p}}\delta B\right]. \end{aligned} \tag{17}$$

To show that Alfvén waves cannot exist, we need to study the electron fluid velocity, which comes from Maxwell's equations and the MHD equations,

$$\delta v_{\mathrm{e}} = \frac{i\omega m_{\mathrm{p}}}{e\rho_{\mathrm{p}}\omega_{\mathrm{h}}}\delta J \times \hat{z}, \tag{18}$$

where $\omega_{\mathrm{h}} = \Omega_c c^2k^2/\omega_p^2$ is the helicon frequency, a characteristic associated with superconductors. These particle velocities tell us the current is

$$\delta J + \frac{i\omega}{\omega_{\mathrm{h}}}\delta J \times \hat{z} = \frac{e\rho_{\mathrm{p}}}{m_{\mathrm{p}}}\left(\frac{\rho_{\mathrm{pp}}}{\rho_{\mathrm{p}}}\delta u + \frac{\rho_{\mathrm{np}}}{\rho_{\mathrm{p}}}\delta v_{\mathrm{n}}\right), \tag{19}$$

and for frequencies far above the helicon frequency,

$$\delta J \times \hat{z} \approx \frac{\omega_{\mathrm{h}}}{i\omega}\frac{e\rho_{\mathrm{p}}}{m_{\mathrm{p}}}\left(\frac{\rho_{\mathrm{pp}}}{\rho_{\mathrm{p}}}\delta u + \frac{\rho_{\mathrm{np}}}{\rho_{\mathrm{p}}}\delta v_{\mathrm{n}}\right) \Rightarrow \delta v_{\mathrm{p}} = \delta u - \frac{\rho_{\mathrm{e}}\rho_{\mathrm{np}}}{{\rho_{\mathrm{p}}}^2}\delta v_{\mathrm{n}}. \tag{20}$$

Substituting this back into equations (15) and (17), we see that

$$\mathrm{i}\omega\delta v_{\mathrm{n}} + 2\Omega\frac{\rho_{\mathrm{np}}}{\rho_{\mathrm{n}}}(\delta u \times \hat{z}) + 2\Omega\left(\frac{\rho_{\mathrm{nn}}}{\rho_{\mathrm{n}}} + \frac{\omega_{\mathrm{Vn}}}{2\Omega} - \frac{c^2k^2}{\omega_{\mathrm{P}}^2}\frac{\rho_{\mathrm{p}}\rho_{\mathrm{np}}^2}{\rho_{\mathrm{n}}\rho_{\mathrm{pp}}}\right)(\delta v_{\mathrm{n}} \times \hat{z}) = 0$$

$$\begin{aligned} 2\Omega\frac{\rho_{\mathrm{np}}}{\rho_{\mathrm{p}}}(\delta v_{\mathrm{n}} \times \hat{z}) + \frac{\rho_{\mathrm{np}}}{\rho_{\mathrm{p}}}\left(\frac{2i\Omega\omega_{\mathrm{h}}}{\omega} - \frac{i\Omega_{\mathrm{C}}\omega_{\mathrm{Vp}}}{\omega}\right)\delta v_{\mathrm{n}} + 2\Omega\left(\frac{\rho_{\mathrm{pp}}}{\rho_{\mathrm{p}}} + \frac{\omega_{\mathrm{Vp}}}{2\Omega} - \frac{c^2k^2}{\omega_{\mathrm{P}}^2} - \frac{\omega_{\mathrm{h}}}{2\Omega}\right)(\delta u \times \hat{z}) \\ + \left(\mathrm{i}\omega - \frac{\mathrm{i}\Omega_{\mathrm{C}}\omega_{\mathrm{Vp}}}{\omega}\frac{\rho_{\mathrm{pp}}}{\rho_{\mathrm{p}}} + \frac{2\mathrm{i}\Omega\omega_{\mathrm{h}}}{\omega}\frac{\rho_{\mathrm{pp}}}{\rho_{\mathrm{p}}}\right)\delta u = 0. \end{aligned} \tag{21}$$

From here, we can solve the first equation for δu, and put the result into the second to get a relatively simple dispersion relation for δv_{n}. We will take some assumptions of scale, namely

$$\left\{\omega, \Omega\right\} \gg \left\{\omega_{\mathrm{h}}, \omega_{\mathrm{vp}}, \omega_{\mathrm{vn}}\right\} \text{ and } {\omega_{\mathrm{p}}}^2 \gg c^2k^2, \tag{22}$$

to simplify the dispersion relation down to

$$\omega^3 \pm 2\Omega\left(\frac{\rho_{\mathrm{nn}}}{\rho_{\mathrm{n}}} + \frac{\rho_{\mathrm{pp}}}{\rho_{\mathrm{p}}}\right)\omega^2 + \left(4\Omega^2\frac{\rho_{\mathrm{nn}}\rho_{\mathrm{pp}} - {\rho_{\mathrm{np}}}^2}{\rho_{\mathrm{n}}\rho_{\mathrm{p}}} - \Omega_{\mathrm{C}}\omega_{\mathrm{Vp}}\frac{\rho_{\mathrm{pp}}}{\rho_{\mathrm{p}}}\right)\omega \mp 2\Omega\Omega_{\mathrm{C}}\omega_{\mathrm{Vp}}\frac{\rho_{\mathrm{nn}}\rho_{\mathrm{pp}} - {\rho_{\mathrm{np}}}^2}{\rho_{\mathrm{n}}\rho_{\mathrm{p}}} = 0, \tag{23}$$

which is solved by

$$\delta u = \frac{\rho_{\mathrm{n}}}{\rho_{\mathrm{np}}}\left(\pm\frac{\omega}{2\Omega} - \frac{\rho_{\mathrm{nn}}}{\rho_{\mathrm{n}}}\right)\delta v_{\mathrm{n}}. \tag{24}$$

Under the assumption that

$$\left\{\rho_{\mathrm{n}}, \rho_{\mathrm{nn}}\right\} \gg \rho_{\mathrm{p}} \text{ and } \rho_{\mathrm{nn}}\rho_{\mathrm{pp}} - {\rho_{\mathrm{np}}}^2 \approx \rho_{\mathrm{n}}\rho_{\mathrm{pp}}, \tag{25}$$

this provides frequencies of the form

$$\omega = \pm\begin{cases} 2\Omega \\ 2\Omega\frac{\rho_{\mathrm{pp}}}{\rho_{\mathrm{p}}} \\ \frac{{\omega_{\mathrm{cv}}}^2}{2\Omega}\frac{\rho_{\mathrm{p}}}{\rho_{\mathrm{pp}}} \end{cases} \tag{26}$$

in the rotating frame — these must all have $\mp\Omega$ added to retrieve the frequencies in the inertial frame. Notice that the third branch features the "cyclotron-vortex" frequency, $\omega_{\mathrm{cv}} = \sqrt{\Omega_C\omega_{\mathrm{vp}}\rho_{\mathrm{pp}}{\rho_{\mathrm{p}}}^{-1}}$, which is totally distinct from the Alfvén frequency. Without an oscillation at the Alfvén frequency, Alfvén waves cannot be excited. Nevertheless, when these superfluid effects couple to the shell of the neutron star, they cause the abrupt changes in Ω we know and love.

Glitches in Pulsar Rotation Frequencies

The idea of a glitch in the rock-steady rotation frequency of a pulsar was first supposed after a sudden *increase* in both $\dot{\Omega}$ and Ω of one part in one hundred and two in one million, respectively, were observed in the Vela pulsar, in the form of the exaggerated example pictured below.

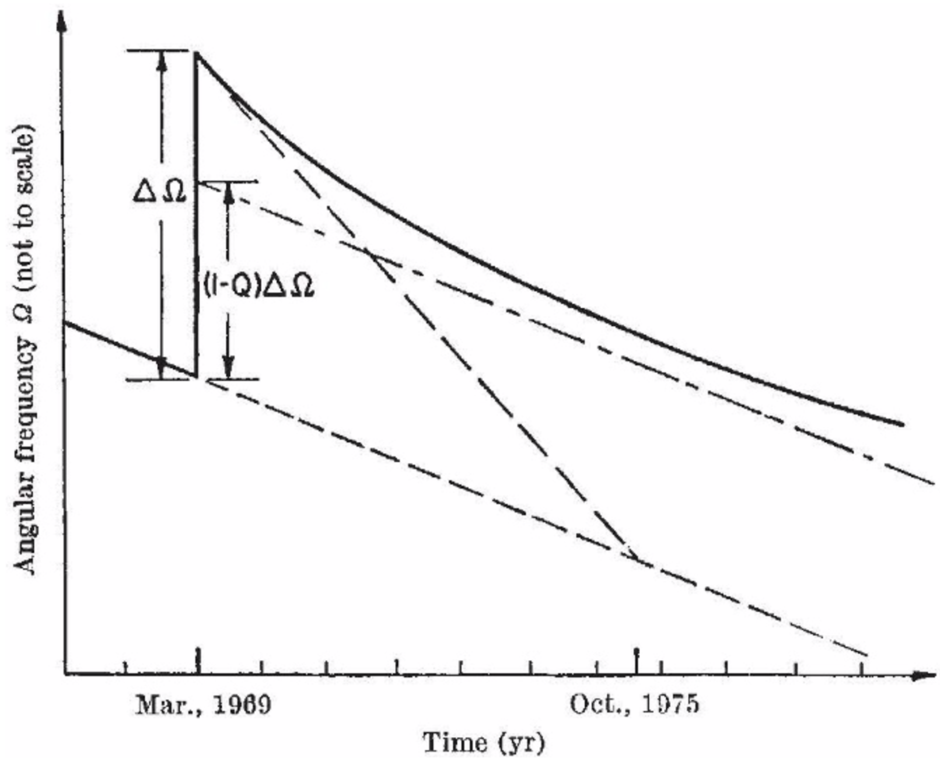

Figure 3: Frequency-exaggerated example of a pulsar glitch from Baym et. al. (1969).

The majority of the rotational kinetic energy of a neutron star resides with the largest part of the moment of inertia, in the superfluid neutron core. The coupling between the core and the crust which generates the observed pulses is primarily dictated by interactions between the charged particles of the core and the vortices as discussed above.

Alpar and Sauls (1988) posit that these interactions arise from the huge hyperlocal magnetic field contained in a quantum of flux from a vortex, *each* around 10^{15}G within $\sim$ 30fm, which have large effects on the rotational velocity of neighboring electrons and protons. Despite the difference in rotational acceleration of protons and electrons, the tremendous transient currents induced force the protons to co-rotate with the electrons. This coupling from both protons and electrons then affects the rotation of the crust, and thus a glitch has occurred.

Earlier, Alpar et. al (1984) discussed the time-scale of this coupling to be

$$\tau_d \sim \frac{\rho/\rho_{\rm c}}{\Omega(x\rho)^{1/6}} \left(\frac{m_{\rm p}}{\delta m_{\rm p}{}^*}\right)^2 \sqrt{\frac{m_{\rm p}{}^*}{m_{\rm p}}}. \tag{27}$$

Then, if we let the crust have moment of inertia I_c and rotation frequency Ω, the same as the pulse frequency, and let the core have moment of inertia I_n and rotation frequency Ω_n, their shared angular momentum can be examined individually,

$$I_c\dot{\Omega} = -N - \frac{I_c}{\tau}\left(\Omega - \Omega_n\right) \quad \text{and} \quad I_n\dot{\Omega}_n = \frac{I_n}{\tau}\left(\Omega - \Omega_n\right), \tag{28}$$

with braking torque N from the coupling discussed above. Note that for long times, $\dot{\Omega} \approx -N/I$ where $I = I_c + I_n$. Furthermore,

$$\Omega_n - \Omega = \frac{N}{I}\frac{I_n}{I_c}\tau. \tag{29}$$

Next, over a post-glitch timescale, we notice that the fractional change in rotation frequency is opposite the fractional change in moment of inertia, that is,

$$\frac{\Delta\Omega}{\Omega} = -\frac{\Delta I_c}{I_c} \quad \text{and} \quad \frac{\Delta\Omega_n}{\Omega_n} = -\frac{\Delta I_n}{I_n}. \tag{30}$$

We can substitute these values into equation (28) and drop small terms to see that

$$\frac{\Delta\dot{\Omega}}{\dot{\Omega}} = \frac{I\Delta\Omega}{\tau N}\left[1 - \frac{I_c\Delta I_n}{I_n\Delta I_c}\right], \tag{31}$$

meaning that changes in moment effect change in $\dot{\Omega}$ with a factor of $\frac{I\Delta\Omega}{\tau N}$. This factor of usually ~ 5000 is responsible for the large difference in fractional change of $\dot{\Omega}$ over Ω.

Conclusion

The study of pulsars is still a relatively young one in astronomy, with their first observation just over fifty years ago. Nonetheless, since they have been such a prominent field of research, the scientific community has made large strides in understanding the physics underlying these astronomical objects.

In this brief review, we have seen what forms the radiation takes; how to relate several observed properties like a pulsar's distance from an observer and its frequency bandwidth; how a pulsar's frequency changes with age; the effects of quantized vortices on the rotation frequencies and how they couple to the radiative outer crust; and related these back to real astronomical observations throughout.

These semi-ideal systems afford us the luxury of studying basic physical concepts in the real world without elaborate experimental apparatuses or the clutter of nonideal theory. The basic science extracted from these pulsars can be applied to numerous other phenomena, both in and outside of plasma physics.

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