24. $(E + m_e c^2 - E')^2 = c^2(p^2 - 2pp'\cos\theta + p'^2) + m_e^2 c^4$

$$E^2 + E'^2 + m_e^2 c^4 + 2Em_e c^2 - 2EE' - 2E'm_e c^2 = c^2 p^2 - 2c^2 pp'\cos\theta + c^2 p'^2 + m_e^2 c^4$$

With $E^2 = c^2 p^2$ and $E'^2 = c^2 p'^2$,

$$E m_e c^2 - EE' - E' m_e c^2 = -EE' \cos\theta$$

$$m_e c^2 (E - E') = EE'(1 - \cos\theta)$$

$$\frac{E - E'}{EE'} = \frac{1}{E'} - \frac{1}{E} = \frac{1}{m_e c^2}(1 - \cos\theta)$$

25. (a) $$\frac{1}{E'} = \frac{1}{E} + \frac{1 - \cos\theta}{m_e c^2} = \frac{1}{11.32\text{ keV}} + \frac{1 - \cos 62.9°}{511.0\text{ keV}} = 0.08940\text{ keV}^{-1}$$

so $E' = 1/0.08727\text{ keV}^{-1} = 11.19\text{ keV}$.

(b) $$K_e = E_e - m_e c^2 = E - E' = 11.32\text{ keV} - 11.19\text{ keV} = 0.13\text{ keV}$$

26. (a) $$\lambda' = \lambda + (h/m_e c)(1 - \cos\theta)$$

$$= 0.02218\text{ nm} + (0.002426\text{ nm})(1 - \cos 90°) = 0.02461\text{ nm}$$

(b) The momenta of the incident and scattered photons are

$$p = \frac{h}{\lambda} = \frac{1}{c}\frac{hc}{\lambda} = \frac{1}{c}\frac{1240\text{ eV}\cdot\text{nm}}{0.02218\text{ nm}} = 5.591\times10^4\text{ eV}/c \quad (x\text{ direction})$$

$$p' = \frac{h}{\lambda} = \frac{1}{c}\frac{hc}{\lambda} = \frac{1}{c}\frac{1240\text{ eV}\cdot\text{nm}}{0.02461\text{ nm}} = 5.039\times10^4\text{ eV}/c \quad (y\text{ direction})$$

(c) $K_e = E - E' = cp - cp' = 5.591\times10^4\text{ eV} - 5.039\times10^4\text{ eV} = 5.52\times10^3\text{ eV}$

(d) Because momentum must be conserved, the x component of the electron's momentum must equal p, and the y component must equal $-p'$:

$$p_{ex} = p = 5.591\times10^4\text{ eV}/c \qquad \text{and} \qquad p_{ey} = -p' = -5.039\times10^4\text{ eV}/c$$

$$p_e = \sqrt{p_{ex}^2 + p_{ey}^2} = \sqrt{(5.591\times10^4\text{ eV}/c)^2 + (5.039\times10^4\text{ eV}/c)^2} = 7.527\times10^4\text{ eV}/c$$

in the direction given by $\theta = \tan^{-1}\frac{p_{ey}}{p_{ex}} = \tan^{-1}\frac{-5.039\times10^4\text{ eV}/c}{5.591\times10^4\text{ eV}/c} = -42.0°$

28. (a) $$\frac{1}{E'} = \frac{1}{E} + \frac{1-\cos\theta}{m_e c^2} = \frac{1}{0.662\text{ MeV}} + \frac{1-\cos 52.2°}{0.511\text{ MeV}} = 2.268\text{ MeV}^{-1}$$

so $E' = 1/2.268\text{ MeV}^{-1} = 0.441\text{ MeV}$.

(b) $$K_e = E - E' = 0.662\text{ MeV} - 0.441\text{ MeV} = 0.221\text{ MeV}$$

30. The energy of the original photon can be found from the sum of the energies of the scattered electron and photon:

$$E = E' + K_e = 2.302\ \text{MeV} + 0.239\ \text{MeV} = 2.541\ \text{MeV}$$

The scattering angle can be found from Eq. 3.46:

$$1-\cos\theta = mc^2\left(\frac{1}{E'}-\frac{1}{E}\right) = (0.511\,\text{MeV})\left(\frac{1}{0.239\,\text{MeV}}-\frac{1}{2.541\,\text{MeV}}\right) = 1.937$$

from which

$$\theta = \cos^{-1}(-0.937) = 160°$$

38. K_e is largest when E' is smallest (because $K_e = E - E'$) and thus when $1/E'$ is largest, which occurs when $\cos\theta = -1$ (that is, when $\theta = 180°$).

$$\frac{1}{E'} = \frac{1}{E} + \frac{2}{m_e c^2} = \frac{m_e c^2 + 2E}{E m_e c^2} \qquad \text{so} \qquad E' = \frac{E m_e c^2}{m_e c^2 + 2E}$$

$$K_e = E - E' = E - \frac{E m_e c^2}{m_e c^2 + 2E} = \frac{2E^2}{m_e c^2 + 2E}$$

43. The initial speed of the atom can be expressed as $v = c(125.0 \text{ m/s})/(2.997 \times 10^8 \text{ m/s}) = 4.171 \times 10^{-7}c$. The initial momentum (which is nonrelativistic at this low speed) is

$$p_\text{i} = mv = \frac{1}{c}mc^2\frac{v}{c} = \frac{1}{c}(1.007825 \text{ u})(931.5\times10^6 \text{ eV/u})(4.171\times10^{-7}) = 391.6 \text{ eV}/c$$

The photon momentum, which is in the opposite direction, has magnitude

$$p = \frac{1}{c}\frac{hc}{\lambda} = \frac{1}{c}\frac{1240 \text{ eV}\cdot\text{nm}}{97 \text{ nm}} = 12.8 \text{ eV}/c$$

The atom's final momentum is $p_\text{f} = p_\text{i} - p = 391.6 \text{ eV}/c - 12.8 \text{ eV}/c = 378.8 \text{ eV}/c$ and its speed is

$$v_\text{f} = \frac{p_\text{f}}{m} = c\frac{p_\text{f}c}{mc^2} = c\frac{378.8 \text{ eV}}{(1.007825 \text{ u})(931.5\times10^6 \text{ eV/u})} = 4.035\times10^{-7}c = 120.9 \text{ m/s}$$

So the change in the speed of the atom is 125.0 m/s – 120.9 m/s = 4.1 m/s.