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Quantum matters: Physics beyond Landau's paradigms*

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Central to our understanding of quantum many particle physics are two ideas due to Landau. The first is the notion of the electron as a well-defined quasiparticle excitation in the many body state. The second is that of the order parameter to distinguish different states of matter. Experiments in a number of correlated materials raise serious suspicions about the general validity of either notion. A growing body of theoretical work has confirmed these suspicions, and explored physics beyond Landau's paradigms. This article provides an overview of some of these theoretical developments.

Keywords:

1. Introduction

Much of our understanding of quantum many electron physics rests on two central ideas which may be traced to Landau. The first is the notion that the electron retains its integrity as a quasiparticle excitation above the quantum ground state of the many particle system. This notion underlies Landau's celebrated Fermi Liquid theory of metals but is also shared by many familiar phases of matter (band insulators, BCS superconductors, spin density waves, ...). This is true even if microscopically the electrons interact reasonably strongly with each other. The other important idea of Landau is that of the order parameter to classify and distinguish phases of matter. Closely related is the notion of spontaneously broken symmetry - indeed the Landau order parameter quantifies the amount of symmetry breaking in any ordered phase. The concept of the order parameter plays an important role in phase transition theory. The universal critical singularities at second order phase transitions are usually attributed to the long wavelength fluctuations of the order parameter degrees of freedom. When combined with general renormalization group ideas this gives a sophisticated theoretical framework - often known as the Landau-Ginzburg-Wilson(LGW) paradigm - for describing phase transition phenomena.

Both of these two notions - the integrity of the electron and the Landau order parameter - are so fundamental that they are routinely taught at the early stages of a solid state physics education. Remarkably in the last several years, a number of experimental developments have challenged the general applicability of either

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of these two basic notions to quantum condensed matter. The best known is the phenomenon of the quantum Hall effect that occurs in a two dimensional electron gas in high magnetic field[1]. The quantum Hall systems are striking and well-established examples of the violation of both of Landau's paradigms. Indeed the electron does not in general survive as a quasiparticle[2] in fractional quantum Hall states nor is the order in such a state captured by a local Landau order parameter[3].

Since the discovery and explanation of the quantum Hall effect, a number of other remarkable phenomena have been found in other correlated electron systems which are poorly understood. Perhaps the most notorious is the problem of high temperature superconductivity but there are many others as well. Examples include a host of "non-fermi liquid" phenomena in the rare-earth intermetallics known as the heavy fermions[4], various kinds of materials near Mott metal-insulator transitions, etc. Many of these long standing problems in condensed matter theory seem not to yield to any conventional thinking. They presumably require new methods of attack, new languages, and perhaps even new conceptual advances. At any rate these experimental discoveries certainly lead to the suspicion that Landau's paradigms may breakdown in serious ways in correlated many electron systems. In the last few years, this suspicion has been strikingly confirmed by a variety of theoretical advances that unquestionably demonstrate the inadequacies of Landau's paradigms for a general understanding of correlated matter.

This article provides an overview of these theoretical developments. We begin by briefly discussing phases of quantum matter that do not fit in with Landau's ideas. We then discuss very recent work on the breakdown of the Landau paradigm at zero temperature 'quantum' phase transitions.

2. Breakdown of Landau paradigms in correlated quantum phases

As mentioned above the fractional quantum Hall effect provides a well-established (and by now old) example of a correlated phase that violates Landau's paradigms. An even older example is provided by one dimensional systems such as (half-integer) quantum spin chains or polyacetylene[5]. A more modern realization of one dimensional physics occurs in the Carbon nanotubes[6]. In these examples, the electron does not retain its integrity as a quasiparticle excitation. Rather the excitations have quantum numbers that are fractions of those of the electron.

One of the most interesting theoretical developments in the last several years is the realization that such 'broken electron' phenomena are *not* restricted to such extreme situations as one dimension or two dimensions in strong magnetic fields. Indeed it has become clear that electrons can break apart in regular solids with strong electron-electron interaction in any spatial dimension. The electron does not survive as a quasiparticle in such phases of matter - instead there are excitations with quantum numbers that are fractions of those of the electron.

A great deal has been learnt about these 'fractionalized' phases in $d \geq 2$ at zero magnetic field theoretically. The structure of the excitation spectrum (or

more generally the structure of the low energy effective field theory) has been elucidated[7,8,9,10,11,12]. These phases have a certain kind of 'order' that is not captured by a local Landau order parameter. Rather the ordering is a global property of the many electron ground state wavefunction[13,8,14] - often referred to as 'topological order'. This kind of order generalizes and is indeed distinct from the old notion of spontaneously broken symmetry. Several concrete and simple microscopic models which display these phenomena exist in both two[15,16,17,18,19,20,21,22] and three spatial dimensions[20,23,24,25]. Prototypical ground state wavefunctions for a number of such fractionalized states can be written and explicitly shown to possess properties (such as topological order) expected on general grounds[13,26]. Finally there even exist some ideas on how to directly detect certain kinds of topological order in experiments[14,27,28].

All of this is spectacular theoretical progress - much of it happened in the last five or so years and builds on important ideas and results[29,13,7,8] from the early days of high-T_c theory. In particular many of the theoretical criticisms levelled against these ideas have now been satisfactorily answered. However there still is no unambiguous identification of such broken electron phenomena in experiments other than the previously established instances (FQHE and $d = 1$).

Where else might it happen? The theoretical understanding provides some hints. It has long been appreciated that frustrated quantum magnets may be a good place to look for such physics. It has also become clear that other promising candidates are not-so-strongly correlated materials. This may be seen explicitly in some of the microscopic models showing fractionalization where it appears in intermediate correlation regimes where neither kinetic nor potential energy overwhelmingly dominates the other[20]. Further support is provided by the observation that in spin systems fractionalization is promoted by multi-particle ring exchange terms[30,10,18]ch terms become increasingly important for the spin physics of Mott insulators as one moves away from the very strong interaction limit (decreasing U in a Hubbard model description). Thus Mott insulators that are not too deeply into the insulating phase or quantum solids such as He-3 or He-4 near melting may be good places to look as well.

3. Breakdown of Landau paradigms at quantum phase transitions

We now turn to the breakdown of Landau's paradigms at zero temperature 'quantum' phase transitions. That this might happen was originally hinted at by various distinct kinds of observations in the literature. First as reviewed in the previous section, Landau order parameters do not necessarily capture the true order in quantum phases. Then it is quite natural that transitions out of such phases are not described by Landau ideas either. For instance continuous transitions exist between distinct quantum Hall states which clearly cannot be described in terms of simple order parameter fluctuations a'la Landau. But what about transitions out of of phases in which Landau order parameters do capture the order? Here at least one might have

hoped for Landau ideas on phase transitions to work. We now review recent work showing that even in this case the Landau paradigm breaks down.

The possibility of such a breakdown is suggested by two different observations. The first is in numerical calculations on various quantum transitions that see a direct second order quantum phase transition between two phases with different broken symmetry characterized by two apparently independent order parameters[31,32]. This is in general forbidden within the Landau approach to phase transitions except at special multicritical points. A similar phenomenon is also seen in experiments[33] on the heavy fermion compound UPt_3. At low temperatures, this is a superconductor (believed to be triplet paired). Upon doping Pd into the Pt site, the superconductivity very quickly disappears and is replaced instead by an antiferromagnetic metal. Within the resolution of the existing experiments these two different kinds of order (superconductivity and antiferromagnetism) seem to be separated by a direct second order transition - within Landau order parameter theory this too would be a special accident. However the surprising frequency with which such 'Landau-forbidden' quantum transitions show up suggests a reexamination of the validity of the Landau paradigm itself.

A second and perhaps more important reason to suspect the general validity of the Landau paradigm comes from a number of fascinating experiments probing the onset of magnetic long range order in the heavy fermion metals[4]. Remarkably the behavior right at the quantum transition between the magnetic and non-magnetic metallic phases is very strikingly different from that of a fermi liquid. The natural assumption is to attribute the non-fermi liquid physics to the universal critical singularities of the quantum critical point. Within the Landau paradigm these will be due to long wavelength long time fluctuations of the natural magnetic order parameter. In other words the hope is that Landau's ideas on phase transitions may perhaps be used to kill Landau's theory of Fermi Liquids. However theories associating the critical singularities with fluctuations of the natural magnetic order parameter in a metallic environment[34] seem to have a hard time explaining the observed non-fermi liquid phenomena. This failure once again fuels the suspicion that perhaps the Landau approach to phase transitions is incorrect. Specifically other phenomena such as the possible loss of Kondo screening of local moments may contribute[35] to and perhaps even dominate the critical singularities[36]. This kind of thinking - particularly the latter possibility - is clearly outside the LGW framework for critical phenomena. In other words *the Landau order parameter (even if present) may distract from the fluctuations responsible for the true critical behavior.*

These suspicions have been strikingly confirmed in recent theoretical work[37,38] on quantum phase transitions in insulating magnets in two spatial dimension. As usual insulating magnets provide a good theoretical laboratory to study phase transition phenomena. A number of results have been found which quite clearly demonstrate the failure of LGW theory at certain (but not all) quantum phase transitions. In all the examples studied so far the critical phenomenology is instead apparently most conveniently described in terms of objects that carry fractional quantum num-

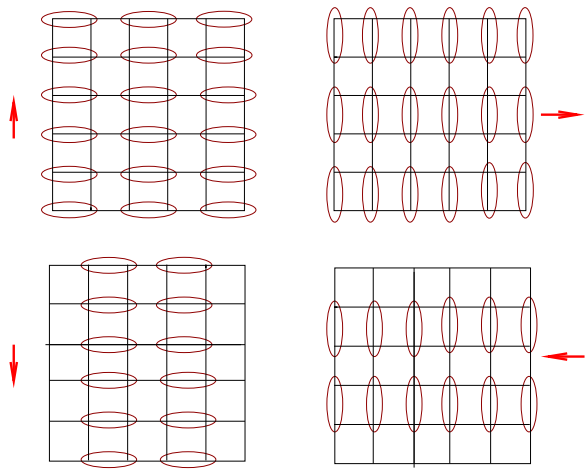

Fig. 1. Schematic picture of a columnar VBS state showing the four degenerate ground states. The encircled lines represent the bonds across which the spins are paired into a valence bond. The four ground states are associated with four different orientations of a Z_4 clock order parameter.

bers and which interact with each other through emergent gauge forces. These fractional objects do not necessarily exist (as good excitations) in the two phases but become useful degrees of freedom at the quantum critical point. This kind of phenomenon has been dubbed 'deconfined' quantum criticality - with a sharp and specific meaning of the term 'deconfined'.

Consider a spin-1/2 antiferromagnet on a two dimensional square lattice described by a Hamiltonian of the general form

$$H = J \sum_{<rr'>} \vec{S}_r . \vec{S}_{r'} + ... \tag{1}$$

The $\vec{S}_r$ are spin-1/2 operators and $J > 0$ is the nearest neighbour exchange constant. The ellipses represent other interactions such as a diagonal exchange or multiparticle ring exchange that may be tuned to drive phase transitions. In the absence of these extra terms the ground state is known to have long ranged Neel order. The corresponding order parameter is a vector in spin space $\vec{N} \sim (-1)^{(x+y)} \vec{S}_r$. For suitable choices of the extra terms it is expected that the ground state will not have long range Neel order even at zero temperature. The simplest of such 'quantum paramagnets' are states known as valence bond solids(VBS) - see Fig. 1 In a cartoon of such states each spin forms a singlet valence bond with one of its neighbours. The resulting dimers stack up in some particular pattern in the VBS ground state. The resulting state clearly has spin rotation symmetry but the pattern of dimer ordering breaks various lattice symmetries. Clearly the order parameter for the VBS state is a spin singlet that transforms non-trivially under the lattice space group operations - it is readily constructed out of the bond energy operators $\vec{S}_r . \vec{S}_{r'}$. The elementary spin-carrying excitations in this phase are gapped spin triplet particles.

In a naive Landau description of the two phases that focuses only on the low energy order parameters, a direct second order transition is not expected except at fine tuned multicritical points. However this naive expectation has been argued to be incorrect. A generic second order transition is possible between these two phases with different broken symmetries. The resulting critical theory is however unusual and *not* naturally described in terms of the order parameter fields of either phase.

The key reason behind this violation of naive Landauesque expectation lies in the observation[39,40] that topological defects in either order parameter carry non-trivial quantum numbers. In particular the defects in one order parameter transform in the same way under the microscopic symmetries as the order parameter for the other phase. Thus when the defects in, say the Neel vector configuration, proliferate they destroy long range Neel order. However the non-trivial quantum numbers they carry induces VBS order in the resulting paramagnet[41].

In the theory of Ref. 37, 38 the natural description of the transition is in terms of spin-1/2 "spinon" fields z_α ($\alpha = 1, 2$ is a spinor index). The Néel order parameter is bilinear in the spinons:

$$\vec{N} \sim z^\dagger \vec{\sigma} z. \tag{2}$$

Here $\vec{\sigma}$ is the usual vector of Pauli matrices and multiplication of the spinor index is implied. The fields z_α create single spin-1/2 quanta, "half" that of the spin-1 quanta created by the Néel field $\vec{N}$. The analysis of Ref. [37,38] shows that the correct critical field theory has the action $\mathcal{S}_z = \int d^2 r d\tau \mathcal{L}_z$, and

$$\begin{aligned} \mathcal{L}_z = \sum_{a=1}^{2} |\left(\partial_\mu - i a_\mu\right) z_a|^2 + s|z|^2 + u\left(|z|^2\right)^2 \\ + \kappa\left(\epsilon_{\mu\nu\kappa}\partial_\nu a_\kappa\right)^2, \end{aligned} \tag{3}$$

This is distinct from both the $O(3)$ universality class in $D = 3$ (as might have been expected based on the Neel order parameter) or the Z_4 universality class (as expected from the Z_4 VBS order parameter). The distinction with the $O(3)$ universality class may be a bit puzzling to field theorists familiar with the CP^1 description of the $O(3)$ non-linear sigma model - the crucial point is that the gauge field in Eqn. 3 above is non-compact. As explained in Ref. 42 with a non-compact gauge field this model does not describe the usual $O(3)$ ordering transition in $D = 3$ - rather it describes the transition[43] in $O(3)$ models wehere 'hedgehog' defects have been suppressed by hand[44]. Ref. 42 also contains detailed numerical calculations of critical exponents for the transition in the model Eqn. 3. The non-compactness of the gauge field leads to an extra emergent conservation law (conserved gauge flux) that helps give precise meaning to the notion of deconfinement at the critical point. This conservation law emerges only at the critical point and does not obtain away from it in either phase.

The critical behavior at this transition is strikingly anamolous - indeed it may be viewed as the moral equivalent of 'non-fermi liquid' behavior in this insulating

context. For instance the magnon spectral function is extremely broad when compared to other quantum transitions - the exponent η is estimated[42] to be ≈ 0.6, bigger by an order of magnitude as compared to the conventional $O(3)$ fixed point. This may roughly be understood as being due to the decay of magnons into the spinon degrees of freedom. A number of other interesting properties - such as the presence of more than one diverging length/time scale - have also been found[37,38].

The Neel-VBS transition is not the only example of this kind of quantum phase transition. A number of other transitions in quantum antiferromagnetism have been shown to have many similarities with the phenomena described above. These include the transition from the VBS state to a gapped 'spin liquid' paramagnet[38] and quantum transitions between two different patterns of VBS ordering on certain lattices[45,46] (for instance in a bilayer honeycomb lattice). Further deconfined critical *phases* described by gapless Dirac-like fermionic spin-1/2 objects coupled to an emergent non-compact $U(1)$ gauge field have been shown to exist as stable quantum phases[47] in two space dimensions. Thus it appears that the phenomenon of deconfined quantum criticality is reasonably common in two dimensional quantum magnets.

Easy plane versions of quantum spin-1/2 models have also been examined and shown to have Landau-forbidden transitions and asociated deconfined quantum critical points[37,38]. These may also be fruitfuly viewed as superfluid-insulator transitions of bosons at half-filling on the square lattice. The case of bosons at a general commensurate filling p/q has been examined recently[48] - again the topological defects have been shown to carry non-trivial quantum numbers which in turn leads to non-trivial order in the insulating phase. Direct second order Landau-forbidden transitions seem possible for a number of special fillings.

4. Conclusions

The developments discussed above provide a theoretically important zeroth order answer to the basic question posed by experiments in modern correlated electron physics: Can Landau's ideas breakdown in quantum matter more generally than in one dimension or the quantum Hall effect? While the theoretical progress at this basic level has been dramatic, we do not at present know what role, if any, it will play in understanding existing experiments on materials such as the cuprates or the heavy fermions. Nevertheless the intuition gleaned from these results will hopefully suggest ways of thinking correctly about such experimental problems.

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